

les **Séries** *d'Exercices*

*Dernière
Mise à jour
28/10/2022*

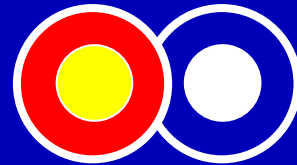
*Nouvelle
édition*

Revue et corrigée

*+ Groupe WhatsApp pour
Discuter les difficultés*

1

**Limites
Continuité**



2 OUARZAZATE 2023

110 Exercices

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Bac SM

**MATHS
2023**

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Ouarzazate 2023*



SÉRIES D'EXERCICES

« 2ème Année Bac – SM »

You're not supposed to create new methods or new techniques. Just understand those that already exist. it's not about intelligence it's about hard work. It's about the amount of work per day dudes.

Projet de livre 2022-2023

Tome 1 : limites et continuité

- **Montrer une limite par la définition**
- **Enlever la forme indéterminée**
- **La continuité à gauche et à droite**
- **Limite au voisinage de plus ou moins l'infini**
- **Prolongement par continuité**
- **L'arc tangente et la racine nième**
- **Montrer qu'une équation admet des solutions**
- **Travailler avec la règle de l'Hôpital**

Professeur Badr Eddine EL FATIHI

Ouarzazate 2022

Vendredi 28 Octobre 2022



1 : Préface

Ce livre est un support d'exercices corrigés conçu en faveur des élèves de la 2ème année Bac SM du Maroc. J'ai y classé 110 exercices pour la leçon intitulée limites et continuité. Les exercices proposés sont riches, variés et contiennent tout type de questions. C'est une plate-forme de travail pour les élèves qui auraient besoin d'un supplément de soutien très particulier. dans ce cadre, l'élève est invité à choisir le type d'exercices là où il se sent faible et de prendre son temps pour renforcé ses apprentissages. Mon objectif est d'aider ces élèves à parvenir à un niveau qui leurs permettrait de passer les devoirs, les examens et tout type de concours d'admission pour les écoles supérieurs avec succès.

Cette série contient entre autre un rappel de cours, les énoncés des exercices et les réponses détaillées qu'on devrait lire attentivement et en profiter au maximum les idées de résolution. J'ai classé dedans encore des moyens et des méthodes hors programme juste pour élargir son équilibre de connaissances. Sachez que, dans les concours d'admission et même dans les examens, la réponse finale compte plus que la méthode suivie. La vitesse de réalisation est aussi importante car vous serez certainement serrés par le temps. D'ailleurs les concours sont formulés sous la forme de questions à choix multiple. Bon courage à tout le monde et à bientôt 😊

2 : Méthodologie du travail

- Considérer d'abord une séance d'exercice comme un jeu, car Apprendre par le jeu est le meilleur moyen existant de nos jours
- Choisir le type d'exercices voulu
- Lisez la question et essayer de trouver la réponse en 10 min en consultant de temps à autre le rappel de cours
- Consulter ma réponse sur ce livre
- Notez les lacunes et difficultés rencontrées
- Retourner pour refaire l'exercice à nouveau
- Passer à un autre exercice

3 : Rappel de cours

Outil N° 1 :

C'est la définition d'une limite d'une fonction que ce soit finie ($l \in \mathbb{R}$) ou infinie, au voisinage d'un point ou au voisinage de plus ou moins l'infini :

$$\blacksquare \lim_{x \rightarrow x_0} f(x) = l \\ \Leftrightarrow \left| \begin{array}{l} (\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ |x - x_0| < \alpha \Rightarrow |f(x) - l| < \varepsilon \end{array} \right.$$

$$\blacksquare \lim_{x \rightarrow x_0} f(x) = +\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ |x - x_0| < \alpha \Rightarrow f(x) > A \end{array} \right.$$

$$\blacksquare \lim_{x \rightarrow +\infty} f(x) = l \\ \Leftrightarrow \left| \begin{array}{l} (\forall \varepsilon > 0) (\exists B > 0) (\forall x \in D_f) : \\ x > B \Rightarrow |f(x) - l| < \varepsilon \end{array} \right.$$

$$\blacksquare \lim_{x \rightarrow +\infty} f(x) = +\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists B > 0) (\forall x \in D_f) : \\ x > B \Rightarrow f(x) > A \end{array} \right.$$

Il suffirait d'apprendre par cœur ces quatre définitions. Et pour en déduire les cas de moins l'infini, il suffit d'effectuer un petit changement de variable de type :

$$\begin{aligned} x \rightarrow -\infty &\Leftrightarrow (-x) \rightarrow +\infty \\ f(x) \rightarrow -\infty &\Leftrightarrow -f(x) \rightarrow +\infty \end{aligned}$$

On obtient ainsi les définitions suivantes :

$$\blacksquare \lim_{x \rightarrow x_0} f(x) = -\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ |x - x_0| < \alpha \Rightarrow f(x) < -A \end{array} \right.$$

$$\blacksquare \lim_{x \rightarrow -\infty} f(x) = l \\ \Leftrightarrow \left| \begin{array}{l} (\forall \varepsilon > 0) (\exists B > 0) (\forall x \in D_f) : \\ x < -B \Rightarrow |f(x) - l| < \varepsilon \end{array} \right.$$

$$\blacksquare \lim_{x \rightarrow -\infty} f(x) = -\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists B > 0) (\forall x \in D_f) : \\ x < -B \Rightarrow f(x) < -A \end{array} \right.$$

Outil N° 2 :

C'est la définition de la limite à droite ou à gauche d'une fonction au voisinage d'un point quelconque :

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x > x_0}} f(x) = l \\ \Leftrightarrow \left| \begin{array}{l} (\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ 0 < x - x_0 < \alpha \Rightarrow |f(x) - l| < \varepsilon \end{array} \right.$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x < x_0}} f(x) = l \\ \Leftrightarrow \left| \begin{array}{l} (\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ -\alpha < x - x_0 < 0 \Rightarrow |f(x) - l| < \varepsilon \end{array} \right.$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x > x_0}} f(x) = +\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ 0 < x - x_0 < \alpha \Rightarrow f(x) > A \end{array} \right.$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x < x_0}} f(x) = +\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ -\alpha < x - x_0 < 0 \Rightarrow f(x) > A \end{array} \right.$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x > x_0}} f(x) = -\infty \\ \Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ 0 < x - x_0 < \alpha \Rightarrow f(x) < -A \end{array} \right.$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ x < x_0}} f(x) = -\infty$$

$$\Leftrightarrow \left| \begin{array}{l} (\forall A > 0) (\exists \alpha > 0) (\forall x \in D_f) : \\ -\alpha < x - x_0 < 0 \Rightarrow f(x) < -A \end{array} \right.$$

Outil N° 3 :

Si la limite d'une fonction numérique existe en un point, alors elle est unique. C'est ce qu'on appelle l'unicité de la limite :

$$\lim_{x \rightarrow x_0} f(x) = \lim_{t \rightarrow x_0} f(t)$$

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Outil N° 4 :

Voici les limites de quelques fonctions usuelles :

$$\blacksquare \lim_{x \rightarrow x_0} P(x) = P(x_0) \quad ; \quad P = \text{polynôme}$$

$$\blacksquare \lim_{\substack{x \rightarrow x_0 \\ Q(x_0) \neq 0}} \frac{P(x)}{Q(x)} = \frac{P(x_0)}{Q(x_0)} \quad ; \quad P, Q = \text{polynômes}$$

$$\blacksquare \lim_{x \rightarrow x_0} \sin(x) = \sin(x_0)$$

$$\blacksquare \lim_{x \rightarrow x_0} \cos(x) = \cos(x_0)$$

$$\blacksquare \lim_{x \rightarrow x_0} \tan(x) = \tan(x_0) \quad ; \quad x_0 \neq \frac{\pi}{2}[\pi]$$

$$\blacksquare \lim_{x \rightarrow x_0} \sqrt{x} = \sqrt{x_0} \quad ; \quad x_0 \geq 0$$

$$\blacksquare \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = 1$$

$$\blacksquare \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$$

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Outil N° 5 :

Opérations sur les limites : en prenant en considération juste les formes déterminées :

$$\blacksquare \lim_{x \rightarrow x_0} (f + g)(x) = \lim_{x \rightarrow x_0} f(x) + \lim_{x \rightarrow x_0} g(x)$$

$$\blacksquare \lim_{x \rightarrow x_0} (f \times g)(x) = \lim_{x \rightarrow x_0} f(x) \times \lim_{x \rightarrow x_0} g(x)$$

$$\blacksquare \lim_{x \rightarrow x_0} \left(\frac{f}{g} \right)(x) = \frac{\lim_{x \rightarrow x_0} f(x)}{\lim_{x \rightarrow x_0} g(x) \neq 0}$$

Outil N° 6 :

Voici une liste de formes déterminées écrites sous cette forme juste pour simplifier la rédaction :

$l + \infty = +\infty$	$l - \infty = -\infty$
$+\infty + \infty = +\infty$	$-\infty - \infty = -\infty$
$\text{nég} \times (+\infty) = -\infty$	$\text{nég} \times (-\infty) = +\infty$
$\text{posi} \times (+\infty) = +\infty$	$\text{posi} \times (-\infty) = -\infty$
$(+\infty)(-\infty) = -\infty$	$(+\infty)(+\infty) = +\infty$
$\frac{1}{0^+} = +\infty$	$\frac{1}{0^-} = -\infty$
$\frac{1}{+\infty} = 0^+$	$\frac{1}{-\infty} = 0^-$
$(-\infty)(-\infty) = +\infty$	

Outil N° 7 :

Voici une liste des formes indéterminées :

$+\infty - \infty$	$0 \times \infty$	$\frac{\infty}{\infty}$	$\frac{0}{0}$
1^∞	∞^0	0^0	

Remarque : la forme indéterminée fondamentale est zéro/zéro. Et toutes les autres formes indéterminées sont des variantes de cette forme là.

$$\rightsquigarrow \text{exemple1 : } 0 \times \infty = 0 \times \frac{1}{0} = \frac{0}{0}$$

$$\rightsquigarrow \text{exemple2 : } \frac{\infty}{\infty} = \frac{1}{\infty} \times \infty = 0 \times \infty = \frac{0}{0}$$

Outil N° 8 :

C'est la règle de l'Hôpital : je suis sûr est certain que cette technique est hors programme mais vous devriez l'apprendre et à la maîtriser pour l'appliquer éventuellement dans le brouillon pour déterminer la valeur de la limite.

■ la forme $\frac{0}{0} \Rightarrow \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$

Exemples :

■ $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$

■ $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{1 + \tan^2(x)}{1} \right) = 1$

■ $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) = \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2}$

■ $\lim_{x \rightarrow 0} \left(\frac{1 + \sin x}{x} \right) = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$

■ $\lim_{x \rightarrow 0} \left(\frac{x - \sin x}{\tan x - x} \right) = \lim_{x \rightarrow 0} \frac{\sin x}{2 \tan x \cdot (1 + \tan^2(x))}$

$= \lim_{x \rightarrow 0} \frac{\cos x}{2(1 + \tan^2(x)) + 2(\tan^2(x))(1 + \tan^2(x))}$

$= \frac{\cos 0}{2(1 + \tan^2(0)) + 2(\tan^2(0))(1 + \tan^2(0))} = \frac{1}{2}$

■ $\lim_{x \rightarrow 0} \left(\frac{\sin(x) - \tan(x)}{x^3} \right) = \frac{-1}{2}$

■ $\lim_{x \rightarrow 2} \left(\frac{\sqrt{x-1} - 1}{x^2 - 4} \right) = \frac{1}{8}$

Je vous laisse le soin de vérifier ces deux dernières limites.

