4 4 OL4 N4 TION4 L4 4 '4 4 ONOMI4 INSTITUT SOUS R4 4 ION4 L 4 4
4 PPLIQU4 4 (4 N4 4) ST4 TISTIQU4 4 T 4 '4 4 ONOMI4 4 PPLIQU4 4
4 P 4044 4 P 244
4 4 K4 R–S4 N4 4 4 L X4 OUN4 4 –4 4 M4 ROUN

4 VRIL 2003

4 ON4 OURS 4 '4 L4 V4 IN4 4 NI4 UR 4 4 S TR4 V4 UX ST4 TISTIQU4 S

VOI4 4

4 ORRI4 4 4 4 L'4 PR4 UV4 4 4 4 4 L4 UL NUM4 RIQU4

4 UR4 4 4 2 H4 UR4 S

Excrcœc

1. Notons $z :\ b + bb$, ˆv^^ $b \in I\!R$ ˆt $b \in I\!R$, lˆs solutˆons ˆ^ l'^quˆtˆon

$$z^2 :\ 1 + b, \tag{0.1}$$

ˆlors lˆs r^^ls b ˆt b v^rˆfiˆnt lˆ syst^mˆ ˆ'^quˆtˆons suˆvˆnt

$$\left\{\begin{array}{lcl} b^2 - b^2 & : & 1 \\ 2bb & : & 1 \\ b^2 + b^2 & : & \sqrt{2} \end{array}\right. \Longleftrightarrow \left\{\begin{array}{lcl} 2b^2 & : & 1 + \sqrt{2} \\ 2b^2 & : & \sqrt{2} - 1 \\ 2bb & : & 1 \end{array}\right.$$

Lˆs r^^ls b ˆt b ^tˆnt ˆ^ mˆmˆ s^^nˆ, on ˆn ˆ^^uˆt lˆs ˆ^ux solutˆons z_1 ˆt z_2 ˆ^ l'^quˆtˆon (0.1) :

$$z_1 \ :\ \sqrt{\frac{1+\sqrt{2}}{2}} + b\sqrt{\frac{\sqrt{2}-1}{2}} :\ (\sqrt{2})^{1/2}\left[\frac{\sqrt{1+\sqrt{2}}}{2^{3/4}} + b\frac{\sqrt{\sqrt{2}-1}}{2^{3/4}}\right] \tag{0.2}$$

$$z_1 \ :\ -\sqrt{\frac{1+\sqrt{2}}{2}} - b\sqrt{\frac{\sqrt{2}-1}{2}} :\ (\sqrt{2})^{1/2}\left[-\frac{\sqrt{1+\sqrt{2}}}{2^{3/4}} - b\frac{\sqrt{\sqrt{2}-1}}{2^{3/4}}\right] \tag{0.3}$$

2. R^prˆs^ntˆtˆon ˆ^ z_1 ˆt ˆ^ z_2 : voˆr ˆn fin ˆ'^xˆr^^^.

3. L[[orm[[xpon[nt[[ll[[u nom[r[[ompl[x[$1+a$[st $1+a4\ \sqrt{2}([os(\pi.4)+as[n(\pi.4))\ 4$ $\big)(\sqrt{2})^{1/2}\ ([\ ^{a\frac{\pi}{8}}))^2$4p[r[ons[qu[nt l[nom[r[[ompl[x[m[s sous [orm[[xpon[nt[[ll[$(\sqrt{2})^{1/2}\ ([\ ^{a\frac{\pi}{8}})$ [st solut[on [[l'[qu[t[on (0.1), [l[st[on[[[[l[l'un[[[[[ux solut[ons trouv[[s [[ns l[qu[st[on 1. 4omm[l[p[rt[[r[[ll[[t l[p[rt[[[m[[[n[[r[[u nom[r[[ompl[x[z_1 sont pos-[t[v[s, z_1 [[on[un [r[um[nt [ompr[s [ntr[0 [t $\pi.2$, p[r[ons[qu[nt z_1 4 $(\sqrt{2})^{1/2}\ ([\ ^{a\frac{\pi}{8}})$.
L[[orm[[xpon[nt[[ll[[[z_1 [t l[r[l[t[on (0.2) [onn[4

$$[os(\pi.4)\ 4\ \frac{\sqrt{1+\sqrt{2}}}{2^{3/5}}.$$
$$s[n(\pi.4)\ 4\ \frac{\sqrt{\sqrt{2}-1}}{2^{3/5}}.$$

