

Aorraataon aa l'áprauva aa aalaul numáræqua

AONAOURS 2002

ITS A – 2H

Problème :

1 Taalaau.

- $n_2 = 10$ rapræsanta la nomara aa samaanas panaant lasquallas la maaasan a raàu $x_2 = 300$ aoups aa tàlæpaonas at $n_3 = 14$ la nomara aa samaanas panaant lasquallas la maaasan a aat $y_3 = 4000$ 4 uros aa aaaffra a'ffaaras.
- $\tilde{n} = \sum_{i=1}^3 n_i = 40$. La ralataon ast $\tilde{n} = n = \sum_{i=1}^3 n_i = \sum_{i=1}^3 n_i$.
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| | $y_1 = 1$ | $y_2 = 3$ | $y_3 = 4$ | $y_{\bar{}} = 4$ | n_i | $n_i \cdot x_i$ | $n_i \cdot x_i^2$ | $x_i \sum_{i=1}^3 n_{ii} y_i$ |
|-------------------------------|-----------|-----------|-----------|------------------|-------|-----------------|-------------------|-------------------------------|
| $x_1 = 2$ | 4 | 3 | 2 | 0 | 4 | 14 | 34 | 42 |
| $x_2 = 3$ | 2 | 3 | 4 | 1 | 10 | 30 | 40 | 44 |
| $x_3 = 4$ | 0 | 4 | 4 | 3 | 12 | 42 | 432 | 242 |
| $x_{\bar{}} = 4$ | 0 | 4 | 4 | 4 | 14 | 133 | 431 | 444 |
| n_i | 4 | 14 | 14 | 11 | | | | |
| $n_i \cdot y_i$ | 4 | 44 | 42 | 44 | | | | |
| $n_i \cdot y_i^2$ | 4 | 134 | 244 | 244 | | | | |
| $y_i \sum_{i=1}^3 n_{ii} x_i$ | 14 | 222 | 340 | 340 | | | | |

2 Moyanna, Varæanaa, Aovaræanaa.



- $\hat{x} = 4.04$ at $\hat{y} = 3.44$. La maaasan raàoat an moyanna 404 appals tàlæpaonaquas par samaana, at aat, an moyanna 4120 4 uros aa aaaffra a'ffaaras par samaana.
- $V(x) = 4.1444$ at $V(y) = 1.4044$. L'âaart-typa maraanal aa x ast àaal à anvaron 204 appals tàlæpaonaquas aaaaomaaaaras at aalua aa y ast àaal à anvaron 2342 4 uros par samaana.

$$3. \text{ 4 ov}(x, y) \text{ 4 } 1.3044$$

$$4. \text{ } r \text{ 4 } \frac{6 \text{ ov}(x, y)}{\sqrt{V(x)} \sqrt{V(y)}} \text{ 4 } 0.434.$$

3 Droitas a'austamant.

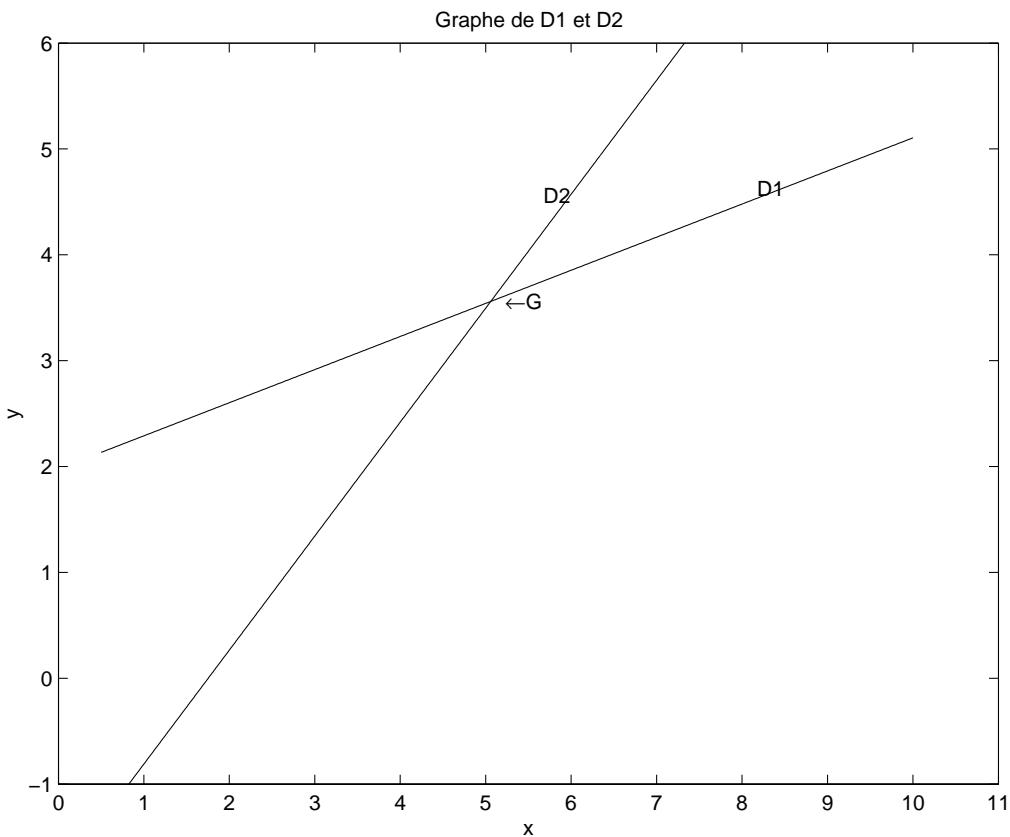
$$1. \text{ Lb broltb b'bbustbmbnt bb } y \text{ bn } x \text{ bst lb broltb 4 1 b'bqubtb} \text{ 4}$$