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Outil N° 9 :

L'utilisation de la calculatrice : cette technique est valable juste dans vos préparations chez-vous à la maison pour avoir une idée sur la valeur de la limite. Voici deux exemples à méditer :

■ $\lim_{x \rightarrow 0} \left(\frac{x - \sin x}{\tan x - x} \right) = f(0,001) \text{ rad} = 0,50003 = \frac{1}{2}$

■ $\lim_{x \rightarrow +\infty} \left(\sqrt{\sqrt{x^4 - x^3}} - x \right) = f(10^8)$
 $= -0,24999 = -\frac{1}{4}$

Outil N° 10 :

La continuité en un point :

$$\boxed{f \text{ est continue en } x_0 \Leftrightarrow \lim_{x \rightarrow x_0} f(x) = f(x_0)}$$

Outil N° 11 :

La continuité en un point implique, et nécessite la continuité à droite et à gauche :

$$\boxed{f \text{ est cont en } x_0 \Leftrightarrow \lim_{x \rightarrow x_0} f(x) = \lim_{x > x_0} f(x) = \lim_{x < x_0} f(x) = f(x_0)}$$

Outil N° 12 :

Prolongement par continuité d'une fonction :

$\tilde{f}$ est un prolongement par continuité de la fonction f en un $x_0 \notin D_f$

$$\Leftrightarrow \begin{cases} \tilde{f}(x) = f(x) & ; \quad x \in D_f \\ \tilde{f}(x_0) = \lim_{x \rightarrow x_0} f(x) = l \in \mathbb{R} \end{cases}$$

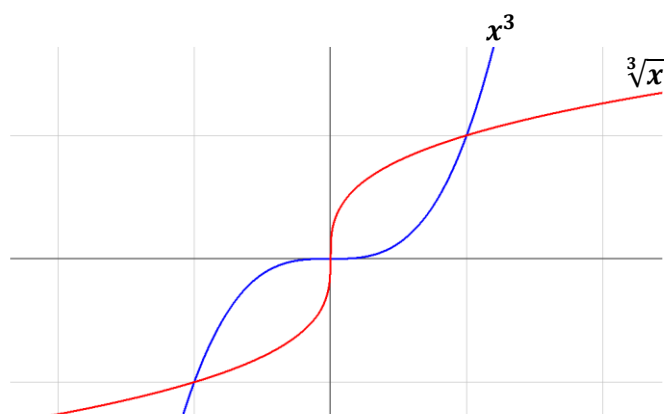
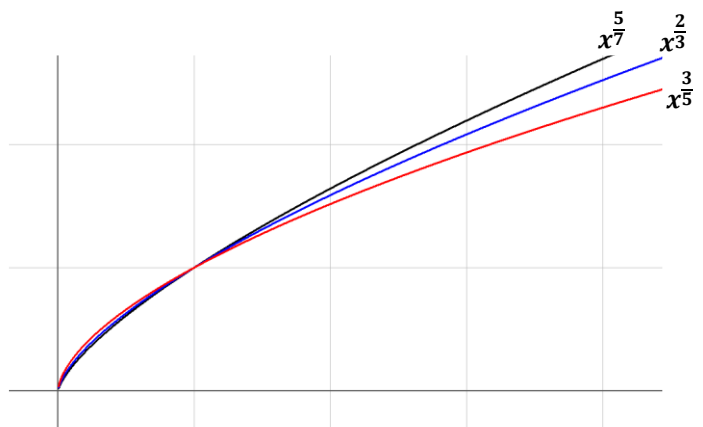
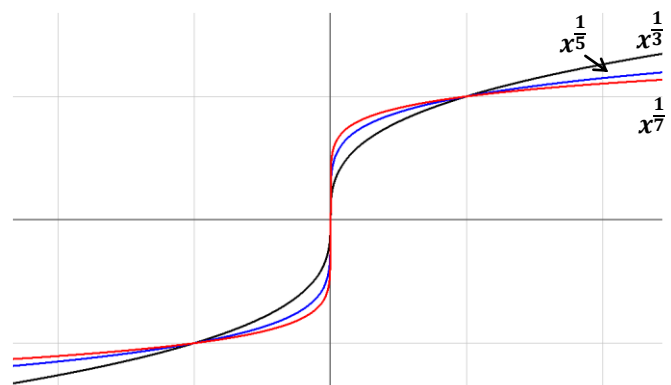
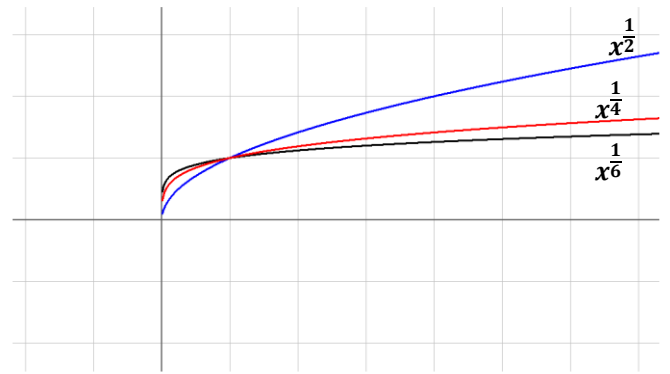
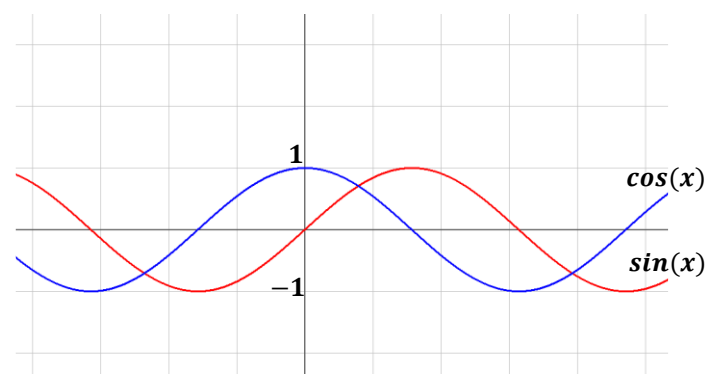
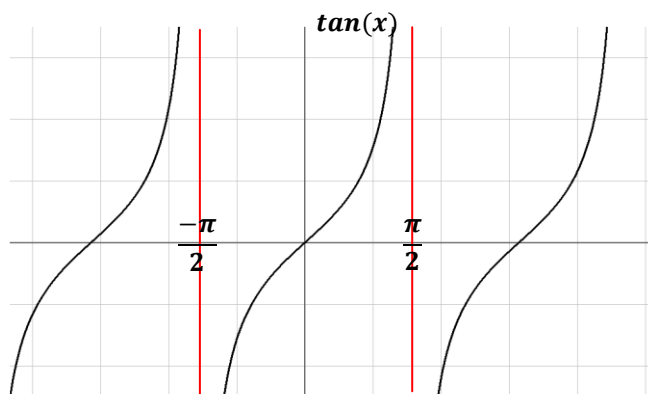
Outil N° 13 :

La continuité sur un intervalle :

- f est continue sur l'intervalle $]a, b[$
 $\Leftrightarrow f$ est continue en x_0 ; $\forall x_0 \in]a, b[$
- f est continue sur l'intervalle $[a, b[$
 $\Leftrightarrow \begin{cases} f \text{ est continue sur }]a, b[\\ \text{et } f \text{ continue en } a^+ \end{cases}$
- f est continue sur l'intervalle $]a, b]$
 $\Leftrightarrow \begin{cases} f \text{ est continue sur }]a, b[\\ \text{et } f \text{ continue en } b^- \end{cases}$
- f est continue sur l'intervalle $[a, b]$
 $\Leftrightarrow \begin{cases} f \text{ est continue sur }]a, b[\\ \text{et } f \text{ continue en } a^+ \text{ et } b^- \end{cases}$

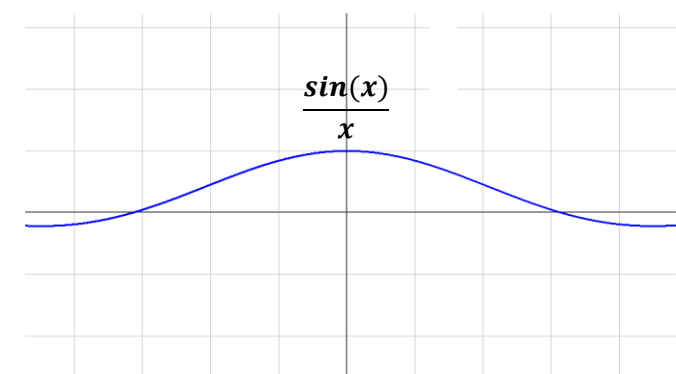
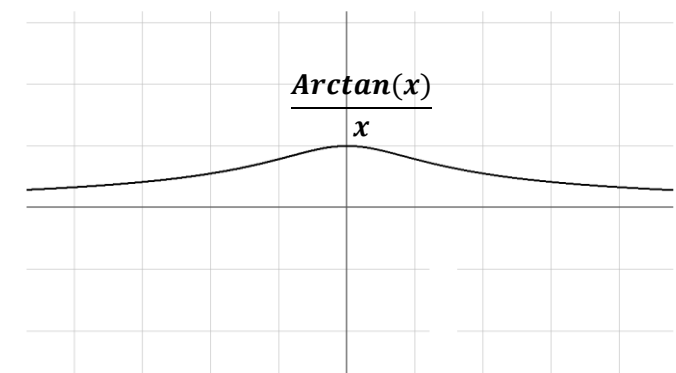
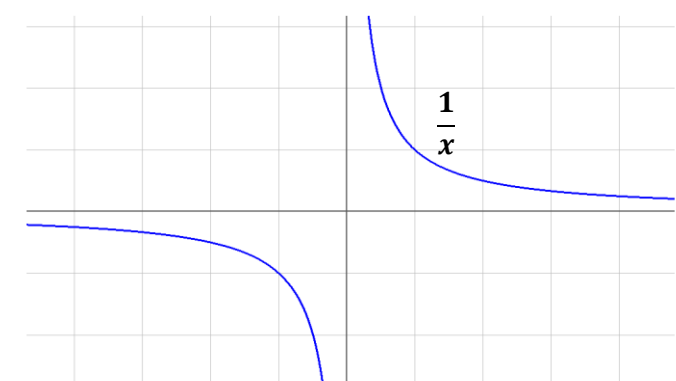
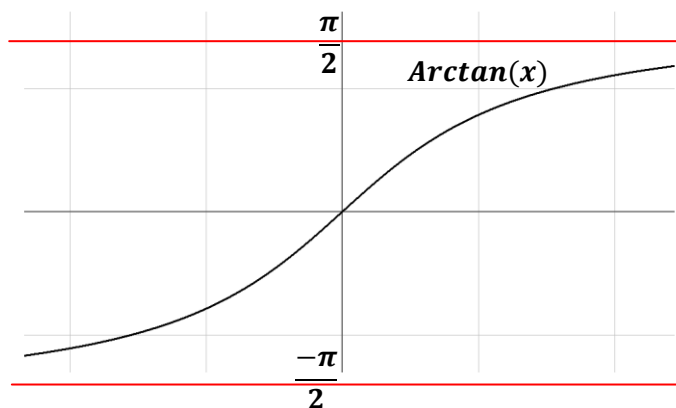
Outil N° 14 :

La mémorisation de l'allure de quelques fonctions usuelles vous permettriez d'en tirer les limites que vous en aurez besoin :



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Outil N° 15 :

Opérations sur les fonctions continues :
soient f et g deux fonctions continues sur un intervalle I et soit $\lambda \in \mathbb{R}$ Alors :

- $f + g$; λf ; $f \times g$ sont continues sur I
- f^n ; $n \in \mathbb{N}^*$ est continue sur I
- $|f|$ est continue sur I
- $\sqrt{f}$ continue sur I avec $f \geq 0$
- f/g est continue sur I avec $g \neq 0$

Outil N° 16 :

La continuité d'une composition :

- $\left| \begin{array}{l} f \text{ cont sur l'intervalle } I \\ g \text{ cont sur l'intervalle } J \\ f(I) \subseteq J \end{array} \right. \Rightarrow g \circ f \text{ est continue sur } I$

Outil N° 17 :

Comment intervertir les signes de limite et image. Autrement-dit, quand aurais-je le droit de dire que la limite de l'image est égale à l'image de la limite ?

