DEMONSTRATION DES FORMULES GEOTECHNIQUES

$$* \quad \gamma_d = \frac{\gamma}{1+\omega} \quad (1)$$

$$\omega = \frac{P_\omega}{P_s} = \frac{P - P_s}{P_s} = \frac{P}{P_s} - 1 = \frac{\frac{P}{V}}{\frac{P_s}{V}} - 1 = \frac{\gamma}{\gamma_d} - 1$$

$$\omega = \frac{\gamma}{\gamma_d} - 1 \iff \gamma_d = \frac{\gamma}{1+\omega}.$$

$$* \quad e = \frac{\gamma_s}{\gamma_d} - 1. \quad (2)$$

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1 = \frac{\frac{V}{P_s}}{\frac{V_s}{P_s}} - 1 = \frac{\frac{1}{\gamma_d}}{\frac{1}{\gamma_s}} - 1 = \frac{\gamma_s}{\gamma_d} - 1.$$

$$* \quad e = \frac{\gamma_s}{\gamma_d} - 1 \iff \gamma_d = \frac{\gamma_s}{1+e} \quad (4)$$

$$* \quad n = 1 - \frac{\gamma_d}{\gamma_s} \quad (8)$$

$$n = \frac{V_v}{V} = \frac{V - V_s}{V} = 1 - \frac{V_s}{V} = 1 - \frac{\frac{V_s}{P_s}}{\frac{V}{P_s}} = 1 - \frac{\frac{1}{\gamma_s}}{\frac{1}{\gamma_d}} = 1 - \frac{\gamma_d}{\gamma_s}$$

$$* \quad n = 1 - \frac{\gamma_d}{\gamma_s} \iff \gamma_s = \frac{\gamma_d}{1-n} \quad (14) \iff \gamma_d = \gamma_s(1-n) \quad (6)$$

$$* \quad e = \frac{\gamma_s(1+\omega)}{\gamma} - 1 \quad (30)$$

$$e = \frac{\gamma_s}{\gamma_d} - 1 \text{ or } \gamma_d = \frac{\gamma}{1+\omega} \text{ donc } e = \frac{\gamma_s}{\frac{\gamma}{1+\omega}} - 1 \Rightarrow e = \frac{\gamma_s(1+\omega)}{\gamma} - 1$$

$$* \quad n = 1 - \frac{\gamma}{\gamma_s(1+\omega)} \quad (31)$$

$$n = 1 - \frac{\gamma_d}{\gamma_s} \text{ or } \gamma_d = \frac{\gamma}{1+\omega} \text{ donc } n = 1 - \frac{\frac{\gamma}{1+\omega}}{\gamma_s} \Rightarrow n = 1 - \frac{\gamma}{\gamma_s(1+\omega)}$$

$$* \ n = \frac{e}{1+e} \quad (7)$$

$$n = \frac{V_v}{V} = \frac{V_v}{V_s+V_v} = \frac{\frac{V_v}{V_s}}{\frac{V_s+V_v}{V_s}} = \frac{e}{\frac{V_s}{V_s}+\frac{V_v}{V_s}} = \frac{e}{1+e}$$

$$* \ e = \frac{n}{1-n} \quad (11)$$

$$e = \frac{V_v}{V_s} = \frac{V_v}{V-V_v} = \frac{\frac{V_v}{V}}{\frac{V-V_v}{V}} = \frac{n}{\frac{V}{V}-\frac{V_v}{V}} = \frac{n}{1-n}$$

$$* \ \gamma = \frac{1+\omega}{1+e}\gamma_s \quad (16)$$

$$(1) \Rightarrow \gamma_d = \frac{\gamma}{1+\omega} \ (a) \ , \ (4) \Rightarrow \gamma_d = \frac{\gamma_s}{1+e} \ (b)$$

$$(a) \text{ et } (b) \Rightarrow \frac{\gamma}{1+\omega} = \frac{\gamma_s}{1+e} \Rightarrow \gamma = \frac{1+\omega}{1+e}\gamma_s$$

