Année Scolaire
2023 -2024
Prof. : M. TEHUA
0546234613

Fomesoutra.com ça soutra !

TRAVAUX DIRIGES DE MATHEMATIQUES
THEME : *Primitive et Intégrale « **vol (1)** »*
NIVEAU : *Terminale C, D, E, F*
Prof:
Email:

Tableau des Primitives Usuelles

Fonction Dérivée *f*	Primitive F
$f(x) = a \quad \forall a \in \mathbb{R}$	$F(x) = ax + k \quad \forall k \in \mathbb{R}$
$f(x) = ax \quad \forall a \in \mathbb{R}$	$F(x) = \frac{1}{2}ax^2 + k \quad \forall k \in \mathbb{R}$
$f(x) = x^n \quad \forall n \in \mathbb{Z}$	$F(x) = \frac{1}{n+1}x^{n+1} + k \quad \forall k \in \mathbb{R}$
$f(x) = x^m \quad \forall m \in \mathbb{Q}^*$	$F(x) = \frac{1}{m+1}x^{m+1} + k \quad \forall k \in \mathbb{R}$
$f(x) = \frac{1}{x^n} \quad \forall n \in \mathbb{N}^*$	$F(x) = -\frac{1}{(n-1)x^{n-1}} + k \quad \forall k \in \mathbb{R}$
$f(x) = -\frac{1}{x^2}$	$F(x) = \frac{1}{x} + k \quad \forall k \in \mathbb{R}$
$f(x) = \sqrt{ax+b} \quad \forall (a,b) \in \mathbb{R}^*$	$F(x) = \frac{2}{3}(ax+b)\sqrt{ax+b} \quad \forall k \in \mathbb{R}$
$f(x) = \sqrt{x}$	$F(x) = \frac{2}{3}x\sqrt{x} + k \quad \forall k \in \mathbb{R}$
$f(x) = \frac{a}{\sqrt{x}} \quad \forall a \in \mathbb{R}^*$	$F(x) = 2a\sqrt{x} + k \quad \forall k \in \mathbb{R}$
$f(x) = \frac{1}{\sqrt{x}}$	$F(x) = 2\sqrt{x} + k \quad \forall k \in \mathbb{R}$
$f(x) = \frac{1}{x}$	$F(x) = ln(x) + k \quad \forall k \in \mathbb{R}$
$f(x) = \frac{ln(x)}{x}$	$F(x) = \frac{1}{2}(ln\,\lvert x \rvert)^2 + k \quad \forall k \in \mathbb{R}$
$f(x) = e^{ax} \quad \forall a \in \mathbb{R}$	$F(x) = \frac{1}{a}e^{ax} + k \quad \forall k \in \mathbb{R}$
$f(x) = e^x$	$F(x) = e^x + k \quad \forall k \in \mathbb{R}$
$f(x) = sin(x)$	$F(x) = -cos(x) + k \quad \forall k \in \mathbb{R}$
$f(x) = sin(ax+b) \quad \forall (a,b) \in \mathbb{R}^*$	$F(x) = -\frac{1}{a}cos(ax+b) + k \quad \forall k \in \mathbb{R}$
$f(x) = cos(x)$	$F(x) = sin(x) + k \quad \forall k \in \mathbb{R}$
$f(x) = cos(ax+b) \quad \forall (a,b) \in \mathbb{R}^*$	$F(x) = \frac{1}{a}sin(ax+b) + k \quad \forall k \in \mathbb{R}$
$f(x) = tan(ax+b) \quad \forall (a,b) \in \mathbb{R}^*$	$F(x) = -\frac{1}{a}ln\lvert cos(ax+b)\rvert + k \quad \forall k \in \mathbb{R}$

f(x) = cotan(ax+b) $\forall(a,b) \in \mathbb{R}^*$	***F(x) =*** $\frac{1}{a} ln\,|sin(ax+b)| + k$ $\quad \forall k \in \mathbb{R}$
f(x) = $1+tan^2x = \frac{1}{cosx^2x}$	***F(x)= tan(x) + k*** $\quad n\ \forall k \in \mathbb{R}$
f(x) = $1+cotan^2x = \frac{1}{sin^2x}$	***F(x) = −cotan(x) + k*** $\quad \forall k \in \mathbb{R}$
f(x) = $\frac{1}{x^2-a^2}$ $\quad \forall a \in \mathbb{R}^*$	***F(x) =*** $\frac{1}{2a} ln\,|\frac{x-a}{x+a}| + k$ $\quad \forall k \in \mathbb{R}$
f(x) = $\frac{1}{\sqrt{x^2 \pm a^2}}$ $\quad \forall a \in \mathbb{R}^*$	***F(x) =*** $x + \sqrt{x^2 \pm a^2} + k$ $\quad \forall k \in \mathbb{R}$