- $\left| \begin{array}{l} f : I \mapsto f(I) \text{ continue sur } I \\ g : J \mapsto g(J) \text{ continue en } \ell \\ \ell = \lim_{x \rightarrow x_0} f(x) \end{array} \right.$

$$\Rightarrow \lim_{x \rightarrow x_0} g(f(x)) = g\left(\lim_{x \rightarrow x_0} f(x)\right)$$

Outil N° 18 :

L'image d'un intervalle de $\mathbb{R}$ par une fonction continue est un intervalle de $\mathbb{R}$.

Soit f une fonction continue et strictement monotone sur un intervalle $I \subseteq \mathbb{R}$.

On calcule l'image de I selon les cas suivants :

$$\blacksquare f([a, b]) = \begin{cases} [f(a), f(b)] & \text{si } f \text{ est } \nearrow \text{ sur } I \\ [f(b), f(a)] & \text{si } f \text{ est } \searrow \text{ sur } I \end{cases}$$

$$\blacksquare f([a, b[) = \begin{cases} \left] \lim_{a^+} f(x), \lim_{b^-} f(x) \right[& \text{si } f \text{ est } \nearrow I \\ \left] \lim_{b^-} f(x), \lim_{a^+} f(x) \right[& \text{si } f \text{ est } \searrow I \end{cases}$$

$$\blacksquare f([a, b]) = \begin{cases} [f(a), \lim_{b^-} f(x)] & \text{si } f \text{ est } \nearrow I \\ \left] \lim_{b^-} f(x), f(a) \right[& \text{si } f \text{ est } \searrow I \end{cases}$$

$$\blacksquare f(]-\infty, a]) = \begin{cases} \left] \lim_{-\infty} f(x), f(a) \right[& \text{si } f \text{ est } \nearrow I \\ [f(a), \lim_{-\infty} f(x)] & \text{si } f \text{ est } \searrow I \end{cases}$$

$$\blacksquare f(]a, +\infty[) = \begin{cases} \left] \lim_{a^+} f(x), \lim_{+\infty} f(x) \right[& \text{si } f \text{ est } \nearrow I \\ \left] \lim_{+\infty} f(x), \lim_{a^+} f(x) \right[& \text{si } f \text{ est } \searrow I \end{cases}$$

$$\blacksquare f(\mathbb{R}) = \begin{cases} \left] \lim_{-\infty} f(x), \lim_{+\infty} f(x) \right[& \text{si } f \text{ est } \nearrow I \\ \left] \lim_{+\infty} f(x), \lim_{-\infty} f(x) \right[& \text{si } f \text{ est } \searrow I \end{cases}$$

Outil N° 19 :

Théorème des valeurs intermédiaires :

Version générale, version monotone, version particulière :

$$\blacksquare \begin{cases} f \text{ cont } [a, b] \\ y \in f([a, b]) \end{cases} \Rightarrow \exists x \in [a, b] ; f(x) = y$$

$$\blacksquare \begin{cases} f \text{ cont } [a, b] \\ y \in f([a, b]) \\ f \text{ strict monot} \end{cases} \Rightarrow \exists! x \in [a, b] ; f(x) = y$$

$$\blacksquare \begin{cases} f \text{ cont } [a, b] \\ f(a) \times f(b) < 0 \end{cases} \Rightarrow \exists x \in]a, b[; f(x) = 0$$

$$\blacksquare \begin{cases} f \text{ cont } [a, b] \\ f(a) \cdot f(b) < 0 \\ f \text{ strict monot} \end{cases} \Rightarrow \exists! x \in]a, b[; f(x) = 0$$

On dira que l'équation $f(x) = 0$ admet une seule solution dans l'intervalle $]a, b[$

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Outil N° 20 :

Théorème de la fonction réciproque :

$$\blacksquare \begin{cases} f \text{ continue } I \\ f \text{ strict monot} \end{cases} \Rightarrow f : I \mapsto f(I) \text{ bjection}$$

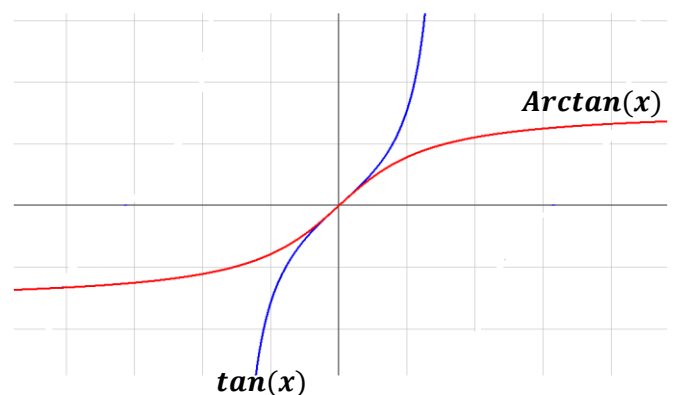
$$\blacksquare f : I \mapsto f(I) \text{ bjection} \Rightarrow \begin{cases} f \text{ et } f^{-1} \text{ ont les} \\ \text{même variations} \end{cases}$$

$$\blacksquare f : I \mapsto f(I) \text{ bject} \Rightarrow \begin{cases} (C_f) \text{ et } (C_{f^{-1}}) \text{ sont} \\ \text{symétriques par} \\ \text{rapport à } (\Delta): y = x \end{cases}$$

Outil N° 21 :

La fonction arc tangente :

$$\blacksquare \begin{aligned} \text{Arctan} : \mathbb{R} &\mapsto \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[\\ y &\mapsto \text{Arctan}(y) \\ \tan(x) &\mapsto x \end{aligned}$$



Voici quelques propriétés fondamentales de la fonction arc tangente :

$$\blacksquare (\forall x \in \mathbb{R}) \left(\forall y \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[\right) ;$$

$$\text{Arctan}(x) = y \Leftrightarrow x = \tan(y)$$

$$\blacksquare (\forall x \in \mathbb{R}) ; \tan(\text{Arctan } x) = x$$

$$\blacksquare \left(\forall x \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[\right) ; \text{Arctan}(\tan x) = x$$

$$\blacksquare (\forall x \in \mathbb{R}) ; \text{Arctan}(-x) = -\text{Arctan}(x)$$

■ $\forall x \in (a, b) \in \mathbb{R}^2 :$

$$\begin{cases} \text{Arctan}(a) = \text{Arctan}(b) & \Leftrightarrow a = b \\ \text{Arctan}(a) < \text{Arctan}(b) & \Leftrightarrow a < b \end{cases}$$

■ $\lim_{x \rightarrow -\infty} \text{Arctan}(x) = \frac{-\pi}{2} ; \lim_{x \rightarrow +\infty} \text{Arctan}(x) = \frac{\pi}{2}$

■ $\lim_{x \rightarrow \pm\infty} \frac{\text{Arctan}(x)}{x} = 0^+ ; \lim_{x \rightarrow 0} \frac{\text{Arctan}(x)}{x} = 1$

■ $(\forall x > 0) ; \text{Arctan}(x) + \text{Arctan}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

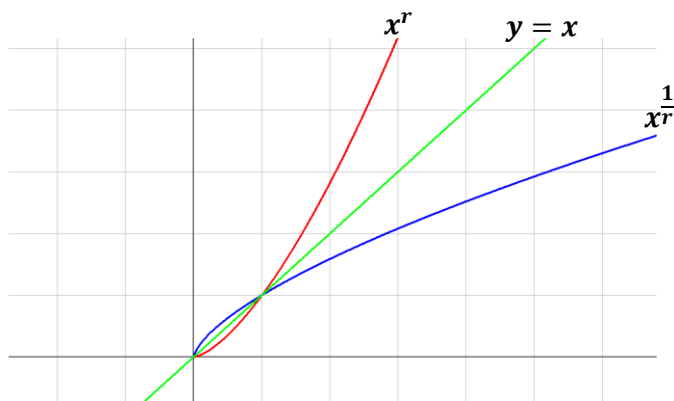
■ $(\forall x < 0) ; \text{Arctan}(x) + \text{Arctan}\left(\frac{1}{x}\right) = \frac{-\pi}{2}$

Outil N° 22 :

La fonction Racine $n^{\text{ième}}$; $n \in \mathbb{N}^*$

■ $\sqrt[n]{\cdot} : \mathbb{R}^+ \mapsto \mathbb{R}^+$
 $y \mapsto \sqrt[n]{y}$
 $x^n \mapsto x$

Remarque : si n est impair alors la fonction racine nième est définie de $\mathbb{R}$ à valeurs dans $\mathbb{R}$ Mais dans la majorité des cas on se restreint au cas de $\mathbb{R}^+$ pour qu'on puisse travailler dans un domaine positif et avoir la liberté en appliquant les règles de calcul comme $\left(x^{\frac{1}{3}}\right)^2 = (x^2)^{\frac{1}{3}} = x^{\frac{2}{3}}$



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Voici quelques propriétés de la fonction racine nième définie sur $\mathbb{R}^+$:

■ $(\forall x \in \mathbb{R}^+) (\forall y \in \mathbb{R}^+) :$
 $\sqrt[n]{x} = y \Leftrightarrow x = y^n$

■ $(\forall x \in \mathbb{R}^+) : \sqrt[n]{x^n} = (\sqrt[n]{x})^n = x$

■ $(\forall x \in \mathbb{R}^+) (\forall y \in \mathbb{R}^+) :$
 $\sqrt[n]{x} = \sqrt[n]{y} \Leftrightarrow x = y$

■ $(\forall x \in \mathbb{R}^+) (\forall y \in \mathbb{R}^+) :$
 $\sqrt[n]{x} < \sqrt[n]{y} \Leftrightarrow x < y$

Outil N° 23 :

Propriétés de la fonction racine nième :
 Soient a et b deux nombres réels positifs et p et n deux entiers naturels supérieurs ou égaux à 2, On a :

■ $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

■ $\sqrt[n]{\frac{1}{a}} = \frac{1}{\sqrt[n]{a}} ; a \neq 0$

■ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} ; b \neq 0$

■ $\sqrt[np]{a^p} = \sqrt[n]{a}$

■ $\sqrt[n]{\sqrt[p]{a}} = \sqrt[np]{a}$

■ $(\sqrt[n]{a})^p = \sqrt[n]{a^p}$

Outil N° 24 :

Passage aux limites dans des inégalités :

■ $\underbrace{f(x)}_{\text{tend vers } l} > \underbrace{g(x)}_{\text{tend vers } l'} \Rightarrow l \geq l'$

■ $\underbrace{f(x)}_{\text{tend vers } l} < \underbrace{g(x)}_{\text{tend vers } l'} \Rightarrow l \leq l'$

$$\blacksquare \quad \underbrace{h(x)}_{\text{tend vers } l'} < \underbrace{f(x)}_{\text{tend vers } l} < \underbrace{g(x)}_{\text{tend vers } l''}$$

$$\Rightarrow l' \leq l \leq l''$$

Outil N° 25 :

Puissances rationnelles d'un nombre strictement positif. Soient $a \in \mathbb{R}^+$ et $r = \frac{p}{q}$
Avec $p \in \mathbb{Z}$ et $q \in \mathbb{N}^*$. On a :

$$a^r = a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

Outil N° 26 :