$$* \ \gamma_s = \frac{\gamma}{(1-n)(1+\omega)} \quad (32)$$

$$(1) \Rightarrow \gamma_d = \frac{\gamma}{1+\omega} \ (a) \ , \ (6) \Rightarrow \gamma_d = \gamma_s(1-n) \ (b)$$

$$(a) \text{ et } (b) \Rightarrow \gamma_s(1-n) = \frac{\gamma}{1+\omega} \Rightarrow \gamma_s = \frac{\gamma}{(1-n)(1+\omega)}$$

$$* \ \gamma_{sat} = \gamma_d + \gamma_\omega\left(1-\frac{\gamma_d}{\gamma_s}\right) \quad (15)$$

$$\gamma_{sat} = \frac{P_{sat}}{V} = \frac{P_s+P_\omega}{V} = \frac{P_s}{V} + \frac{\gamma_\omega \times V_v}{V} = \gamma_d + \gamma_\omega \times n = \gamma_d + \gamma_\omega\left(1-\frac{\gamma_d}{\gamma_s}\right)$$

$$* \ \gamma' = \gamma_d - \gamma_\omega(1-n) \quad (21)$$

$$\gamma' = \gamma_{sat} - \gamma_\omega \quad \text{or} \quad \gamma_{sat} = \gamma_d + \gamma_\omega\left(1-\frac{\gamma_d}{\gamma_s}\right) \ \text{donc} \ \gamma' = \gamma_d + \gamma_\omega\left(1-\frac{\gamma_d}{\gamma_s}\right) - \gamma_\omega$$

$$\gamma' = \gamma_d + \gamma_\omega\left(1-\frac{\gamma_d}{\gamma_s}-1\right) = \gamma_d + \gamma_\omega\left(\frac{\gamma_d}{\gamma_s}\right) = \gamma_d - \gamma_\omega(1-n)$$

14 * $\omega_{sat} = \dfrac{e \times \gamma_w}{\gamma_s}$ (17)

$$\omega_{sat} = \frac{P_{wsat}}{P_s} = \frac{\gamma_w \times V_v}{P_s} = \frac{\frac{V_v \times \gamma_w}{V_s}}{\frac{P_s}{V_s}} = \frac{e \times \gamma_w}{\gamma_s}$$

* $\omega_{sat} = \dfrac{n \times \gamma_w}{\gamma_d}$ (19)

$$\omega_{sat} = \frac{P_{wsat}}{P_s} = \frac{\gamma_w \times V_v}{P_s} = \frac{\frac{V_v \times \gamma_w}{V}}{\frac{P_s}{V}} = \frac{n \times \gamma_w}{\gamma_d}$$

* $\omega_{sat} = \gamma_w \left(\dfrac{1}{\gamma_d} - \dfrac{1}{\gamma_s} \right)$ (9)

(17) $\Rightarrow$ $\omega_{sat} = \dfrac{e \times \gamma_w}{\gamma_s}$ or (2) $\Rightarrow$ $e = \dfrac{\gamma_s}{\gamma_d} - 1$ donc on a

$$\omega_{sat} = \gamma_w \times \frac{1}{\gamma_s} \times \left(\frac{\gamma_s}{\gamma_d} - 1 \right) = \gamma_w \left(\frac{\cancel{\gamma_s}}{\cancel{\gamma_s}\gamma_d} - \frac{1}{\gamma_s} \right) = \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right)$$

* $S_r = \dfrac{\omega}{\omega_{sat}}$ (10)

$S_r = \dfrac{V_w}{V_v}$ or $V_w = \dfrac{\omega \times P_s}{\gamma_w}$ car $\omega = \dfrac{P_w}{P_s} = \dfrac{\gamma_w \times V_w}{P_s}$ on a donc

$S_r = \dfrac{\omega \times P_s}{\gamma_w \times V_v}$ or $\omega_{sat} = \dfrac{P_{wsat}}{P_s} = \dfrac{\gamma_w \times V_v}{P_s}$ donc $\dfrac{P_s}{\gamma_w \times V_v} = \dfrac{1}{\omega_{sat}}$ alors

$S_r = \omega \times \dfrac{1}{\omega_{sat}} = \dfrac{\omega}{\omega_{sat}}$.