EXERCICE 0

Dans chaque cas déterminer la primitive de la fonction proposée :

A(x) = x^2-4x +2 ; B(x) = $(2x-1)^3$; C(x) = (-x+6)(x^2-12x+7)^2 ; D(x) = $\frac{3}{x-2} + \frac{7}{(2x+1)^3}$

E(x) = cos4x -3x +10 ; F(x) = $\frac{8x}{x^2+1} + \frac{5}{\sqrt{3x-1}}$; G(x) = $\frac{xsinx+cosx}{x^2}$; H(x) = $\sin^3 x.\cos x$

I(x) = $4xe^{3x^2+1} + 3e^{\sqrt{2x-3}}$; J(x) = x^3-2x^2 +3 ; K(x) = $x(x^2+1)^2$; L(x) = 2cos(3x+1)

M(x) = $(x+1)^3$; N(x) = $\frac{1}{(3x-1)^2}$; O(x) = $\frac{2x-1}{(x^2-x+1)^2}$; P(x) = $\frac{x}{\sqrt{x^2-1}}$; Q(x) = $\frac{e^x}{e^x+2}$

R(x) = $3x^4 + x - 2x^2 - 1$; S(x) = sin$x$. cosx^3 ; T(x) = sin($wt+\varphi$) ; U(x) = $\sqrt{x} - \frac{2}{x^3}$

V(x) = $\frac{4x+2}{x^2+x+1}$; W(x) = $2 + \frac{3}{x-1} - \frac{4}{(x+1)^2}$; X(x) = $\frac{\ln x}{x} + e^{-3x+1}$; Y(x) = $\frac{1}{x\ln x} + \frac{1}{x^2}e^{\frac{1}{x}}$

EXERCICE 1

Dans chaque cas déterminer la primitive de la fonction proposée :

A(t) = $\frac{t}{2-t}$; B(t) = $\frac{t^2-t+3}{t^2}$; C(t) = $\frac{t+3}{(t+1)(t+2)}$; D(t) = $\frac{2t}{t-5}$;E(t) = $tant^2 - 8$; F(t) = sin^3t

G(t) = $cos^2t - 3t + 17$; H(t) = $\frac{t^2+3t+3}{(2t+3)^2}$ I(t) = $4t^3+t^2$ -4 ; J(t) = $(t^2-3)^3$

K(t) = $\frac{3}{2}t^3 - \frac{4}{3}t^2 - 1$; L(t) = $\sqrt{-3t+2}$; M(t) = $\frac{3}{t^2} - \frac{1}{\sqrt{t}}$; N(t) = 3(-t-2)(3t+2)^2

O(t) = cos(-6t+5) ; P(t) = 3sin(1-4t) ; Q(t) = $\frac{1}{3cos^2 3t}$; R(t) = $\frac{3}{(3t+1)^3}$; S(t) = $\frac{-6}{\sqrt{-6t+5}}$

T(t) = (4t-4)$(2t^2-4t+1)^2$; U(t) = $\frac{-5}{(7t+3)^3}$; V(t) = (t-1)(-3t^2+6t+7)^2;W(t) = $\frac{1}{\sqrt{-8t+1}}$

X(t) = $\frac{sinx - cosx}{(cosx+sinx)^3}$; Y(t) = 3cos(2t+1)$sin^3(2t+1) - 1 - tan^2 t$; Z(t) = $sin^3 t$

$Z_1(t) = cos^5 \frac{3}{2}t$;$Z_2(t) = sin^3 t.cos^3 t$;$Z_3(t) = sin^3(-t+3)$;$Z_4(t) = cos^5(\frac{3}{2}t+1)$

$Z_5(t) = cos^3 2t.sin2t$;$Z_6(t)$ =t$\sqrt{t^2+1}$;$Z_7(t)\frac{t-1}{\sqrt{t^3-3t+1}}$;$Z_8(t)$=(t-1)$\sqrt{t^2-2t+5}$