Propriétés des puissances rationnelles :
Soient $r, r' \in \mathbb{Q}$ et $a, b \in \mathbb{R}^+$ On a :

$a^r \times a^{r'} = a^{r+r'}$	$(ab)^r = a^r \times b^r$
$(a^r)^{r'} = a^{rr'}$	$a^{-r} = \frac{1}{a^r}$
$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$	$\frac{a^r}{a^{r'}} = a^{r-r'}$

Outil N° 27 :

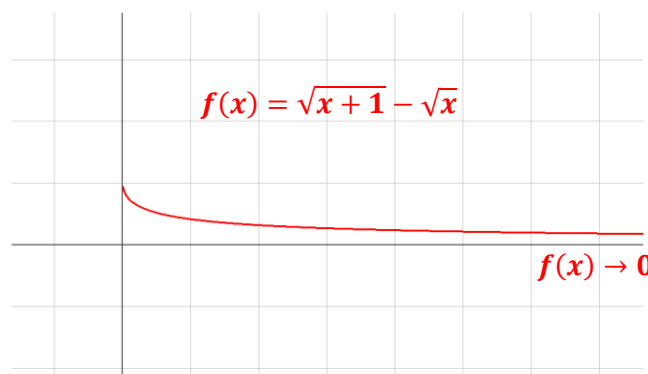
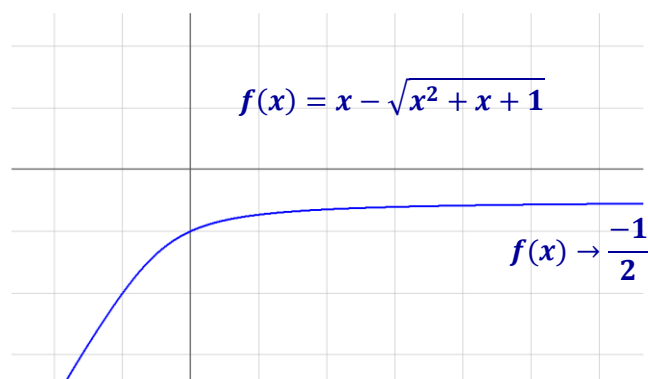
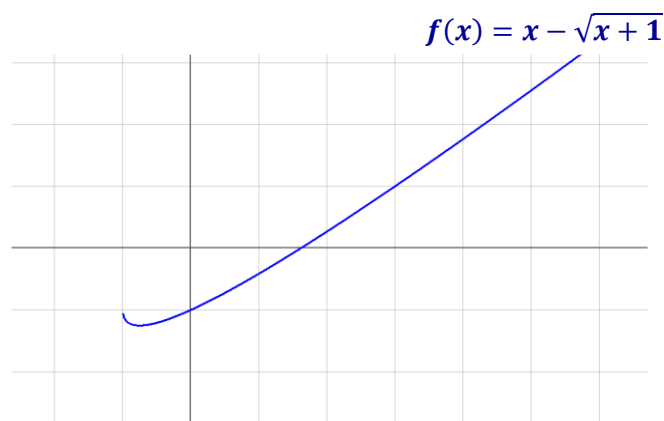
L'utilisation des inégalités pour déterminer des limites (critère de comparaison) :

$$\blacksquare \quad \underbrace{h(x)}_{\substack{\text{tend vers } l \\ x \rightarrow x_0}} \leq f(x) \leq \underbrace{g(x)}_{\substack{\text{tend vers } l \\ x \rightarrow x_0}} \Rightarrow \lim_{x \rightarrow x_0} f(x) = l$$

$$\blacksquare \quad f(x) \leq \underbrace{g(x)}_{\substack{\text{tend vers } -\infty \\ x \rightarrow x_0}} \Rightarrow \lim_{x \rightarrow x_0} f(x) = -\infty$$

$$\blacksquare \quad \underbrace{h(x)}_{\substack{\text{tend vers } +\infty \\ x \rightarrow x_0}} \leq f(x) \Rightarrow \lim_{x \rightarrow x_0} f(x) = +\infty$$

Annexe : quelques graphes



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4 : Série d'exercices

Exercice N° 1 :

Montrer, en utilisant la définition d'une limite d'une fonction numérique que :

- 1) $\lim_{x \rightarrow 0} (x^2 + x) = 0$
- 2) $\lim_{x \rightarrow 0} \left(\frac{x}{2x + 1} \right) = 0$
- 3) $\lim_{x \rightarrow 0} \left(\frac{3x^2}{x^2 + 1} \right) = 0$
- 4) $\lim_{x \rightarrow 0} (2x + x^2 - x^3) = 0$

Exercice N° 2 :

Démontrer les limites suivantes en utilisant la définition d'une limite d'une fonction :

- 1) $\lim_{x \rightarrow 1} (3x^2 - 5x + 1) = -1$
- 2) $\lim_{x \rightarrow -1} \left(\frac{2x + 1}{x - 1} \right) = \frac{1}{2}$
- 3) $\lim_{x \rightarrow 2} \sqrt{4x + 1} = 3$
- 4) $\lim_{x \rightarrow +\infty} \left(-1 + \frac{1}{3\sqrt{x}} \right) = -1$

Exercice N° 3 :

Soit f la fonction définie sur $\mathbb{R}$ par :

$$f(x) = \frac{2x}{1 + x^2}$$

- 1) Montrer que $(\forall x \in \mathbb{R}) : |f(x) - 1| \leq (x - 1)^2$

- 2) En déduire que $\lim_{x \rightarrow 1} f(x) = 1$

Exercice N° 4 :

Soit g la fonction définie sur $\mathbb{R}^+$ par :

$$g(x) = \frac{\sqrt{x}}{1 + \sqrt{x}}$$

- 1) Mq : $(\forall x \in \mathbb{R}^+) ; \left| g(x) - \frac{1}{2} \right| \leq \frac{1}{2} |x - 1|$
- 2) Conclure que : $\lim_{x \rightarrow 1} g(x) = \frac{1}{2}$

Exercice N° 5 :

Soit h la fonction définie ainsi :

$$h(x) = \frac{x - 1}{2x + 1}$$

- 1) Trouver un réel k tel que :
 $\forall x \neq \frac{-1}{2} ; |x + 1| \leq \frac{1}{3} \Rightarrow |h(x) - 2| \leq k|x + 1|$
- 2) Montrer que : $\lim_{x \rightarrow -1} h(x) = 2$

Exercice N° 6 :

Soit : $f(x) = x^3 - 2x^2 + 2x + 1$

- 1) Trouver un $\lambda \in \mathbb{R}$ tel qu'on ait :
 $\forall x \in \mathbb{R} ; |x| \leq 1 \Rightarrow |f(x) - 1| \leq \lambda|x|$
- 2) Montrer que : $\lim_{x \rightarrow 0} f(x) = 1$

Exercice N° 7 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow -\infty} \frac{5x^2 + x}{(x - 2)^2}$	2) $\lim_{x \rightarrow 3^+} \left(\frac{2x}{3 - x} \right)$
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3) $\lim_{x \rightarrow +\infty} \left(\frac{x^{2018}}{x^{2019} + 1} \right)$	4) $\lim_{x \rightarrow +\infty} \left(\frac{3 - \sqrt{x}}{x} \right)$
5) $\lim_{x \rightarrow 1} \frac{x}{(1-x)^2}$	6) $\lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x}}{x^2 + 2} \right)$

Exercice N° 8 :

Calculer chacune des limites suivantes :

- 1) $\lim_{x \rightarrow +\infty} (x^2 - x)$
- 2) $\lim_{x \rightarrow -\infty} (2 - x + x^3)$
- 3) $\lim_{x \rightarrow -\infty} 2x^3 + (x^2 - 1)(1 - 3x)$
- 4) $\lim_{x \rightarrow -\infty} x^3(1 - 2x)^5$
- 5) $\lim_{x \rightarrow -\infty} (-2x^4 - x^2 + x + 1)$
- 6) $\lim_{x \rightarrow +\infty} (1 - 2x^2)(1 + 3x)$

Exercice N° 9 :

Calculer chacune des limites suivantes :

- 1) $\lim_{x \rightarrow 0} \left(\frac{\sin(\pi x)}{x} \right)$; 2) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos(2x)}{x^2} \right)$
- 3) $\lim_{x \rightarrow 0} \left(\frac{\sin^2(x)}{3x^2} \right)$; 4) $\lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{\tan(3x)} \right)$
- 5) $\lim_{x \rightarrow 0} \left(\frac{\sin(7x)}{\sin x} \right)$; 6) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{\sin x} \right)$

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Exercice N° 10 :

Soit f la fonction définie ainsi :

$$\begin{cases} f(x) = \frac{\sin(x-1)}{2(x^2-x)} & ; x > 1 \\ f(x) = \frac{x-1}{|2x-1|-1} & ; x \leq 1 \end{cases}$$

La fonction f admet-elle une limite en 1 ?

Exercice N° 11 :

On considère la fonction f définie ainsi :

$$\begin{cases} f(x) = \frac{x^5 - x^4 + x^3 + 3}{x+1} & ; x \neq -1 \\ f(-1) = 12 \end{cases}$$

Étudier la continuité de la fonction f en zéro

Exercice N° 12 :

Résoudre les équations suivantes :

- 1) ■ : $\text{Arctan}(3x) = \frac{\pi}{8}$
- 2) ■ : $\text{Arctan}(x^2 + 2) = \text{Arctan}(3x)$
- 3) ■ : $\text{Arctan}(x^2 - x) = \frac{3\pi}{4}$
- 4) ■ : $\text{Arctan}(x) = \frac{\pi}{4} + 2\text{Arctan}\left(\frac{1}{4}\right)$
- 5) ■ : $\text{Arctan}(x) + \text{Arctan}(2x) = \frac{\pi}{3}$
- 6) ■ : $\text{Arctan}(\sqrt{x}) = \frac{-\pi}{4}$

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Exercice N° 13 :

Pour chacune des fonctions suivantes, déterminer le domaine de définition puis étudier la continuité sur chaque sous intervalle du domaine de définition :

$$f(x) = \tan\left(\frac{\pi}{2x-1}\right) ; \quad g(x) = \sin\left(\cos\frac{\pi}{x}\right)$$

$$h(x) = \frac{x^2 - \sqrt{2-x}}{|x+1| - 2} ; \quad k(x) = \frac{1 - \cos(2\pi x)}{x(x-1)}$$

$$\begin{cases} u(x) = \frac{1 - \cos\sqrt{|x|}}{|x|} & ; \quad x \neq 0 \\ u(0) = \frac{1}{2} \end{cases}$$

$$v(x) = \frac{x}{\tan(\pi x)}$$

Exercice N° 14 :

Calculer les limites suivantes :

$$\lim_{x \rightarrow -2} (x^2 - 7x - 1) ; \quad \lim_{x \rightarrow -1} (x^{2018} - x^{2017} + 2)$$

$$\lim_{x \rightarrow 3} \left(\frac{2x^3 - 3x - 9}{x - 1} \right) ; \quad \lim_{x \rightarrow \frac{3}{2}} \left(\frac{x^2 + x}{2x - 1} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \sin x ; \quad \lim_{x \rightarrow \frac{-\pi}{4}} \tan x$$

Exercice N° 15 :

Calculer chacune des limites suivantes :

$$\lim_{x \rightarrow 3} \frac{-x^2 + x + 6}{x^2 - 4x + 3} ; \quad \lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + 11x - 6}{x^5 - 32}$$

$$\lim_{x \rightarrow -1} \left(\frac{2 - \sqrt{1-3x}}{x^2 - 1} \right) ; \quad \lim_{x \rightarrow 3^+} \left(\frac{\sqrt{x^2 - 9}}{x - 3} \right)$$

$$\lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x^2 - x}}{x} \right) ; \quad \lim_{x \rightarrow 1^-} \left(\frac{\sqrt{1 - \sqrt{x}}}{x - 1} \right)$$

