

Correction primitives

a) $f(x) = \frac{x+1}{(x^2+2x)^3} = \frac{1}{2} \cdot \frac{2x+2}{(x^2+2x)^3} = \frac{1}{2} \frac{u'(x)}{u^3(x)} = \frac{1}{2} u'(x) u^{-3}(x) = \frac{1}{2} \times \frac{1}{-2} \times (-2) u'(x) u^{-3}(x),$

$$u(x) = x^2 + 2x, n-1 = -3, n = -2, F(x) = -\frac{1}{4} (x^2 + 2x)^{-2} = -\frac{1}{4(x^2 + 2x)^2}.$$

b) $f(x) = \frac{x}{x^2-1} = \frac{1}{2} \times \frac{2x}{x^2-1} = \frac{1}{2} \times \frac{u'(x)}{u(x)}$ avec $u(x) = x^2 - 1$, $F(x) = \frac{1}{2} \ln u(x) = \frac{1}{2} \ln(x^2 - 1) + k.$

c) $f(x) = x - 1 + \frac{\ln x}{x} = x - 1 + \frac{1}{x} \times \ln x = x - 1 + \frac{1}{2} \times 2u'(x) \times u(x)$ avec $u(x) = \ln x$,

$$F(x) = \frac{x^2}{2} - x + \frac{1}{2} u^2(x) = \frac{x^2}{2} - x + \frac{1}{2} (\ln x)^2 + k.$$

d) $f(x) = (\sin^2 x - 3 \sin x + 8) \cdot \cos x = \cos x \times \sin^2 x - 3 \cos x \times \sin x + 8 \cos x ;$

$$u(x) = \sin^3 x, u'(x) = 3 \cos x \sin^2 x, v(x) = \sin^2 x, v'(x) = 2 \cos x \sin x, w(x) = \sin x, w'(x) = \cos x.$$

$$F(x) = \frac{1}{3} \sin^3 x - \frac{3}{2} \times \sin^2 x + 8 \times \sin x + k.$$

$$F\left(\frac{3\pi}{2}\right) = 0 \Leftrightarrow \frac{1}{3} \sin^3 \frac{3\pi}{2} - \frac{3}{2} \times \sin^2 \frac{3\pi}{2} + 8 \times \sin \frac{3\pi}{2} + k = 0 \Leftrightarrow -\frac{1}{3} - \frac{3}{2} - 8 + k = 0 \Leftrightarrow k = \frac{2+9+48}{6} = \frac{59}{6}.$$

$$F(x) = \frac{1}{3} \sin^3 x - \frac{3}{2} \sin^2 x + 8 \sin x + \frac{59}{6}.$$

e) $f(x) = \frac{x^3 + 5x^2 + 7x + 4}{x^2 + 2x + 1} = \frac{(x+3)(x^2+2x+1)+1}{x^2+2x+1} = x+3 + \frac{1}{x^2+2x+1} = x+3 + \frac{1}{(x+1)^2}.$

$$F(x) = \frac{x^2}{2} + 3x - \frac{1}{x+1}.$$