* $e = \dfrac{\omega \times \gamma_s}{S_r \times \gamma_w}$ (13)

(17) $\Rightarrow$ $\dfrac{e \times \gamma_w}{\gamma_s} = \omega_{sat}$ et (10) $\Rightarrow$ $S_r = \dfrac{\omega}{\omega_{sat}}$ donc $\omega_{sat} = \dfrac{\omega}{S_r}$

par suite $\dfrac{e \times \gamma_w}{\gamma_s} = \dfrac{\omega}{S_r}$ $\Leftrightarrow$ $e = \dfrac{\omega \times \gamma_s}{S_r \times \gamma_w}$

* $e \times S_r = \omega \times G_s$ (3)

(13) $\Rightarrow$ $e = \dfrac{\omega \times \gamma_s}{S_r \times \gamma_w}$ or $G_s = \dfrac{\gamma_s}{\gamma_w}$ donc $e = \dfrac{\omega}{S_r} \times G_s$

$\Rightarrow$ $e \times S_r = \omega \times G_s$

$* \quad \gamma' = \gamma_d \left(1 - \frac{\gamma_w}{\gamma_s}\right) \quad (20)$, $\gamma' = \gamma_d \cancel{+} \gamma_w (1-n) = \gamma_d \cancel{+} \gamma_w \left(\frac{\gamma_d}{\gamma_s}\right) = \gamma_d \left(1 \cancel{+} \frac{\gamma_w}{\gamma_s}\right)$

$$* \quad S_r = \frac{\omega}{\gamma_w\left(\frac{1}{\gamma_d} - \frac{1}{\gamma_s}\right)} \quad (12)$$

$(10) \Rightarrow S_r = \frac{\omega}{\omega_{sat}}$ or $(9) \Rightarrow \omega_{sat} = \gamma_w\left(\frac{1}{\gamma_d} - \frac{1}{\gamma_s}\right)$ donc

$$S_r = \frac{\omega}{\gamma_w\left(\frac{1}{\gamma_d} - \frac{1}{\gamma_s}\right)}$$

$$* \quad \omega = \frac{n \times \gamma_w \times S_r}{\gamma_s (1-n)} \quad (23)$$

$(10) \Rightarrow S_r = \frac{\omega}{\omega_{sat}} \Leftrightarrow \omega = \omega_{sat} \times S_r$ et $\omega_{sat} = \frac{e \times \gamma_w}{\gamma_s}$ (17) donc

$\omega = \frac{e \times \gamma_w \times S_r}{\gamma_s}$ or $(11) \Rightarrow e = \frac{n}{1-n}$ alors $\omega = \frac{n \times \gamma_w \times S_r}{\gamma_s (1-n)}$

$$* \quad \gamma' = (\gamma_s - \gamma_w)(1-n) \quad (18)$$

$(21) \Rightarrow \gamma' = \gamma_d - \gamma_w (1-n)$ or $(6) \Rightarrow \gamma_d = \gamma_s (1-n)$. donc

$$\gamma' = \gamma_s (1-n) - \gamma_w (1-n) \Rightarrow \gamma' = (1-n)\left[\gamma_s - \gamma_w\right]$$

$$* \quad \gamma_d = \frac{\gamma_s \times \gamma'}{\gamma_s - \gamma_w} \quad (5)$$