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Exercice N° 16 :

Soit f la fonction définie ainsi :

$$\begin{cases} f(x) = \frac{2x^2 - x + 1}{x - 2} & ; \quad x \geq 1 \\ f(x) = \frac{x^3 - 1}{x^2 - 1} & ; \quad x < 1 \end{cases}$$

Calculer les limites ainsi proposées :

$\lim_{x \rightarrow +\infty} f(x)$	$\lim_{x \rightarrow -\infty} f(x)$	$\lim_{x \rightarrow 1^+} f(x)$
$\lim_{x \rightarrow 1^-} f(x)$	$\lim_{x \rightarrow 2^+} f(x)$	$\lim_{x \rightarrow 2^-} f(x)$

Exercice N° 17 :

On considère la fonction f définie sur $\mathbb{R}$:

$$\begin{cases} f(x) = 2x - 3 & ; \quad x < 2 \\ f(x) = x^2 - 3 & ; \quad x \geq 2 \end{cases}$$

- 1) Justifier la continuité de f sur $\mathbb{R}$.
- 2) Déterminer : $f([-2,4])$ et $f(]-\infty, 1])$

Exercice N° 18 :

Étudier la continuité en zéro des fonctions suivantes :

$$\begin{cases} f(x) = \left(\frac{\sqrt{x} - \sqrt{1+x^2}}{x+1} \right) & ; \quad x \geq 0 \\ f(x) = \left(\frac{\cos x - \sqrt{1+\sin x}}{x} \right) & ; \quad x < 0 \end{cases}$$

$$\begin{cases} g(x) = \left(\frac{\sin x - \tan x}{\sqrt{x}} \right) & ; \quad x > 0 \\ g(x) = \left(x \cdot \sin\left(\frac{1}{x}\right) \right) & ; \quad x < 0 \\ g(0) = 0 \end{cases}$$

Exercice N° 19 :

On considère la fonction g définie par :

$$\begin{cases} g(x) = \left(\frac{ax^2 - ax}{x^2 - 5x + 4} \right) & ; \quad x > 1 \\ g(x) = \left(\frac{x^3 - 1}{\sqrt[3]{x} + x - 2} \right) & ; \quad x < 1 \end{cases}$$

Déterminer la valeur de a pour que la fonction g soit prolongeable par continuité en 1.

Exercice N° 20 :

On considère la fonction f définie par :

$$\begin{cases} f(x) = \left(\frac{\sqrt{3 + \cos x} - 2}{x^2} \right) & ; \quad x \neq 0 \\ f(0) = \frac{-1}{8} \end{cases}$$

Étudier la continuité de f en zéro.

Exercice N° 21 :

Déterminer le réel a pour que la fonction proposée soit continue en zéro.

$$\begin{cases} f(x) = \left(\frac{\cos^3(x) - 1}{\sin^2(x)} \right) & ; \quad x \neq 0 \\ f(0) = a \end{cases}$$

Exercice N° 22 :

Déterminer le réel a pour que la fonction f définie ci-dessous, soit continue en $\pi/2$:

$$\begin{cases} f(x) = \left(\frac{\sqrt{\sin x} - 1}{x - \frac{\pi}{2}} \right) & ; \quad x \neq \frac{\pi}{2} \\ f\left(\frac{\pi}{2}\right) = a \end{cases}$$

Exercice N° 23 :

Soit a un réel strictement positif, on considère la fonction g définie par :

$$\begin{cases} g(x) = \left(\frac{x^2 + \sqrt{x+a} - \sqrt{a}}{x} \right) & ; \quad \begin{cases} x \neq 0 \\ x \geq -a \end{cases} \\ g(0) = \frac{1}{2\sqrt{a}} \end{cases}$$

Montrer que g est continue en 0 puis préciser la limite de la fonction g quand x tend vers plus l'infini.

Exercice N° 24 :

Soit g la fonction numérique définie par :

$$\begin{cases} g(x) = \left(\frac{x + \tan(2x)}{\sin(3x)} \right) & ; \quad x \neq 0 \\ g(0) = 1 \end{cases}$$

Étudier la continuité de la fonction g au point zéro.

Exercice N° 25 :

Soit f la fonction numérique définie par :

$$\begin{cases} f(x) = (3 - x^2) & ; \quad x \leq 0 \\ f(x) = \left(\frac{x^2 - 3}{2x - 1} \right) & ; \quad x > 0 \end{cases}$$

Étudier la continuité de la fonction f au point d'abscisse zéro.

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Exercice N° 26 :

Soit g la fonction numérique définie par :

$$\begin{cases} g(x) = \left(\frac{x^2 - 1}{|x - 1|} \right) & ; \quad x \neq 1 \\ g(1) = 2 \end{cases}$$

Étudier la continuité de la fonction g au point d'abscisse 1.

Exercice N° 27 :

Dans chacun des cas suivants, étudier la continuité de la fonction f au point a .

$$\blacksquare \quad f(x) = \frac{|x^2 - 5| - 4}{\sqrt{x} - 1} \quad ; \quad a = 1$$

$$\blacksquare \quad f(x) = \frac{|x^2 - 5| - 4}{\sqrt{x} - 1} \quad ; \quad a = \sqrt{5}$$

$$\blacksquare \quad \begin{cases} f(x) = \frac{x^3 - 8}{\sqrt{x^2 + 5} - 3} & ; \quad x \neq 2 \\ f(2) = 18 \end{cases} \quad ; \quad a = 2$$

Exercice N° 28 :

Dans chacun des cas suivants, étudier la continuité de la fonction g au points d'abscisse a .

$$\begin{cases} g(x) = (x^2 - 9) \sin\left(\frac{1}{x-3}\right) & ; \quad x \neq 3 \\ g(3) = 0 \end{cases} \quad ; \quad a = 3$$

$$\begin{cases} g(x) = \frac{(1 - \tan x)^2}{1 + \cos(4x)} & ; \quad x \neq \frac{\pi}{4} \\ g\left(\frac{\pi}{4}\right) = \frac{1}{2} \end{cases} \quad ; \quad a = \frac{\pi}{4}$$

Exercice N° 29 :

Étudier la limite de la fonction f au point a dans les cas suivants :

$$1) \quad f(x) = \frac{|x - 1| \cdot x}{x^2 - 1} \quad ; \quad a = 1$$

$$2) \quad f(x) = \frac{(x + 1)^2}{|x^2 - 1|} \quad ; \quad a = -1$$

$$3) \quad \begin{cases} f(x) = \sin x & ; \quad x \geq \pi \\ f(x) = \cos x & ; \quad x < \pi \end{cases} \quad ; \quad a = \pi$$

$$4) \quad \begin{cases} f(x) = \frac{x^2 - 2x}{x + 2} & ; \quad x \geq 0 \\ f(x) = E(x) & ; \quad x < 0 \end{cases} \quad ; \quad a = 0$$

Exercice N° 30 :

Calculer les limites suivantes :

$$1) \quad \lim_{x \rightarrow 0} \left(\frac{\sqrt{1 + x^2}}{x^2} \right)$$

$$2) \quad \lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{x^2 + 1} \right)$$

$$3) \quad \lim_{x \rightarrow +\infty} \left(\frac{1}{x} + 2 + \sin\left(\frac{1}{x}\right) \right)$$

$$4) \quad \lim_{x \rightarrow 0^-} \left(\frac{\sqrt{1 - x + x^2}}{x^3} \right)$$

$$5) \quad \lim_{x \rightarrow 0^+} \left(\frac{1 + x^2}{\sqrt{x}} \right)$$

$$6) \quad \lim_{x \rightarrow 0} \left(\frac{2}{x} - 1 + \cos\left(\frac{2}{x}\right) \right)$$

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Exercice N° 31 :

Calculer chacune des limites suivante :

1) $\lim_{x \rightarrow -1} \frac{3}{(x+1)^2}$

2) $\lim_{x \rightarrow -1} \left(\frac{\tan^2 x + 1}{(x+1)^2} \right)$

3) $\lim_{x \rightarrow 1^+} \frac{x^3 - 1}{(x-1)^3}$

4) $\lim_{x \rightarrow 2^-} \frac{1}{x-2} - \left| \sin \left(\frac{2}{(x-2)^2} \right) \right|$

5) $\lim_{x \rightarrow 1^-} \left(1 + \frac{1}{\sqrt{x}} \right) \left(\frac{1}{(x-1)^{2009}} \right)$

6) $\lim_{\substack{x \rightarrow -4 \\ x < -4}} \frac{E(x)}{(x+4)^3}$

Exercice N° 32 :

Calculer les limites suivantes :

1) $\lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x} \right) \left(\frac{3}{\sqrt{x}} - 1 \right)$

2) $\lim_{x \rightarrow 0} \left(\frac{\cos x - 1}{x} \right)$

3) $\lim_{x \rightarrow 1} \left(\frac{\sqrt{x} - 1}{x - 1} \right)$

4) $\lim_{x \rightarrow -\infty} x^2 (2 + \sin x)$

5) $\lim_{x \rightarrow +\infty} \left(\frac{5x^2 - 1}{3x^2 + 4} \right)$

6) $\lim_{x \rightarrow +\infty} (\sqrt{x} - 1 + \cos x)$

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Exercice N° 33 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 0} (x^2 + x^4) \cdot \sin \left(\frac{1}{x} \right)$

2) $\lim_{x \rightarrow +\infty} \frac{E(x)}{x}$

3) $\lim_{x \rightarrow 0} \frac{x+1}{\sqrt{1+x^2}}$

4) $\lim_{x \rightarrow +\infty} \frac{|\sin x|}{\sqrt{1+x^2}}$

Exercice N° 34 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x^2 + 1} \right)$

2) $\lim_{x \rightarrow +\infty} \left(\frac{x + \sin x}{x^2 + \cos x} \right)$

3) $\lim_{x \rightarrow +\infty} \left(\frac{2x + \cos x}{3x + \sin x} \right)$

4) $\lim_{x \rightarrow +\infty} \left(\frac{E(\sqrt{x})}{x} \right)$

5) $\lim_{x \rightarrow -\infty} \left(\frac{\sin x}{x^2 + 1} \right)$

6) $\lim_{x \rightarrow +\infty} \left(\frac{2 - \cos x}{1 + \sqrt{x}} \right)$

Exercice N° 35 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 0} \left(x^2 + x + \frac{1}{x^2} \right)$

$$2) \lim_{x \rightarrow 1^+} \left(\frac{x^3 + 1}{(x-1)^3} \right)$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{x^3 + 1}{(x-1)^3} \right)$$

$$4) \lim_{x \rightarrow +\infty} (\sqrt{x} + x^2)$$

$$5) \lim_{x \rightarrow -\infty} (x^3 - x^2 + x + 1)$$

$$6) \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x}}{x^2 + 2} \right)$$

Exercice N° 36 :

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow +\infty} (-3x^2 + x + 1)$$

$$2) \lim_{x \rightarrow -\infty} \left(\frac{3x + 5}{x^2 + 1} \right)$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{x^2 - 4x}{2x^2 + 1} \right)$$

$$4) \lim_{x \rightarrow +\infty} \left(\frac{x^2 - 7}{x^3 + 2x + 1} \right)$$

$$5) \lim_{x \rightarrow -\infty} \left(\frac{x^3 + x^2 + x + 1}{x^3 + 3} \right)$$

$$6) \lim_{x \rightarrow +\infty} \left(\frac{x^4 + x^2 + 1}{x - 1} \right)$$

Exercice N° 37 :

