4VRIL 2004

4ON4OURS IN4ÀNI4URS 4 4S TR4V4Ua ST4TISTIQU4S

ITS Voæ C

CORRICĆ CC L'ĆPRCUVC CC CCLCUL NUMĆRIQUC

Excrcœc 1

a. Pour tout $n \in \mathbb{N}$, x_{n+1} s'oataænt an aæoutant à x_n la ràal posataæ $\frac{1}{(n+1)`}$. La suata ast par aonsàquant una suata aroæsanta.

a. Pour tout $m \in \mathbb{N}^\star$, at m . n, on oataænt

$$\begin{aligned} |x_m - x_n| \quad 4 \quad & x_m - x_n \\ 4 \quad & \sum_{k=n+1}^{m} \frac{1}{e`} \end{aligned}$$

a.

$$x_m - x_n \quad 4 \quad \frac{1}{(n+1)`}\Big)1 + \frac{1}{n+2} + \frac{1}{(n+2)(n+3)} + \ldots + \frac{1}{(n+2)(n+3)\ldots(m)}\Big)$$

a.

$$x_m - x_n \quad . \quad \frac{1}{(n+1)`}\Big)1 + \frac{1}{n+1} + \frac{1}{(n+1)^2} + \ldots + \frac{1}{(n+1)^{m-n-1}}\Big)$$

a.

$$S_{m-n-1} \quad 4 \quad \frac{1 - \frac{1}{(n+1)^{m-n}}}{1 - \frac{1}{(n+1)}}$$

a

$$\begin{aligned} \mathrm{lam}_{m\to+\infty} S_{m-n-1} \quad 4 \quad & \frac{1}{1 - \frac{1}{(n+1)}} \\ 4 \quad & \frac{1}{n} + 1 \end{aligned}$$

a.

1.

$$|e - x_n| \quad 4 \quad \sum_{k=n+1}^{+\infty} \frac{1}{e`}$$

2. 4n r[pr[n[nt l[s qu[st[ons **a.**, **a.** [t **a.**, on o[t[[nt

$$0 \;\; . \;\; a - x_n \; . \quad \frac{1}{n`n}$$

3. 4n pr[n[nt n 4 10, [t [n ut[l[s[nt (0.1), on o[t[[nt un [n[[[r[m[nt [[[p[r

$$x_{10} \;\; . \;\; a \; . \quad x_{10} + \frac{1}{(10)`10}. \tag{0.1}$$

où x_{10} 4 2.41424140114on [n [[[u[t qu[

$$2.4142414011 \; . \;\; a \; . \;\; 2.414241444.$$

L[v[l[ur [ppro[[[[[[[a [10^{-9} pr[s [st a 4 2.414241444.

Excrcœc 2 L'ut[l[s[t[on [u T[[or[m[[[s 4[[ro[ss[m[nts 4[n[s r[qu[[r[l[[[r[v[[[l[t[[[l[[on[t[on sur un [nt[rv[ll[$[a.a[$. a . a, a [t a r[[ls. Pr[nons l'[nt[rv[ll[[10000. 10001[[t l[[on[t[on a $4x \to \sqrt{x}$. L[[on[t[on a [st [[r[v[[l[sur [10000. 10001[[[[[r[v[[l[[on[t[on $a' \; 4x \in [10000.\,10001[\to \dfrac{1}{2\sqrt{x}}$. L[T[[or[m[[[s 4[[ro[ss[m[nts 4[n[s nous [ssur[l'[x[st[n[[['un r[[l $a \in$[10000. 10001[t[l qu[

$$\begin{aligned} \frac{\sqrt{10001}-\sqrt{10000}}{10001-10000} \;\; &4 \;\; \frac{1}{2\sqrt{a}} \\ &\Leftrightarrow \\ \frac{\sqrt{10001}-100}{1} \;\; &4 \;\; \frac{1}{2\sqrt{a}}. \end{aligned} \tag{0.2}$$