EXERCICE 2

Déterminer la primitive F da la fonction *f* vérifiant les conditions indiquées :

f(x) = $x^3 + x^2 - 1$ et F(0) = 7 *f*(x) = $(x-3)^6$ et F(3) = 0

f(x) = $\frac{2}{(-x+3)^3}$ et F(0) = 0 *f*(x) = $\frac{x-1}{\sqrt{x^2-6x+5}}$ et F(0) = $\sqrt{5}$

f(x) = $\frac{\sin x}{\cos^2 x}$ et F($\frac{\pi}{3}$) = 0 *f*(x) = $\frac{\sin x - \cos x}{\cos^3 x}$ et F(π) = 1

f(x) = sin x.cos$^4 x$ et F(π) = 0

EXERCICE 3

1)-Les proportions suivantes sont-elles vraies ou fausses :

a)$\int_0^x 2dt = 2x$ b)$\int_{-1}^2 xdx = \frac{3}{2}$ c)$\int_0^1 3x^2dx = 3$ d)$\int_1^2 \frac{1}{x}dx = \ln 2$ e)$\int_0^1 e^x = e$

2)-Calculer les intégrales suivantes :

A = $\int_0^2(3x^2 - 4x + 5)\mathrm{d}x$; B = $\int_2^4(x^3 - 4x^2 + x)\mathrm{d}x$; C = $\int_2^4(x^3 + 3x^2 - 1)\mathrm{d}x$;

D = $\int_0^\pi sin3x\mathrm{d}x$;E = $\int_1^2 \frac{x}{2+x}\mathrm{d}x$; F =$\int_0^\pi(cos2x + sin2x)\mathrm{d}x$;G =$\int_0^{\frac{\pi}{3}}(cos3x + sin3x)\mathrm{d}x$

$H = \int_2^{e^2} \frac{1}{x\ln x}\mathrm{d}x$ I = $\int_2^1\left(x + \frac{1}{\sqrt{x}} + \frac{1}{x} - \frac{1}{x^2}\right)dx$; J = $\int_2^0 \sqrt{2x}dx$; K = $\int_{\ln 2}^{\ln 3} e^{-3x}\,dx$;

L = $\int_{\ln 2}^{\ln 4} e^{-2x}dx$; M = $\int_0^{\frac{\pi}{2}} sin^2 x\mathrm{d}x$ N = $\int_0^1(x^2 + 2x)\mathrm{d}x$; O = $\int_{\ln 2}^{\ln 3} e^{2x+1}\mathrm{d}x$;

P = $\int_0^{\frac{\pi}{3}} tanx\mathrm{d}x$; Q = $\int_{-1}^0 3e^{\frac{1}{2}x+3}\mathrm{d}x$; R = $\int_1^2(\frac{3}{x^2} + \frac{4}{x})\mathrm{d}x$;;S = $\int_0^2\left(\frac{2x^3-x^2+1}{x}\right)\mathrm{d}x$;

T = $\int_0^{2\pi}(x^3 + sinx)\mathrm{d}x$; U = $\int_0^2 \frac{1}{\sqrt{x+2}}dx$; V = $\int_0^1 \frac{e^x+1}{e^x+x-e}dx$; W = $\int_0^1 \frac{x}{1+x}\mathrm{d}x$

X = $\int_0^1 \frac{x-2}{2x+3}\mathrm{d}x$; Y = $\int_{-2}^1 \sqrt{x-3}\,dx$; Z = $\int_1^1 \frac{(lnx)^4}{x}dx$; Γ = $\int_0^\pi \sin(x + \frac{\pi}{4})\mathrm{d}x$

α=$\int_0^4(x-2)(x^2-4x+1)^2dx$; β=$\int_0^1 \frac{2x}{1+x^2}dx$;$\gamma$=$\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}\mathrm{d}x$;$\Delta = \int_0^{\frac{\pi}{2}}(2 - 3sinx)\mathrm{d}x$

$\delta = \int_{ln2}^{ln3} e^{2x}dx$; $\varphi = \int_0^1 xe^{x^2+1}dx$; $\phi = \int_0^1 \frac{2e^{2x}+3}{e^{2x}+3x}dx$ $\rho = \int_0^{\frac{\pi}{6}} sin^2 5x\mathrm{d}x$;