$(18) \Rightarrow \gamma' = (\gamma_s - \gamma_w)(1-n)$ or $1 - n = \frac{\gamma_d}{\gamma_s}$ alors $\gamma' = (\gamma_s - \gamma_w)\frac{\gamma_d}{\gamma_s}$

$$\frac{\gamma_d}{\gamma_s} = \frac{\gamma'}{\gamma_s - \gamma_w} \Rightarrow \gamma_d = \frac{\gamma_s \times \gamma'}{\gamma_s - \gamma_w}$$

$$* \quad n = \frac{\gamma_s - \gamma_{sat}}{\gamma_s - \gamma_w} \quad (28)$$

$(18) \Rightarrow (\gamma_s - \gamma_w)(1-n) = \gamma'$ donc $n = \frac{(\gamma_s - \gamma_w) - \gamma'}{\gamma_s - \gamma_w}$ avec $\gamma' = \gamma_{sat} - \gamma_w$

$$n = \frac{\gamma_s - \gamma_w - \gamma_{sat} + \gamma_w}{\gamma_s - \gamma_w} = \frac{\gamma_s - \gamma_{sat}}{\gamma_s - \gamma_w}$$

$$* \quad e = \frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega} \quad (22)$$

$$(18) \Rightarrow \gamma' = (\gamma_s - \gamma_\omega)(1-n)$$

$$\gamma_{sat} - \gamma_\omega = (\gamma_s - \gamma_\omega)\frac{\gamma_d}{\gamma_s}$$

$$\frac{\gamma_{sat} - \gamma_\omega}{\gamma_s - \gamma_\omega} = \frac{\gamma_d}{\gamma_s}$$

$$\frac{\gamma_s - \gamma_\omega}{\gamma_{sat} - \gamma_\omega} = \frac{\gamma_s}{\gamma_d}$$

$$\frac{\gamma_s \cancel{- \gamma_\omega} - \gamma_{sat} \cancel{+ \gamma_\omega}}{\gamma_{sat} - \gamma_\omega} = \frac{\gamma_s}{\gamma_d} - 1$$

$$\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega} = e$$

$$* \quad \gamma' = \frac{\gamma_d(\gamma_s - \gamma_\omega)}{\gamma_s} \quad (33)$$

$(18) \Rightarrow \gamma' = (\gamma_s - \gamma_\omega)(1-n)$ avec $1-n = \dfrac{\gamma_d}{\gamma_s}$

$$\gamma' = (\gamma_s - \gamma_\omega)\frac{\gamma_d}{\gamma_s}$$

$$\gamma' = \frac{\gamma_d(\gamma_s - \gamma_\omega)}{\gamma_s}$$

$$* \quad \gamma' = \frac{\gamma_s - \gamma_\omega}{1+e} \quad (27)$$

$(18) \Rightarrow \gamma' = (\gamma_s - \gamma_\omega)(1-n) = (\gamma_s - \gamma_\omega)\dfrac{\gamma_d}{\gamma_s}$ or $1+e = \dfrac{\gamma_s}{\gamma_d}$ donc

$$\gamma' = \frac{\gamma_s - \gamma_\omega}{1+e}$$

$$* \quad S_r = \frac{\omega \times \gamma_s \times \gamma}{\gamma_\omega[\gamma_s(1+\omega) - \gamma]} \quad (24)$$

$(23) \Rightarrow \omega = \dfrac{n \times \gamma_\omega \times S_r}{\gamma_s(1-n)} \Leftrightarrow S_r = \dfrac{\gamma_s(1-n) \times \omega}{n \times \gamma_\omega}$ avec $\overset{(11)}{e} = \dfrac{n}{1-n}$ donc $S_r = \dfrac{\gamma_s \times \omega}{e \times \gamma_\omega}$

avec $(2) \Rightarrow e = \dfrac{\gamma_s}{\gamma_d} - 1$ on a : $S_r = \dfrac{\gamma_s \times \omega}{\gamma_\omega\left(\frac{\gamma_s}{\gamma_d} - 1\right)}$ or $\gamma_d = \dfrac{\gamma}{1+\omega}$ alors $S_r = \dfrac{\omega \times \gamma_s}{\gamma_\omega\left(\frac{\gamma_s(1+\omega)}{\gamma} - 1\right)}$