4omm[$a \in$[10000. 10001[, on [$\frac{1}{2\sqrt{c}}$. $\frac{1}{2\sqrt{10000}}$ 4 $\frac{1}{200}$. 4n ut[l[s[nt (0.2) on o[t[[nt l'[n[[[r[m[nt [[$\sqrt{10001}$ su[v[nt 4

$$100 \; . \;\; \sqrt{10001} \; . \;\; 100 + \frac{1}{200} \; 4 \;\; 100.\,004.$$

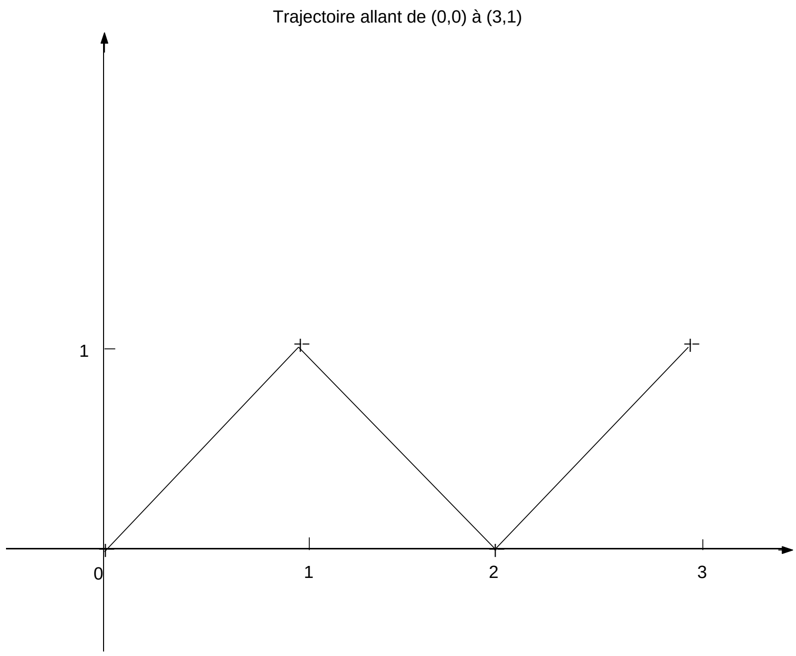
L[v[l[ur [ppro[[[[[[$\sqrt{10001}$ [10^{-3} [st $\sqrt{10001}$ 4 100. 004.

Excrcœc 3

1. S[tous l[s [l[[nts sont s[rv[s, [[ns l[[[[ss[[l y [(40 -40)4 20 [[ll[ts [[10 [uros [t 40 [[ll[ts [[20 [uros.

2.

a. Tr[[[[[to[r[poss[[l[[[ns un r[p[r[ort[o[on[l [ll[nt [[(0. 0) 4 (0. S_0) [(3. 1) 4 (3. S_3 4 1).



a. L[[u[[[[t[[r p[ut s[rv[r tous l[s [l[[nts s[[′ [u[un [nst[nt d pour $d \in \}1.\ldots.100\}$, l[tr[[[[to[r[n[tr[v[rs[l'[x[[[s [[s[[ss[s v[rs [[s v[l[urs [[S_i n[′[[t[v[s.

a. Pour qu[l[tr[[[[to[r[[tt[[[n[l[po[nt (40. 10), [l [[ut qu[$p+q$ 4 40 [t $p-q$ 4 10, on [n [[′[u[t qu[p 4 30 [t qu[q 4 20.

a. 4ompt[r l[nom[r[[[tr[[[[to[r[s [ll[nt [[(0.0) 4 $(0.S_0)$ [′ (100.20) 4 $(100.S_{100}$ 4 $20)$ [ons[st[[′ [ompt[r l[nom[r[[[[om[[n[[sons qu[l'on p[ut [[[r[[n pr[n[nt 40 [′l[′m[nts [[ns un [ns[m[l[[[100 [′l[′m[nts. P[r [ons[′qu[nt [[nom[r[[st [′[[l [′ . $\frac{60}{100}$.