$\sigma = \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} sin^4 x\mathrm{d}x$; $\psi = \int_0^{\frac{\pi}{3}} cos^4 x. sin^2 x\mathrm{d}x$; $\varepsilon = \int_0^{\frac{\pi}{3}} (cos^4 x - sin^2 x)\mathrm{d}x$

EXERCICE 4

Calculer les intégrales ci-après :

A = $\int_0^1 (4x^3 - 3x^2 + 2x - 5)\mathrm{d}x$;B = $\int_{-\pi}^{\pi} \sin(-2x+\pi)\,dx$;C=$\int_0^{-4} \sqrt{4x+9}\,dx$;

D =$-4\int_{\sqrt{2}}^{\sqrt{3}} \frac{5}{x^2}dx$; E = $\int_{-1}^{2} (x+2)\sqrt{x+2}\,dx$; F = $\int_0^2 \frac{dx}{\sqrt{x-1}}$; G = $\int_0^2 \frac{-x+1}{\sqrt{x^2-2x+5}}dx$;

H=$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{sinx-cosx}{(cosx+sinx)^2}dx$;I=$\int_0^1 (x^2-1)(x^3-3x1)^3\mathrm{d}x$;J= $\int_0^1 (-2x1)\sqrt{5x^2-5x+4}\,dx$;

K = $\int_0^2 \sqrt{4-x^2}\,dx$L = $\int_0^{\frac{\pi}{2}} cosx e^{sinx}dx$; M = $\int_{-1}^{0} \frac{x}{\sqrt{x^2+3}}dx$; N = $\int_2^3 \frac{x-1}{\sqrt{x^2-2x+1}}dx$;

O = $\int_1^6 \sqrt{x^2+2}\,dx$; P= $\int_2^2 \sqrt{2x+1}\,dx$Q = $\int_0^2 \frac{dx}{\sqrt{x+2}}$; R = $\int_0^2 \frac{dx}{\sqrt{x}}$; S = $\int_0^1 \frac{1}{(1+x)^3}dx$;

T = $\int_{-2}^{1} \frac{4}{(x-2)^2}dx$; U = $\int_{-1}^{2} \frac{3}{2+x}\mathrm{d}x$; V = $\int_1^2 \left(\frac{x^2+x-2}{x}\right)\mathrm{d}x$;W = $\int_0^1 (4e^x + e)dx$;

X=$\int_0^1 (-e^x + 2x)dx$.Y=$\int_0^{\ln 2} (e^{2x} + e^x - 6)\mathrm{d}x$;Z=$\int_{\ln 2}^{\ln 5} \frac{1}{3}e^x\mathrm{d}x$; $\theta = \int_{\pi}^{0} sin2xdx$

α=$\int_1^e \frac{\ln x}{x}dx$;β=$\int_0^1 (\frac{e^x}{e^x+2} + \frac{4e^{2x}}{e^{2x}+4} + \frac{e^{-x}-2}{e^x} + \frac{e^{3x}}{1+e^{3x}} + \frac{e^{2x}}{1+e^x} + \frac{e^x}{e^{-x}+1})dx$;

$\delta = \int_0^1 e^x(e^x-3)dx$;ψ = $\int_{-2}^{1} e^x(e^x-1)dx$; $\varrho = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (3cosx - sinx)dx$

$\lambda = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^4 x\cos x + \sin x\sin 2x + \cos^3 x\text{-}3\cos x\sin x + \sin^4 x + \cos^4 x)\mathrm{d}x$

EXERCICE 5

Calculer les intégrales suivantes :

A = $\int_1^4 |x\text{-}3|\mathrm{d}x$ B = $\int_2^{-1} |1\text{-}x|\mathrm{d}x$ C = $\int_{-2}^{3} |x\text{-}1|\mathrm{d}x$ D = $\int_{-2}^{2} |1\text{-}x^2|\mathrm{d}x$

E =$\int_{-1}^{2} |e^x - 1|\mathrm{d}x$ F = $\int_{-3}^{0} |x^2 - x - 2|\mathrm{d}x$ G = $\int_{-4}^{5} |x^2 - 9|\mathrm{d}x$ H = $\int_0^{\pi} |\cos 2x|\mathrm{d}x$

EXERCICE 6

A l'aide d'une intégration par partie, calculer les intégrales ci-après :

A = $\int_{-2}^{1} xe^{x}dx$; B = $\int_{1}^{2} x\ln x dx$; C = $\int_{1}^{2} \frac{\ln x}{x^2}dx$; D = $\int_{0}^{\frac{\pi}{2}} x^3 cosx dx$