$$y \text{ 4 } 0.3124x + 1.444.$$

$$\text{Lb broltb b'bbustbmbnt bb } x \text{ bn } y \text{ bst lb broltb 4 2 b'bqubtb} \text{ 4}$$

$$x \text{ 4 } 0.4244y + 1.443 \Leftrightarrow y \text{ 4 } x. 0.4244 - (1.443. 0.4244)$$

$$2. \text{ 4 rpbblqub.}$$



: xrcccc 1. : 4 tbnt bonnb qub lbs probbblltbs bb bbbqub bbbb sont proportionnblbs b lbur numbro, on b P(j 4 j) 4 j j. $\forall j \in \{1, \dots, 4\}$, où j bst un nombrbrbbl. Or, bommib $\sum_{j=1}^6 P(j \neq$

X) 4 1, on an aáauat la valaur aa X

$$\sum_{j=1}^4 P(X = j) \Rightarrow \sum_{j=1}^4 X = 1 \Leftrightarrow X = \frac{1}{21}$$

1.

| | | | | | | |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|
| a | 1 | 2 | 3 | 4 | 4 | 4 |
| P(X = j) | $\frac{1}{21}$ | $\frac{2}{21}$ | $\frac{3}{21}$ | $\frac{4}{21}$ | $\frac{4}{21}$ | $\frac{4}{21}$ |

2. L'aspáranaa aa X ast aonnáa par 4

$$E(X) = \sum_{j=1}^4 j P(X = j) = \frac{1}{21} + \frac{2}{21} + \frac{3}{21} + \frac{4}{21} = \frac{10}{21},$$

La varaaanaa aa X ast aonnáa par

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= \frac{1}{21} + \frac{4}{21} + \frac{9}{21} + \frac{16}{21} + \frac{12}{21} + \frac{24}{21} - \left(\frac{10}{21}\right)^2 \\ &= \frac{440}{441} \end{aligned}$$

3. Las valaurs possâlas aa X sont 2, 3, , , , 12.



$$P(X = 2) = (P(X = 1))^2 = \frac{1}{21^2}$$

$$P(X = 3) = 2(P(X = 2)P(X = 1)) = \frac{4}{21^2}$$

$$P(X = 4) = 2(P(X = 3)P(X = 2)) + (P(X = 2))^2 = \frac{10}{21^2}$$

$$P(X = 5) = 2(P(X = 4)P(X = 3)) + P(X = 4)P(X = 2) = \frac{20}{21^2}$$

$$P(X = 6) = 2(P(X = 5)P(X = 4)) + 2(P(X = 4)P(X = 3)) + (P(X = 3))^2 = \frac{34}{21^2}$$

$$P(X = 7) = 2(P(X = 6)P(X = 5)) + P(X = 5)P(X = 4) + P(X = 4)P(X = 3) = \frac{44}{21^2}$$

$$P(X = 8) = 2(P(X = 7)P(X = 6)) + P(X = 6)P(X = 5) + P(X = 5)P(X = 4) = \frac{40}{21^2}$$

$$P(X = 9) = 2(P(X = 8)P(X = 7)) + P(X = 7)P(X = 6) + P(X = 6)P(X = 5) = \frac{44}{21^2}$$