4. L[qu[st[on pr[[[[[nt[[mpl[qu[qu[

$$(1+a)^5\ \ 4\ \ 2^5\ [\ ^{a2\pi}. \tag{0.4}$$

4 'un [utr[[ot[[n ut[l[s[nt l[[ormul[[u [[n]m[[[N[wton, on o[t[[nt 4

$$\begin{aligned}(1+a)^5\ \ 4\ \ &\sum_{a5\ 0}^{5}.\ {}_5^a d^a\\ 4\ \ &(.\ {}_5^0-.\ {}_5^2+.\ {}_5^5-.\ {}_5^5+.\ {}_5^5)+a(.\ {}_5^1-.\ {}_5^3+.\ {}_5^5-.\ {}_5^5).\end{aligned} \tag{0.4}$$

4 [s r[l[t[ons (0.4) [t (0.4), on [[[u[t qu[

$$.\ {}_5^0-.\ {}_5^2+.\ {}_5^5-.\ {}_5^5+.\ {}_5^5\ 4\ 2^5\qquad [t\qquad .\ {}_5^1-.\ {}_5^3+.\ {}_5^5-.\ {}_5^5\ 4\ 0.$$

4. 4 '[pr[s l[s [ormul[s ['4 ul[r, on o[t[[nt

$$\begin{aligned}[os^5(a)\ \ 4\ \ &\Big)\frac{[^{aa}+[^{-aa}}{2}\Big)^5\\ 4\ \ &\frac{.\ {}_5^0[^{5aa}+.\ {}_5^5[^{-5aa}+.\ {}_5^1[^{2aa}+.\ {}_3^3[^{-2aa}+.\ {}_5^2}{2^5}\\ 4\ \ &\frac{1}{2^3}[[os(4a)+4[os(2a)+3[.\end{aligned}$$

$$\begin{aligned}s[n^3(a)\ \ 4\ \ &\Big)\frac{[^{aa}-[^{-aa}}{2a}\Big)^3\\ 4\ \ &\frac{.\ {}_3^0[^{3aa}-.\ {}_3^3[^{-3aa}-.\ {}_3^1[^{aa}+.\ {}_3^2[^{-aa}}{(2a)^3}\\ 4\ \ &\frac{1}{2^2}[-s[n(3a)+3s[n(a)[\end{aligned}$$

4. 4 n utalasant la aat qua aos(4x) ast la partæa ráalla au nomara aomplaxa a^{i4x}, at an utalasant la aormula aa Moavra , on aávaloppa a^{i4x} at sa partæa rálla ast la valaur aa aos(4x),

$$\begin{aligned} \mathrm{a}^{i4x} \quad & 4 \quad (\mathrm{aos(x)} + i\mathrm{san(x)})^4 \\ & 4 \quad . \ {}^{4}_{4}\mathrm{aos(x)}^4 + i. \ {}^{4}_{4}\mathrm{aos(x)}^4\mathrm{san(x)} - . \ {}^{4}_{4}\mathrm{aos(x)}^4\mathrm{san(x)}^2 - i. \ {}^{3}_{4}\mathrm{aos(x)}^3\mathrm{san(x)}^3 + \\ & . \ {}^{2}_{4}\mathrm{aos(x)}^2\mathrm{san(x)}^4 + i. \ {}^{1}_{4}\mathrm{aos(x)san(x)}^4 - . \ {}^{0}_{4}\mathrm{san(x)}^4 \\ & 4 \quad \Big)\mathrm{aos(x)}^4 - . \ {}^{4}_{4}\mathrm{aos(x)}^4\mathrm{san(x)}^2 + . \ {}^{2}_{4}\mathrm{aos(x)}^2\mathrm{san(x)}^4 - \mathrm{san(x)}^4\Big) + \\ & i\Big). \ {}^{4}_{4}\mathrm{aos(x)}^4\mathrm{san(x)} - . \ {}^{3}_{4}\mathrm{aos(x)}^3\mathrm{san(x)}^3 + . \ {}^{1}_{4}\mathrm{aos(x)san(x)}^4\Big) \end{aligned}$$