E = $\int_{1}^{2}(x+1)\ln x dx$; F = $\int_{0}^{\frac{\pi}{2}} xcosxdx$; G = $\int_{0}^{1} x^2 e^{2x+1}dx$; H = $\int_{1}^{0} e^{-x}\ln(1+e^{x})$dx

I = $\int_{1}^{0}(x+1)\, e^{2x+1}\, dx$; J = $\int_{0}^{\frac{\pi}{2}} xsinxdx$; K = $\int_{1}^{e} \sqrt{x}\ln x dx$; L = $\int_{1}^{e} x\sqrt{-x+3}\, dx$

M = $\int_{0}^{\frac{\pi}{2}}(2x+1)cosxdx$; N = $\int_{0}^{\pi}(x-1)sin3xdx$; O = $\int_{0}^{2}(2x+1)e^{2x}dx$

P = $\int_{0}^{\pi} cosx\, e^{x}dx$; Q = $\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} cos3x\, e^{2x}dx$; R = $\int_{-1}^{0} (1+x)e^{-x}\, dx$; S = $\int_{0}^{1} x^2\, e^{3x}\, dx$

T = $\int_{0}^{1}(3x^2-x+1)\, e^{3x}\, dx$; U = $\int_{0}^{1} x^2\, e^{x}\, dx$; V = $\int_{0}^{\frac{\pi}{2}} x^2 sin2xdx$; W = $\int_{0}^{\frac{\pi}{2}} x^2 cosxdx$

X = $\int_{0}^{\frac{\pi}{6}} xsin2xdx$; Y = $\int_{-1}^{2} \frac{xdx}{\sqrt{x+2}}$; Z = $\int_{0}^{4} x\sqrt{4x+9}\, dx$; $\lambda = \int_{-2}^{1}(x-5)\sqrt{2-x}\, dx$

α = $\int_{-1}^{2}(x+2)\sqrt{x+1}\, dx$; β = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cos(x+1)\, dx$; $\varphi = \int_{0}^{1} xe^{x+1}dx$

EXERCICE 7

A l'aide d'un changement de variable calculer les intégrales suivantes :

A = $\int_{0}^{1} x\sqrt{x+1}\, dx$

B = $\int_{2}^{3} x\sqrt{1-x}\, dx$

C = $\int_{0}^{\frac{1}{2}} x^2\sqrt{2x+1}\, dx$

D = $\int_{\frac{5}{3}}^{-2} \frac{x^2}{(3x-4)^5}dx$

E = $\int_{-}^{2} \frac{x}{\sqrt{x+2}}dx$

F = $\int_{-2}^{1}(x+1)\sqrt{x^2-2}\, dx$

G = $\int_{-1}^{2} \frac{x+2}{\sqrt{x^2-1}}dx$

H = $\int_{0}^{1}(x+2)\sqrt{x+2}\, dx$

EXERCICE 8

Pour chaque cas ci-après déterminer :

1) A+B et A-B
2) En déduire la valeur de A et B

a) A = $\int_0^{\pi} cos^2xdx$ et B = $\int_0^{\pi} sin^2xdx$

b) A = $\int_0^{\pi}(2x+1)cos^2xdx$ et B = $\int_0^{\pi}(2x+1)sin^2xdx$

c) A = $\int_0^{\pi} e^{-2x}cos^2xdx$ et B = $\int_0^{\pi} e^{-2x}sin^2xdx$

d) A = $\int_0^{\pi} e^{2x}cos^2xdx$ et B = $\int_0^{\pi} e^{2x}sin^2xdx$

e) A = $\int_0^{\pi}(x+1)cos^2xdx$ et B = $\int_0^{\pi}(x+1)sin^2xdx$

f) A = $\int_0^{\pi} x^2cos^2xdx$ et B = $\int_0^{\pi} x^2sin^2xdx$

EXERCICE 9

Calculer la valeur moyenne de la fonction f sur k :

1) $f(x) = x^2+1$ avec k = [0 ; 10]
2) $f(x) = \frac{1}{\sqrt{x+1}}$ avec k = [8; 15]
3) $f(x) = \frac{1}{x+2}$ avec k = [e-2; e^2-2]
4) $f(x) = cos2x$ avec k = [0 ; π]

BON TRAVAIL !!!