$$S_r = \frac{\omega \times \gamma_s}{\gamma_\omega\left(\frac{\gamma_s(1+\omega) - \gamma}{\gamma}\right)} = \frac{\omega \times \gamma_s \times \gamma}{\gamma_\omega[\gamma_s(1+\omega) - \gamma]}$$

$$* \quad \gamma = \gamma_d + n \times \gamma_w \times S_r \quad (29)$$

$(23) \Rightarrow w = \dfrac{n \times \gamma_w \times S_r}{\gamma_s (1-n)}$ or $\gamma_s (1-n) = \gamma_d$ alors $w = \dfrac{n \times \gamma_w \times S_r}{\gamma_d}$

$$w = \frac{n \times \gamma_w \times S_r}{\gamma_d}$$

$$1+w = \frac{n \times \gamma_w \times S_r}{\gamma_d} + 1$$

$$1+w = \frac{n \times \gamma_w \times S_r + \gamma_d}{\gamma_d}$$

$$\gamma_d (1+w) = \gamma_d + n \times \gamma_w \times S_r$$

or $(1) \Rightarrow \gamma_d = \dfrac{\gamma}{1+w} \Leftrightarrow \gamma = \gamma_d (1+w)$. donc

$$\gamma = \gamma_d + n \times \gamma_w \times S_r$$

$$* \quad \gamma = (1-n)\gamma_s + n \times \gamma_w \times S_r \quad (34)$$

$(29) \Rightarrow \gamma = \gamma_d + n \times \gamma_w \times S_r$ or $\gamma_d = \gamma_s (1-n)$. donc $\gamma = (1-n)\gamma_s + n \times \gamma_w \times S_r$

$$* \quad \gamma = \frac{\gamma_s + e \times \gamma_w \times S_r}{1+e} \quad (26)$$

$(29) \Rightarrow \gamma = \gamma_d + n \times \gamma_w \times S_r$, $(4) \Rightarrow \gamma_d = \dfrac{\gamma_s}{1+e}$ et $(7) \Rightarrow n = \dfrac{e}{1+e}$ donc

$\gamma = \dfrac{\gamma_s}{1+e} + \dfrac{e \times \gamma_w \times S_r}{1+e}$ alors $\gamma = \dfrac{\gamma_s + e \times \gamma_w \times S_r}{1+e}$

$$* \quad S_r = \frac{\gamma_s \times \gamma_{sat} + \gamma \times \gamma_w - \gamma_s \times \gamma_w - \gamma_s \times \gamma}{\gamma_w (\gamma_{sat} - \gamma_s)} \quad (25)$$

$(23) \Rightarrow w = \dfrac{n \times \gamma_w \times S_r}{\gamma_s (1-n)}$ avec $(11) \Rightarrow e = \dfrac{n}{1-n}$

$w = \dfrac{e \times \gamma_w \times S_r}{\gamma_s} \Leftrightarrow S_r = \dfrac{w \times \gamma_s}{e \times \gamma_w}$

or $(1) \Rightarrow \gamma_d = \dfrac{\gamma}{1+\omega} \Leftrightarrow \omega = \dfrac{\gamma}{\gamma_d} - 1$. donc

$$S_r = \frac{\left(\frac{\gamma}{\gamma_d} - 1\right) \times \gamma_s}{e \times \gamma_\omega}$$ et $(22) \Rightarrow e = \dfrac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}$

$$S_r = \frac{\left(\frac{\gamma}{\gamma_d} - 1\right) \times \gamma_s}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)}$$ avec $(5) \Rightarrow \gamma_d = \dfrac{\gamma_s \times \gamma'}{\gamma_s - \gamma_\omega}$