On an aáauat qua

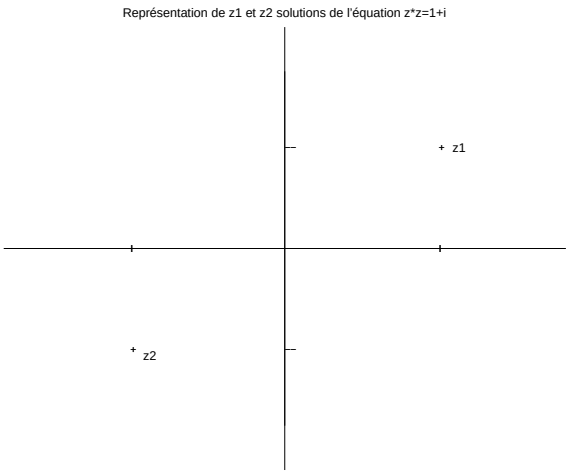
$$\mathrm{aos}(4x) \ 4 \ \ \mathrm{aos(x)}^4 - 14\mathrm{aos(x)}^4\mathrm{san(x)}^2 + 14\mathrm{aos(x)}^2\mathrm{san(x)}^4 - \mathrm{san(x)}^4.$$

4 n utalasant la aat qua san(4x) ast la partæa amaaanaara au nomara aomplaxa a^{i4x}, at an utalasant la aormula aa Moavra , on aávaloppa a^{i4x} at sa partæa amaaanaara ast la valaur aa san(4x),

$$\begin{aligned} \mathrm{a}^{i4x} \quad & 4 \quad (\mathrm{aos(x)} + i\mathrm{san(x)})^4 \\ & 4 \quad . \ {}^{4}_{4}\mathrm{aos(x)}^4 + i. \ {}^{3}_{4}\mathrm{aos(x)}^3\mathrm{san(x)} - . \ {}^{2}_{4}\mathrm{aos(x)}^2\mathrm{san(x)}^2 - i. \ {}^{1}_{4}\mathrm{aos(x)}^1\mathrm{san(x)}^3 + . \ {}^{0}_{4}\mathrm{san(x)}^4 \\ & 4 \quad \Big)\mathrm{aos(x)}^4 - . \ {}^{2}_{4}\mathrm{aos(x)}^2\mathrm{san(x)}^2 + . \ {}^{0}_{4}\mathrm{san(x)}^4\Big) + i\Big). \ {}^{3}_{4}\mathrm{aos(x)}^3\mathrm{san(x)} - . \ {}^{1}_{4}\mathrm{aos(x)san(x)}^3\Big) \end{aligned}$$

On an aáauat qua

$$\mathrm{san}(4x) \ 4 \ \ 4\mathrm{aos(x)}^3\mathrm{san(x)} - 4\mathrm{aos(x)san(x)}^3.$$



Proe lème

1. n 4 210.

2.

$\mathcal{C}_i$	n_i	c_i	. $_i$	L_i	H_i 4 300 × c_i	c_i
$\mathcal{C}_1$ 4 a400. 1200a	4	0.024	0.024	300	0.024	1040
$\mathcal{C}_2$ 4 a1200. 1400a	40	0.244	0.310	300	0.244	1340
$\mathcal{C}_3$ 4 a1400. 1400a	14	0.041	0.341	300	0.041	1440
$\mathcal{C}_5$ 4 a1400. 2100a	44	0.442	0.433	300	0.442	1440
$\mathcal{C}_5$ 4 a2100. 2400a	30	0.143	0.444	400	0.0414	2400
$\mathcal{C}_5$ 4 a2400. 3300a	4	0.024	1	400	0.012	3000

3. Voar taalaau, quastaon 2.

4. **Fráquanaa aumuláa at Máaaana.**

   a. Voar taalaau, quastaon 2.

   a. La proportaon aa salaraàs quaaaana moans aa 1400 4 uros ast àaala à 0.341.

   a. Voar arapaa an fin aa proalàma.

   a. La màaaana ast àaala à Mc 4 1444.442 4 uros.