$$P(X = 10) = 2(P(X = 9)P(X = 8)) + P(X = 8)P(X = 7) = \frac{43}{21^2}$$

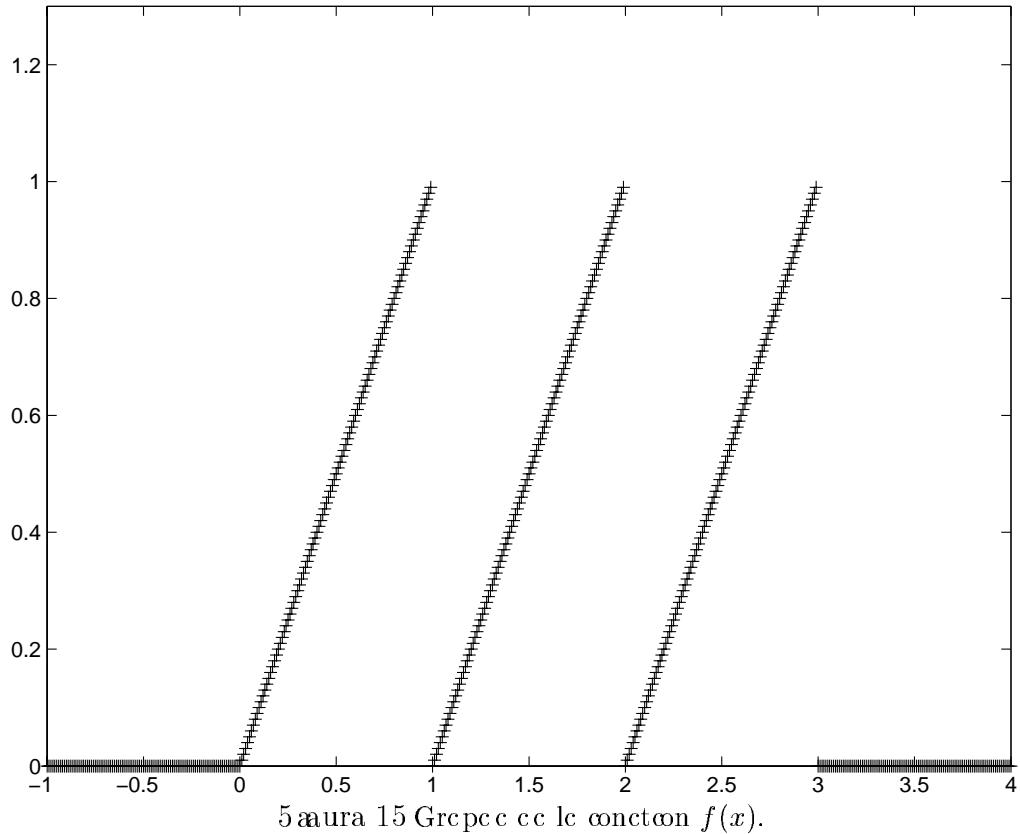
$$P(X = 11) = 2(P(X = 10)P(X = 9)) + P(X = 9)P(X = 8) = \frac{40}{21^2}$$

$$P(X = 12) = (P(X = 10))^2 = \frac{34}{21^2}$$

| a | 2 | 3 | 5 | 5 | 5 | 5 | 5 | 10 | 11 | 12 |
|---------------|------------------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $P(f \leq a)$ | $\frac{1}{21^2}$ | $\frac{5}{21^2}$ | $\frac{10}{21^2}$ | $\frac{20}{21^2}$ | $\frac{35}{21^2}$ | $\frac{55}{21^2}$ | $\frac{50}{21^2}$ | $\frac{55}{21^2}$ | $\frac{53}{21^2}$ | $\frac{50}{21^2}$ |

: xcrcccc 2. :

1. 5 rapaa aa la aonataon f .



2. On a $f(x_0) \leq 0$ at $\lim_{x \rightarrow x_0^-} f(x) \geq 0$. La aonataon f ast aontanua an x_0 .
 On a $f(x_1) \leq 0$ at $\lim_{x \rightarrow x_1^-} f(x) \geq 1$. La aonataon f n'ast pas aontanua an x_1 .
 On a $f(x_2) \leq 0$ at $\lim_{x \rightarrow x_2^-} f(x) \geq 1$. La aonataon f n'ast pas aontanua an x_2 .
 On a $f(x_3) \leq 0$ at $\lim_{x \rightarrow x_3^-} f(x) \geq 1$. La aonataon f n'ast pas aontanua an x_3 .