$$S_r = \frac{\left(\frac{\gamma}{\gamma_d} - 1\right) \times \gamma_s}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)} = \frac{\left(\frac{\gamma}{\frac{\gamma_s \times \gamma'}{\gamma_s - \gamma_\omega}} - 1\right) \times \gamma_s}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)}$$

$$S_r = \frac{\frac{\gamma \times \gamma_s}{\frac{\gamma_s \times \gamma'}{\gamma_s - \gamma_\omega}} - \gamma_s}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)} = \frac{\frac{\gamma \times \gamma_s \times (\gamma_s \times \gamma_\omega)}{\gamma_s \times \gamma'} - \gamma_s}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)}$$

$$S_r = \frac{\frac{\gamma \times \gamma_s \times (\gamma_s - \gamma_\omega) - \gamma_s \times (\gamma_s \times \gamma')}{\gamma_s \times \gamma'}}{\gamma_\omega \times \left(\frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_\omega}\right)} = \frac{\frac{\gamma \times \gamma_s \times (\gamma_s - \gamma_\omega) - \gamma_s \times (\gamma_s \times \gamma')}{\gamma_s \times \gamma'} \times (\gamma_{sat} - \gamma_\omega)}{\gamma_\omega (\gamma_s - \gamma_{sat})}$$

or $\gamma' = \gamma_{sat} - \gamma_\omega$, on a par suite .

$$S_r = \frac{\frac{\gamma \times \gamma_s \times (\gamma_s - \gamma_\omega) - \gamma_s \times (\gamma_s \times \gamma')}{\gamma_s \times \gamma'} \times \gamma'}{\gamma_\omega (\gamma_s - \gamma_{sat})}$$

$$S_r = \frac{\frac{\gamma \times \gamma_s \times (\gamma_s - \gamma_\omega) - \gamma_s \times (\gamma_s \times \gamma')}{\gamma_s}}{\gamma_\omega (\gamma_s - \gamma_{sat})}$$

$$S_r = \frac{\dfrac{\gamma_s\left[\gamma \times (\gamma_s - \gamma_\omega) - (\gamma_s \times \gamma')\right]}{\gamma_s}}{\gamma_\omega\,(\gamma_s - \gamma_{sat})} = \frac{\gamma \times (\gamma_s - \gamma_\omega) - \gamma_s \times \gamma'}{\gamma_\omega\,(\gamma_s - \gamma_{sat})}$$

avec $\gamma' = \gamma_{sat} - \gamma_\omega$. on obtient :

$$S_r = \frac{\gamma \times (\gamma_s - \gamma_\omega) - \gamma_s \times (\gamma_{sat} - \gamma_\omega)}{\gamma_\omega\,(\gamma_s - \gamma_{sat})}$$ en développant on a :

$$S_r = \frac{\gamma \times \gamma_s - \gamma \times \gamma_\omega - \gamma_s \times \gamma_{sat} + \gamma_s \times \gamma_\omega}{\gamma_\omega\,(\gamma_s - \gamma_{sat})}$$ Mettons les termes en ordre selon la formule finale.

$$S_r = \frac{-\gamma_s \times \gamma_{sat} - \gamma \times \gamma_\omega + \gamma_s \times \gamma_\omega + \gamma \times \gamma_s}{\gamma_\omega\,(\gamma_s - \gamma_{sat})}$$ Multiplions le numérateur et le dénominateur par **-1**

$$S_r = \frac{+\gamma_s \times \gamma_{sat} + \gamma \times \gamma_\omega - \gamma_s \times \gamma_\omega - \gamma \times \gamma_s}{\gamma_\omega\,(\gamma_{sat} - \gamma_s)}$$

$$S_r = \frac{\gamma_s \times \gamma_{sat} + \gamma \times \gamma_\omega - \gamma_s \times \gamma_\omega - \gamma \times \gamma_s}{\gamma_\omega\,(\gamma_{sat} - \gamma_s)}$$