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow -\infty} (3x^2 - x + 4)$$

$$2) \lim_{x \rightarrow 0^-} \sqrt{3 - \frac{1}{x}}$$

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$$3) \lim_{x \rightarrow 1} \left(\frac{\sqrt{2x-1} - \sqrt{x^2-x+1}}{\sqrt{x}-1} \right)$$

$$4) \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2 + 1}{x - 1}}$$

$$5) \lim_{x \rightarrow 1} \left(\frac{x^3 + 2x - 3}{x^2 + 2x - 3} \right)$$

$$6) \lim_{x \rightarrow 9} \left(\frac{\sqrt{x+7} - \sqrt{x}-1}{\sqrt{x+16} - \sqrt{x}-2} \right)$$

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Exercice N° 38 :

Calculer les limites suivantes :

$$1) \lim_{\substack{x \rightarrow -1 \\ x > -1}} \left(\frac{x+1}{|2x+3|-1} \right)$$

$$2) \lim_{x \rightarrow 2} \left(\frac{x^2 - 2x}{x - \sqrt{x+2}} \right)$$

$$3) \lim_{\substack{x \rightarrow 1 \\ x < 1}} \left(\frac{\sqrt{x+2} - \sqrt{1+2x}}{\sqrt{1-x}} \right)$$

$$4) \lim_{\substack{x \rightarrow -1 \\ x < -1}} \left(\frac{\sqrt{x^2+x+3x+3}}{x+1} \right)$$

$$5) \lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{x + \sqrt{x^2+x}}{\sqrt{x^2+x+1}-1} \right)$$

$$6) \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{\sqrt{x+1} - \sqrt{3x+1}} \right)$$

Exercice N° 39 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow +\infty} (2x - \sqrt{x})$

2) $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 + 3x - 7} + 2x + 5)$

3) $\lim_{x \rightarrow +\infty} (\sqrt{2x^2 + 1} - \sqrt{x^2 - x - 2})$

4) $\lim_{x \rightarrow +\infty} (\sqrt{5x^2 + x - 1} - 4x + 3)$

5) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 1})$

6) $\lim_{|x| \rightarrow +\infty} (\sqrt{x^2 + x + 1} - x - 3)$

Exercice N° 40 :

Calculer les limites suivantes :

1) $\lim_{x \rightarrow +\infty} (\sqrt{x^3 + 3x^2 + 4} - x^2 + 2)$

2) $\lim_{x \rightarrow -\infty} (\sqrt{3x^2 - 6x - 1} + 2x - 5)$

3) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + x + 1} + xm) ; m \in \mathbb{R}$

4) $\lim_{x \rightarrow -\infty} (\sqrt{1 - x^3} + x - 1)$

5) $\lim_{x \rightarrow -\infty} (x + \sqrt{ax^2 + bx + c})$

Exercice N° 41 :

Calculer chacune des limites :

1) $\lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{\sin(3x)} \right)$

2) $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\sqrt{x} \sin x - \cos x}{x - \frac{\pi}{6}} \right)$

3) $\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{\tan x} \right)$

4) $\lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{\sin(4x)} \right)$

5) $\lim_{x \rightarrow 0} \left(\frac{\cos(\pi x) - 1}{x} \right)$

6) $\lim_{x \rightarrow 1} \left(\frac{\tan(x - 1)}{x^2 - 1} \right)$

Exercice N° 42 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos(6x)}{\sin(4x) \cdot \tan(3x)} \right)$

2) $\lim_{x \rightarrow 1} \left(\frac{\tan(\pi x)}{x - 1} \right)$

3) $\lim_{x \rightarrow 0} \left(\frac{\sin^3(2x)}{x^3} \right)$

4) $\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right)$

5) $\lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{x^3} \right)$

6) $\lim_{x \rightarrow 0^+} \left(\frac{\cos \sqrt{x} - 1}{x} \right)$

Exercice N° 43 :

Calculer les limites suivantes :

1) $\lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sin x - \sqrt{3} \cos x}{x - \frac{\pi}{3}} \right)$

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- 2) $\lim_{x \rightarrow +\infty} \left(x^2 - \sin\left(\frac{1}{x}\right) \right)$
- 3) $\lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x^3} \right)$
- 4) $\lim_{x \rightarrow +\infty} \left(\frac{1 + \sin x}{x^2(2 + \cos x)} \right)$
- 5) $\lim_{x \rightarrow -\infty} \left(1 + \frac{x}{2 + \sqrt{x^4 + 1}} \right)$
- 6) $\lim_{x \rightarrow +\infty} \left(\frac{2E(x) + (x - E(x))^2}{x^2} \right)$

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Exercice N° 44 :

Calculer chacune des limites suivantes :

- 1) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{2x} - 2}{x^2 + 3x - 10} \right)$
- 2) $\lim_{x \rightarrow -\infty} \left(x + \sqrt{x^2 + 1} \right)$
- 3) $\lim_{x \rightarrow 1} \left(\frac{2x^3 + 3x^2 - 4x - 1}{x^3 - 1} \right)$
- 4) $\lim_{x \rightarrow +\infty} \left(\sqrt{x+1} - \sqrt{x} \right)$
- 5) $\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x} - x \right)$
- 6) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{x+2} - 2}{\sqrt{2x+5} - 3} \right)$

Exercice N° 45 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 1} - 3x \right)$
- 2) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{4x^2 + x^3}}{|2x + x^3|} \right)$
- 3) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$
- 4) $\lim_{x \rightarrow +\infty} \left(\frac{x}{\sqrt{x+1}} - \frac{x}{\sqrt{x-1}} \right)$
- 5) $\lim_{x \rightarrow 0} \left(\frac{x + \sqrt{|x|}}{x - \sqrt{|x|}} \right)$
- 6) $\lim_{x \rightarrow +\infty} \left(x\sqrt{\frac{x}{x-1}} - x - 1 \right)$

Exercice N° 46 :

Calculer les limites ainsi proposées :

- 1) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\tan x} \right)$
- 2) $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$
- 3) $\lim_{x \rightarrow 0} \left(\frac{x(1 - \cos x)}{\sin(3x) - 3 \sin x} \right)$
- 4) $\lim_{x \rightarrow 0^+} \left(\frac{1 - \cos x}{\sqrt{x}} \right)$

Exercice N° 47 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow -2} \left(\frac{-1}{3}x^2 - 5x + 7 \right)$

$$2) \lim_{x \rightarrow 3} \left(\frac{1}{(3-x)^2} \right)$$

$$3) \lim_{x \rightarrow +\infty} (x^3 + x)$$

$$4) \lim_{x \rightarrow \sqrt{2}} (\sqrt{2} x^3 - 3x^2 - x)$$

$$5) \lim_{x \rightarrow 0} \left(\frac{2}{x} + \frac{1}{x^2} \right)$$

$$6) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}} \right)$$

Exercice N° 48 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow +\infty} \left(\frac{x^5 + x^4 + 2}{x^2 - 1} \right)$$

$$2) \lim_{x \rightarrow -\infty} \left(\frac{x-1}{x^2 + x + 5} \right)$$

$$3) \lim_{x \rightarrow -\infty} \frac{(x+1)^2 \cdot (2x-7)^2}{4x^3 + x + 5}$$

$$4) \lim_{x \rightarrow +\infty} \left(\frac{1-3x}{4x+7} \right)$$

$$5) \lim_{x \rightarrow -\infty} \left(\frac{-4x^3 + 5x + 9}{7x^3 - 6} \right)$$

$$6) \lim_{|x| \rightarrow +\infty} \frac{\sqrt{5} x^2 (2-x^2)^3}{(x^4-1)^2}$$

Exercice N° 49 :

Calculer chacune des limites suivantes :

$$1) \lim_{|x| \rightarrow +\infty} \left(\frac{3x - x^4 + x(1-5x^2)}{(x^2+1)(2-3x^3)} \right)$$

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$$2) \lim_{x \rightarrow 3} \left(\frac{x-3}{x^2-2x-3} \right)$$

$$3) \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{2x^2 + x - 3} \right)$$

$$4) \lim_{x \rightarrow 2} \left(\frac{4x^3 - 5x - 22}{x^2 - x - 2} \right)$$

$$5) \lim_{x \rightarrow 2} \left(\frac{2x^3 - 7x^2 + 4x + 4}{x^3 - x^2 - 8x + 12} \right)$$

$$6) \lim_{x \rightarrow 3} \left(\frac{x^4 + 3x^3 - 7x^2 - 27x - 18}{x^4 - 3x^3 - 7x^2 + 27x - 18} \right)$$

Exercice N° 50 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow -\sqrt{3}} \left(\frac{x + \sqrt{3}}{3 - x^2} \right)$$

$$2) \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$$

$$3) \lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x^2 - 4} \right)$$

$$4) \lim_{x \rightarrow +\infty} \sqrt{2x+9}$$

$$5) \lim_{x \rightarrow +\infty} (x - \sqrt{x})$$

$$6) \lim_{x \rightarrow -\infty} \sqrt{3-x}$$

Exercice N° 51 :

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow +\infty} (x + \sqrt{x^2 + 1})$$

$$2) \lim_{x \rightarrow +\infty} (x - \sqrt{4x^2 + 1})$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{4x^2 + 3x - 1}}{x + 5} \right)$$

$$4) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{x} \right)$$

$$5) \lim_{x \rightarrow +\infty} (\sqrt{x+1} - \sqrt{x})$$

$$6) \lim_{x \rightarrow +\infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right)$$

Exercice N° 52 :

Soit g la fonction numérique définie par :

$$\begin{cases} g(x) = (x + \alpha)(x + 2) & ; \quad x \leq 1 \\ g(x) = (x + \alpha)(x + \beta) & ; \quad x > 1 \end{cases}$$

Trouver α et β pour que $\lim_{x \rightarrow 1} g(x) = 4$

Exercice N° 53 :

$$\text{Soit } f(x) = \frac{2 + \sin\left(\frac{1}{x}\right)}{x^2}$$

- 1) Montrer que : $(\forall x \in \mathbb{R}^*) ; f(x) \geq \frac{1}{x^2}$
- 2) Calculer $\lim_{x \rightarrow 0} f(x)$

Exercice N° 54 :

$$\text{Soit } g(x) = \frac{\cos x}{x}$$

- 1) Montrer que : $(\forall x \in \mathbb{R}_+^*) ; |g(x)| \leq \frac{1}{x}$
- 2) Déterminer la limite $\lim_{x \rightarrow +\infty} g(x)$

Exercice N° 55 :

$$\text{Soit } h(x) = x^2 \sin\left(\frac{1}{x^3}\right)$$

- 1) Montrer que : $(\forall x \in \mathbb{R}^*) ; |h(x)| \leq x^2$
- 2) Déterminer la limite $\lim_{x \rightarrow 0} h(x)$

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Exercice N° 56 :

$$\text{Soit } K(x) = x^2 - 3 \sin x$$

- 1) Montrer que : $(\forall x \in \mathbb{R}) ; K(x) \geq x^2 - 3$
- 2) Dédire $\lim_{x \rightarrow +\infty} K(x)$ et $\lim_{x \rightarrow -\infty} K(x)$

Exercice N° 57 :

$$\text{Soit } \begin{cases} f(x) = \frac{\sin x}{\sqrt{x}} & ; \quad x > 0 \\ f(x) = \frac{\cos x - |x + 1|}{x} & ; \quad x < 0 \end{cases}$$

- 1) Mq : $\forall x \leq -1 ; |f(x) - 1| \leq \frac{-2}{x}$
- 2) En déduire la limite $\lim_{x \rightarrow -\infty} f(x)$
- 3) Calculer la limite $\lim_{x \rightarrow +\infty} f(x)$