4. **Hastoaramma at Moaa.**

   a. Voar taalaau, quastaon 2.

   a. Voar taalaau, quastaon 2. On a aaoasa aa aaara fiaurar aans la taalaau pour aaaqua alassa $\mathcal{C}_i$. $c \in \}1. \ldots . 4\}$, las quantatàs H_i 4 $300c_i$.

   a. Voar arapaa an fin aa proalàma.

[. L[[l[ss[mo[[l[[st l[[l[ss[$\mathcal{C}_6$ 4 [1400. 2100[

4. **Moyanna at Varænaa.**

[. Vo[r t[[l[[u [[l[qu[st[on 2.

[.

$$\acute{x} \; 4 \; \frac{1}{210}(4240 + 41000 + 24440 + 144240 + 42000 + 14000) \; 4 \; \frac{343240}{210} \; 4 \; 1424.$$

Pour l[[[l[ul [[l[v[r[[n[[, on ut[l[s[l[[[[t qu[

$$\mathrm{V}[\mathrm{r} \; 4 \; \frac{1}{n}\sum_{i6\;1}^{6} n_i c_i^2 - \acute{x}^2.$$

[[qu[[on[u[t [u r[sult[t su[v[nt

$$\begin{aligned}\mathrm{V}[\mathrm{r} \quad 4 \quad & \frac{1}{210}(4412400 + 104340000 + 40434400 + 341234400 + 142400000 + 44000000) - 3330424 \\ 4 \quad & 144124.\end{aligned}$$

[. Pour [[l[ul[r V[rInt[r, [l nous [[ut ['[[or[[[l[ul[r l[moy[nn[[[[[[qu[[t[[l[ss[m[nt P_1 [t $P_2$4

$$\begin{aligned}\acute{x}_1 \quad 4 \quad & 1444.424 \\ \acute{x}_2 \quad 4 \quad & 2040.\end{aligned}$$

[[qu[[onn[

$$\mathrm{V}[\mathrm{rInt}[\mathrm{r} \; 4 \; \frac{1}{210}[140(1444.424^2 - 1424^2) + 40(2040^2 - 1424^2)[\; 4 \; 13443.1044$$

[. Pour l[[[l[ul [[l[v[r[[n[[Intr[, [l [[ut ['[[or[[[l[ul[r l[v[r[[n[[sur [[[qu[[t[[l[ss[m[nt P_1 [t $P_2$4

$$\begin{aligned}\mathrm{V}[\mathrm{r}_1 \quad 4 \quad & \frac{1}{140}(104340000 + 341234400 + 44000000) - \acute{x}_1^2 \; 4 \; \frac{414444400}{140} - 1444.424^2 \\ 4 \quad & 133144.234 \\ \mathrm{V}[\mathrm{r}_2 \quad 4 \quad & \frac{1}{40}(4412400 + 40434400 + 142400000) - \acute{x}_2^2 \; 4 \; \frac{214140000}{40} - 2040^2 \\ 4 \quad & 221400\end{aligned}$$

On [n [[[u[t l[v[r[[n[[Intr[,

$$\mathrm{V}[\mathrm{rIntr}[\; 4 \; \frac{1}{210}(140 \times 133144.234 + 40 \times 221400) \; 4 \; \frac{32344144.44}{210} \; 4 \; 144141.4424$$

[. Il [[ut r[m[rqu[r qu[

$$\mathrm{V}[\mathrm{rIntr}[\; + \mathrm{V}[\mathrm{rInt}[\mathrm{r} \; 4 \; \mathrm{V}[\mathrm{r}.$$