4xcrcœc 4

On pos[u 4 $\ln(x)$ [t v' 4 $\frac{2x}{(1+x^2)^2}$ pour [[[r[un[[nt[′[r[t[on p[r p[rt[[s 4

$$
\begin{aligned}
\int_1^2 \frac{x\ln(x)}{(1+x^2)^2}dx \quad 4 \quad & -\frac{1}{2}[\ln(x)\frac{1}{(1+x^2)}[_1^2 + \frac{1}{2}\int_1^2 \frac{1}{x(1+x^2)}dx \\
4 \quad & -\frac{1}{2}\ln(2)\frac{1}{(4)} + \frac{1}{2}\int_1^2 (\frac{1}{x} - \frac{x}{(1+x^2)})dx \\
4 \quad & -\frac{1}{10}\ln(2) + \frac{1}{2}[\ln(x) - \frac{1}{2}\ln(1+x^2)[_1^2 \\
4 \quad & -\frac{1}{10}\ln(2) + \frac{1}{2}\ln(2) + \frac{1}{4}\ln(2) - \frac{1}{4}\ln(4) \\
4 \quad & \frac{13}{20}\ln(2) - \frac{1}{4}\ln(4).
\end{aligned}
$$

4xcrcœc 4

a. L[nom[r[tot[l [[[[[n[s o[s[rv[′s [st [′[[l [′ n 4 441

a.

4 ff^^t^^s o^s^rv^s	^^^m^tr^ . 10 ^m	10 ^m $\leq$ ^^^m^tr^ $\leq$ 20 ^m	^^^m^tr^ . 20 ^m	M^r^^s ^^s l^^n^s
4	14	24	4	41
4	24	24	22	44
4	34	34	14	40
4	24	11	14	42
4	34	40	24	104
4	24	33	23	42
4	23	23	14	44
H	23	24	14	43
M^r^^s ^^s ^olonn^s	221	214	143	n 4 441

d. Pour ^^^qu^ ^^s^ ^u t^^l^^u su^v^nt, on ^^t^rm^n^ n_{jj} s^ trouv^nt ^ l'^nt^rs^^t^on ^^ l^ l^^n^ j $(1 \leq j \leq 4)$ ^t ^^ l^ ^olonn^ j $(1 \leq j \leq 3)$ 4

4 ff^^t^^s t^^or^qu^s	^^^m^tr^ . 10 ^m	10 ^m $\leq$ ^^^m^tr^ $\leq$ 20 ^m	^^^m^tr^ . 20 ^m
4	14,344	14,044	12,442
4	24,424	24,012	14,444
4	34,234	33,414	22,141
4	14,444	14,421	12,444
4	34,444	34,443	24,444
4	31,141	30,424	20,142
4	24,344	23,403	14,442
H	23,444	23,430	14,404

d. L^ ^^l^ul ^^ Δ ^st

$$\begin{aligned}
\Delta \quad 4 \quad & \sum_{j=1}^{=} \sum_{j=1}^{3} \frac{(N_{jj} - n_{jj})^2}{n_{jj}} \\
4 \quad & 0.1 + 0.004 + 0.223 + 1.444 + 0.004 + 0.444 + 0.044 + 0.034 + 4.204 + 0.444 + \\
& +0.004 + 3.441 + 0.034 + 0.144 + 0.034 + 0.004 + 4.443 + 0.444 + 0.444 + \\
& +0.344 + 0.014 + 0.343 + 0.320 + 0.014 \ 4 \ 14.444. \qquad (0.3)
\end{aligned}$$

d. S^ on o^s^rv^ un^ ^r^n^^ v^l^ur ^^ Δ ^^fin^^ p^r (0.3), on ^ur^ t^n^^n^^ ^ ^on^lur^ qu'^l y ^ un^ ^^p^n^^n^^ ^ntr^ l'^mpl^^^m^nt ^^ l^ zon^ ^t l^ ^l^ss^ ^^ ^^^m^tr^ ^^s ^^^n^s pu^squ^ ^^l^ s^^n^fi^ qu'^l y ^ un^ ^^ff^r^n^^ ^ntr^ l^s ^^str^^ut^ons t^^or^qu^ ^t o^s^rv^^.

d 4omm^ Δ 4 14.444 . 23.444, ^lors on ^^^^pt^ l'^ypot^^s^ ^'^n^^p^n^^n^^ ^ntr^ l'^mpl^^^m^nt ^^ l^ zon^ ^t l^ ^l^ss^ ^^ ^^^m^tr^ ^^s ^^^n^s.