Exercice N° 58 :

$$\text{Soit } f(x) = \frac{\sqrt{1 + \sin x} - 1}{x}$$

- 1) Calculer la limite $\lim_{x \rightarrow 0} f(x)$
- 2) Montrer que : $\forall x \in \mathbb{R}^* ; |f(x)| \leq \frac{1}{|x|}$
- 3) Dédire : $\lim_{x \rightarrow +\infty} f(x)$ et $\lim_{x \rightarrow -\infty} f(x)$

Exercice N° 59 :

$$\text{Soit } f(x) = E(x) + \sin x$$

- 1) Mq : $(\forall x \in \mathbb{R}) : x - 2 < f(x) \leq x + 1$
- 2) En déduire les limites suivantes :
 $\lim_{x \rightarrow +\infty} f(x) ; \lim_{x \rightarrow -\infty} f(x) ; \lim_{x \rightarrow +\infty} \frac{f(x)}{x} ; \lim_{x \rightarrow -\infty} \frac{f(x)}{x}$
- 3) Etudier la limite de f en zéro

Exercice N° 60 :

$$\text{soit } f(x) = x^2 \cdot E\left(\frac{1}{x}\right)$$

- 1) Montrer que : $\forall x \in \mathbb{R}^* ; |f(x) - x| < x^2$

2) En déduire la limite $\lim_{x \rightarrow 0} f(x)$

3) Calculer : $\lim_{x \rightarrow -\infty} f(x)$; $\lim_{x \rightarrow +\infty} f(x)$

Exercice N° 61 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 1} \left(\frac{x\sqrt{x} - 1}{x^2 - 1} \right)$

2) $\lim_{x \rightarrow -4} \left(\frac{x^3 + 64}{3x^2 + 14x + 8} \right)$

3) $\lim_{x \rightarrow -1} \left(\frac{\sqrt{1-3x} - 2}{x^2 + 4x + 3} \right)$

4) $\lim_{\substack{x \rightarrow -1 \\ x > -1}} \left(\frac{2x^2 - 2}{\sqrt{x+1}} \right)$

5) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x^4 + 1} - 1}{x} \right)$

6) $\lim_{x \rightarrow 1^-} \left(\frac{\sqrt{1-x^2}}{x-1} \right)$

Exercice N° 62 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{x^2-1} \right)$

2) $\lim_{x \rightarrow 3} \left(\frac{\sqrt{2x+3} - x}{x^2 - 3x} \right)$

3) $\lim_{x \rightarrow 1^+} \left(\frac{\sqrt{x+3} - \sqrt{3x+1}}{\sqrt{x-1}} \right)$

4) $\lim_{x \rightarrow 9} \left(\frac{\sqrt{x} - 3}{\sqrt{x+7} - 4} \right)$

5) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{2x+1} - \sqrt{x+3}}{\sqrt{x^2-1} - \sqrt{2x^2-5}} \right)$

6) $\lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x^2 - x} - x}{\sqrt{x^2 + x + 1} - 1} \right)$

Exercice N° 63 :

Calculer les limites suivantes :

1) $\lim_{x \rightarrow 1^+} \left(\frac{\sqrt{2x-1} - \sqrt{x-1} - 1}{x-1} \right)$

2) $\lim_{x \rightarrow 1} \left(\frac{\sqrt{x} - 3x + 2}{x-1} \right)$

3) $\lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{x-3} \right)$

4) $\lim_{x \rightarrow 2} \left(\frac{2 - \sqrt{3x-2}}{\sqrt{2x+5} - 3} \right)$

5) $\lim_{x \rightarrow \frac{-1}{2}} \left(\frac{\sqrt{4x+6} + x^2 - 3x - 3}{2x+1} \right)$

6) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{x-1} + \sqrt{x+2} - 3}{x-2} \right)$

Exercice N° 64 :

Calculer chacune des limites suivantes :

1) $\lim_{\substack{x \rightarrow \frac{\pi}{2} \\ x < \frac{\pi}{2}}} \left(\frac{\sqrt{\cos x} - 1 + \sin x}{\cos x - \cos(3x)} \right)$

2) $\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\cos x - \sin x}{1 - \sqrt{2} \cos x} \right)$

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- 3) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos(3x)}{\sin^2(5x)} \right)$
- 4) $\lim_{x \rightarrow \pm\infty} (x^2 + 2x \sin x)$
- 5) $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 + \cos(2x)}{1 - \sin x} \right)$
- 6) $\lim_{x \rightarrow 0} \left(x \cdot \sin\left(\frac{1}{x}\right) \cdot \cos\left(\frac{1}{x}\right) \right)$

Exercice N° 65 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow 0} \left(\frac{\sqrt{\cos x} - \cos x}{\sin(2x) \cdot \tan(3x)} \right)$
- 2) $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\sqrt{3} \sin x - \cos x}{6x - \pi} \right)$
- 3) $\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan x - 1}{2 \sin x - \sqrt{2}} \right)$
- 4) $\lim_{x \rightarrow 1} \left(\frac{\sin(x-1)}{x^2 - 1} \right)$
- 5) $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{\frac{\sin x}{1 + \cos x}} - 1}{x - \frac{\pi}{2}} \right)$

Exercice N° 66 :

Soit a et b deux réels tels que $0 < a < b$

- 1) Calculer : $\lim_{x \rightarrow 0^-} \frac{\sqrt{\cos(ax) - \cos(bx)}}{x}$
- 2) Déterminer α et β pour qu'on ait :
 $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x + 1} - \alpha x - \beta) = 0$

Exercice N° 67 :

$$\text{Soit } \begin{cases} f(x) = \frac{\sin(2x)}{x} - 2 & ; \quad x > 0 \\ f(x) = \frac{x(x+1)}{x^2 - 4} & ; \quad \begin{matrix} x \leq 0 \\ x \neq -2 \end{matrix} \end{cases}$$

- 1) Calculer la limite $\lim_{x \rightarrow 0} f(x)$
- 2) Calculer la limite $\lim_{x \rightarrow -\infty} f(x)$
- 3) Mq : $\forall x > 0 ; \frac{-1}{x} - 2 \leq f(x) \leq \frac{1}{x} - 2$
- 4) Calculer la limite $\lim_{x \rightarrow +\infty} f(x)$
- 5) Calculer : $\lim_{\substack{x \rightarrow -2 \\ x > -2}} f(x)$ et $\lim_{\substack{x \rightarrow -2 \\ x < -2}} f(x)$

Exercice N° 68 :

Calculer chacune des limites suivantes :

- 1) $\lim_{x \rightarrow -\infty} x(x + \sqrt{x^2 + 1})$
- 2) $\lim_{x \rightarrow +\infty} (\sqrt{x} + 4x^2 - x + 5)$
- 3) $\lim_{x \rightarrow +\infty} \left(\frac{x + 2\sqrt{x}}{x - 3} \right)$
- 4) $\lim_{x \rightarrow -\infty} \sqrt{\frac{x+5}{2x-4}}$
- 5) $\lim_{x \rightarrow -\infty} \sqrt{x^2 - 3x + 5}$
- 6) $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x+1}}{\sqrt{x+1}}$

Exercice N° 69 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x} - 1}{\sqrt{x} + 3} \right)$; 2) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} \right)$

$$3) \lim_{x \rightarrow -\infty} (\sqrt{4x^2 - 3x} + 2x - 5)$$

$$4) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x+2}}{x^2} \right) ; \quad 5) \lim_{x \rightarrow -\infty} \sqrt{\frac{x^3 + 1}{2x + 3}}$$

$$6) \lim_{x \rightarrow -\infty} (x + 7 + \sqrt{4 - 2x})$$

Exercice N° 70 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{x^4}{x^2 - x}} + 2x \right)$$

$$2) \lim_{x \rightarrow -\infty} \left(\frac{x - 2}{\sqrt{x^2 + 5x}} \right)$$

$$3) \lim_{x \rightarrow 1} \left(\frac{-2}{|\sqrt{x} - 1|} \right)$$

$$4) \lim_{x \rightarrow +\infty} \left(\sqrt{\sqrt{x^4 - x^3}} - x \right)$$

$$5) \lim_{x \rightarrow 0^+} \left(\frac{x - \sqrt{x}}{x + \sqrt{x}} \right)$$

$$6) \lim_{x \rightarrow 1^+} \left(\frac{x^2 - \sqrt{x}}{\sqrt{x} - 1} \right)$$

Exercice N° 71 :

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow +\infty} (\sqrt{3x^2 + x + 4} - 2x + 1)$$

$$2) \lim_{x \rightarrow +\infty} (\sqrt{9x^2 + 4x} - 3x + 8)$$

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$$3) \lim_{x \rightarrow -\infty} \left(\frac{x - 2}{\sqrt{x^2 + 5x}} \right)$$

$$4) \lim_{x \rightarrow -\infty} \left(\frac{2 - 7x}{3x + 5} \right) \sqrt{1 - 2x}$$

$$5) \lim_{x \rightarrow 4} \left(\frac{\sqrt{x+5} - \sqrt{x} - 1}{\sqrt{x+12} - \sqrt{x} - 2} \right)$$

Exercice N° 72 :

Calculer chacune des limites :

$$1) \lim_{\substack{x \rightarrow \frac{1}{3} \\ x > \frac{1}{3}}} \left| \frac{x^2 - 6x}{3x - 1} \right|$$

$$2) \lim_{\substack{x \rightarrow \frac{-3}{2} \\ x > \frac{-3}{2}}} \left(\frac{3|x - 5| + 2}{4x^2 - 9} \right)$$

$$3) \lim_{x \rightarrow -4} \frac{x + 5}{|x^2 + 4x|}$$

$$4) \lim_{x \rightarrow 0} \left(\frac{x^2 + |x|}{x^2 - |x|} \right)$$

$$5) \lim_{x \rightarrow +\infty} \frac{2x - 3}{|-5x + 7|}$$

$$6) \lim_{x \rightarrow 4} \left(\frac{|x^2 - 2x| - 8}{x^2 - 5x + 4} \right)$$

Exercice N° 73 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow 4^-} \left(\frac{x}{x - 4} \right)$$

$$2) \lim_{x \rightarrow 3^-} \left(\frac{2x^2 - x + 1}{(3 - x)(-1 - x)} \right)$$

$$3) \lim_{x \rightarrow 0^+} (x\sqrt{x} + 2)$$

$$4) \lim_{x \rightarrow 2^+} \left(\frac{2-3x}{2-x} \right)$$

$$5) \lim_{x \rightarrow 3^+} \left(\frac{-7}{\sqrt{x}-3} \right)$$

$$6) \lim_{x \rightarrow 2^+} \left(\frac{x^3-8}{x^2-4x+4} \right)$$

Exercice N° 74 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow 1^-} \frac{(x-1)^2}{(x^2-1)^5} ; \quad 2) \lim_{x \rightarrow 0^+} \left(\frac{x^3-1}{x^2-x} \right)$$

$$3) \lim_{x \rightarrow 3^+} \left(\frac{\sqrt{x^2-9} + \sqrt{x} - \sqrt{3}}{\sqrt{x}-3} \right)$$

$$4) \lim_{\substack{x \rightarrow -2 \\ x > -2}} \left(\frac{4x^2-x+5}{x^2-4} \right)$$

$$5) \lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{x^3-1} \right)$$

$$6) \lim_{x \rightarrow 0^+} \left(\frac{2-\sqrt{x^2+4}}{\sqrt{x}-\sqrt{2x^2}} \right)$$

Exercice N° 75 :

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow 0} \left(\frac{\sin(2x) - 2 \sin x}{x^3} \right)$$

$$2) \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos \sqrt{x}}{\sin x} \right)$$

$$3) \lim_{x \rightarrow 0} \left(\frac{1 - \cos(4x)}{\tan(2x) \cdot \sin x} \right)$$

$$4) \lim_{x \rightarrow +\infty} x^2 \left(1 - \cos \left(\frac{1}{x} \right) \right)$$

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$$5) \lim_{x \rightarrow 0} \left(\frac{2 - \sqrt{x+4}}{\tan(5x)} \right)$$

$$6) \lim_{x \rightarrow 0} \left(\frac{\sin(x) - \tan(x)}{x^3} \right)$$

Exercice N° 76 :

Calculer chacune des limites :

$$1) \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \left(\frac{1 + \sin x}{\cos x} \right)$$

$$2) \lim_{x \rightarrow \left(\frac{-\pi}{2}\right)^+} \tan x ; \quad 4) \lim_{x \rightarrow \left(\frac{-\pi}{2}\right)^-} \tan x$$

$$3) \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cos x}{2x - \pi} \right) ; \quad 5) \lim_{x \rightarrow 1} \left(\frac{\sin(\pi x)}{x-1} \right)$$

Exercice N° 77 :

Pour chacun des cas suivants, montrer que la fonction f admet un prolongement par continuité au point x_0 puis donner ce prolongement :

$$1) f(x) = \frac{x^3-1}{x-1} ; \quad x_0 = 1$$

$$2) f(x) = \frac{\sqrt{1+\sin x} - 1}{x} ; \quad x_0 = 0$$

$$3) f(x) = \frac{x^3 - 2x^2 + 3x + 6}{x+1} ; \quad x_0 = -1$$

$$4) f(x) = \frac{x \cdot \sin x}{\cos x - 1} ; \quad x_0 = 0$$

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Exercice N° 78 :

Pour chacun des cas suivants, montrer que la fonction f admet un prolongement par continuité au point x_0 puis donner ce prolongement :

$$1) \quad f(x) = \frac{|x^2 + 4x| - 3}{x + 3} \quad ; \quad x_0 = -3$$

$$2) \quad f(x) = \frac{x^3 - a^3}{x - a} \quad ; \quad x_0 = a \in \mathbb{R}$$

$$3) \quad f(x) = \frac{(x - 2)^2}{4x - x^3} \quad ; \quad x_0 = 2$$

$$4) \quad f(x) = (x - 1) \cdot \sin\left(\frac{1}{x - 1}\right) \quad ; \quad x_0 = 1$$

Exercice N° 79 :

Dans chacun des cas suivants étudier la continuité de la fonction f sur D_f :

$$1) \quad f(x) = \sin\left(\frac{2x + 1}{x^2 - 1}\right)$$

$$2) \quad f(x) = \cos\left(\sqrt{x^2 + 1}\right)$$

$$3) \quad f(x) = \sqrt{\frac{x - 3}{x + 2}}$$

$$4) \quad f(x) = \sqrt{1 - \sin x}$$

Exercice N° 80 :

Dans chacun des cas suivants étudier la continuité de la fonction f sur D_f .

$$1) \quad f(x) = \cos(2x^2 - 3x + 4)$$

$$2) \quad f(x) = \tan\left(\frac{\pi}{x}\right)$$

$$3) \quad f(x) = \sqrt{\frac{x}{1 + \sin^2(x)}}$$

$$4) \quad f(x) = \cos(\tan^2(x))$$

Exercice N° 81 :

Calculer chacune des limites suivantes :

$$1) \quad \lim_{x \rightarrow +\infty} \left(x - 2\sqrt{x} + \frac{1}{x} \right)^3$$

$$2) \quad \lim_{x \rightarrow 0} \tan\left(\frac{\pi \sin x}{3x}\right)$$

$$3) \quad \lim_{x \rightarrow -\infty} \cos\left(\frac{\pi x + 1}{x + 2}\right)$$

$$4) \quad \lim_{x \rightarrow +\infty} \cos\left(\sin\left(\frac{1}{x}\right)\right)$$

$$5) \quad \lim_{x \rightarrow +\infty} \cos\left(\pi \sqrt{\frac{x - 1}{x + 1}}\right)$$

$$6) \quad \lim_{x \rightarrow 0} \sin\left(\pi \left(\frac{1 - \cos x}{x^2}\right)\right)$$

Exercice N° 82 :

Pour chacun des cas suivants, montrer que la fonction f est continue sur l'intervalle I . puis déterminer $f(I)$.

$$1) \quad f(x) = x^2 + 2 \quad ; \quad I = [-1, 3]$$

$$2) \quad f(x) = \frac{x - 4}{x - 2} \quad ; \quad I = [5, 8]$$

$$3) \quad f(x) = 2x \sqrt{x + 1} \quad ; \quad I = [3, 5]$$

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$$4) \quad f(x) = \tan x \quad ; \quad I = \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[$$

$$5) \quad \begin{cases} f(x) = x + 3 & ; \quad x \leq 2 \\ f(x) = x^2 + 1 & ; \quad x > 2 \end{cases} \quad ; \quad I = [-3, 5]$$

Exercice N° 83 :

Montrer que chacune des équations suivantes admet au moins une solution dans I :

$$1) \quad x^4 + x^2 + 4x - 1 = 0 \quad ; \quad I = [0, 1]$$

$$2) \quad 2 \cos x - x = 0 \quad ; \quad I = [0, \pi]$$

$$3) \quad \tan x + x^2 = 2 \quad ; \quad I = \left[\frac{\pi}{4}, \frac{\pi}{3} \right]$$

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Exercice N° 84 :

Dans chacun des cas suivants, montrer que la fonction f réalise une bijection de I sur un intervalle J à déterminer puis déterminer une expression de $f^{-1}(x)$ pour $x \in J$:

$$1) \quad f(x) = x^2 - 2x + 5 \quad ; \quad I = [1, +\infty[$$

$$2) \quad f(x) = 4x - x^2 \quad ; \quad I =]-\infty, 2[$$

$$3) \quad f(x) = \sqrt{x^2 - x} - x \quad ; \quad I =]-\infty, 0]$$

$$4) \quad f(x) = \frac{x}{x^2 + 2} \quad ; \quad I = [0, \sqrt{2}]$$

Exercice N° 85 :

Résoudre dans $\mathbb{R}$ les équations et inéquations ainsi proposées :

$$1) \quad (E) : \quad \text{Arctan}(x) + \text{Arctan}(3x) = \frac{\pi}{3}$$

$$2) \quad (E) : \quad \text{Arctan}(2x) + \text{Arctan}(x - 1) \leq 0$$

$$3) \quad \text{Arctan}(x) = \text{Arctan}\left(\frac{1}{2}\right) + \text{Arctan}\left(\frac{1}{3}\right)$$

$$4) \quad (E) : \quad \text{Arctan } x + \text{Arctan}(2x) > \frac{\pi}{3}$$

Exercice N° 86 :

Calculer chacune des limites suivantes :

$$1) \quad \lim_{x \rightarrow 0^+} (x + 1) \text{Arctan}\left(\frac{1}{x}\right)$$

$$2) \quad \lim_{x \rightarrow 0^-} (x + 1) \text{Arctan}\left(\frac{1}{x}\right)$$

$$3) \quad \lim_{x \rightarrow -\infty} x \left(\frac{\pi}{2} + \text{Arctan}(x) \right)$$

$$4) \quad \lim_{x \rightarrow 2} \left(\frac{\text{Arctan}(x - 2)}{x^2 - 4} \right)$$

Exercice N° 87 :

1) Étudier la continuité des fonctions :

$$f(x) = \sqrt[3]{\text{Arctan}(x)} \quad \text{et} \quad g(x) = \sqrt[4]{\frac{x}{x-1}}$$

2) calculer les limites suivantes :

$$\lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) \quad ; \quad \lim_{x \rightarrow 0} \left(\frac{\sqrt[3]{x+1} - 1}{\sqrt[4]{x+1} - 1} \right)$$

$$\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x} - x \right)$$

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 1} - \sqrt[3]{x^2 + 1} \right)$$

3) Résoudre dans $\mathbb{R}$ l'équation :

$$\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2} = \sqrt[3]{4x}$$

Exercice N° 88 :

Simplifier les nombres suivants :

$$A = \frac{\sqrt[4]{32} \times \sqrt[6]{27} \times \sqrt[4]{108}}{\sqrt[4]{6}}$$

$$B = \frac{(125)^{\frac{2}{9}} \times (625)^{\frac{1}{4}} \times (25)^{\frac{5}{2}}}{(5)^{\frac{17}{3}}}$$

$$C = \frac{\left(7^{\frac{2}{5}}\right)^{\frac{1}{2}} \times \left(3^{-\frac{2}{3}}\right)^{\frac{3}{2}} \times (21)^{\frac{3}{4}}}{\left(7^{-\frac{3}{2}}\right)^{\frac{1}{3}} \times (243)^{\frac{2}{3}} \times (63^{-2})^{-\frac{1}{6}}}$$

Exercice N° 89 :

Résoudre dans $\mathbb{R}$ les équations :

$$1) (E) : \sqrt[3]{3+x} - \sqrt[3]{3-x} = \sqrt[6]{4x^2}$$

$$2) (E) : 2x\sqrt{x} - 3x \cdot \sqrt[4]{\frac{1}{x}} = 20$$

$$3) (E) : \sqrt{x+1} - \sqrt[3]{x} = 1$$

$$4) (E) : \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) = \frac{\pi}{4}$$

Exercice N° 90 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x^2} - 1}{\sqrt[4]{x} - 1} \right)$$

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$$2) \lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2} - x \right)$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt[4]{x} - \sqrt[3]{x+1}}{\sqrt{x} - \sqrt[6]{x+1}} \right)$$

$$4) \lim_{x \rightarrow 0^+} \frac{1}{x} \left(\operatorname{Arctan}\left(\frac{1}{x}\right) - \frac{\pi}{2} \right)$$

Exercice N° 91 :

Soit f une fonction numérique vérifiant :

$$\forall x \in \mathbb{R} ; 1 + x - x^2 \leq f(x) \leq 1 + x - x^2 + x^4$$

Calculer les limites suivantes :

$$1) \lim_{x \rightarrow +\infty} (f(x) + x^2)$$

$$2) \lim_{x \rightarrow 0} \left(\frac{f(x) - 1}{x} \right)$$

$$3) \lim_{x \rightarrow 0} \left(\frac{f(x) - 1 - x}{x^2} \right)$$

Exercice N° 92 :

On considère la fonction numérique g définie par :

$$\begin{cases} g(x) = (2x + \pi) \tan x & ; x \in \left] -\pi, \frac{-\pi}{2} \right[\\ g(x) = \frac{1 - \cos^3 x}{x \cdot \tan x \cdot \cos^2 x} & ; x \in \left] \frac{-\pi}{2}, 0 \right[\\ g(x) = \frac{3\sqrt{1+x^4} - x}{2+x} & ; x \in [0, +\infty[\end{cases}$$

1) Calculer les limites suivantes :

$$\lim_{x \rightarrow +\infty} g(x) ; \lim_{\left(\frac{-\pi}{2}\right)^+} g(x) ; \lim_{\left(\frac{-\pi}{2}\right)} g(x)$$

2) Établir la continuité de g en zéro.

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Exercice N° 93 :

Déterminer les deux réels a et b pour que la fonction f définie sur $\mathbb{R}$ par :

$$\begin{cases} f(x) = \frac{x^2 + x - a}{x - 2} & ; \quad x > 2 \\ f(x) = \frac{2x + b}{3} & ; \quad x \leq 2 \end{cases}$$

Soit continue au point $x_0 = 2$.

Exercice N° 94 :

Déterminer les réels a et b et c pour que la fonction f définie sur $\mathbb{R}$ par :

$$\begin{cases} f(x) = \frac{3x^2 - 2bx + 1}{2x^2 + ax - a - 2} & ; \quad x < 1 \\ f(x) = \frac{-2x^2 + 3x + 3}{x^2 + 1} & ; \quad x > 1 \\ f(1) = \frac{2 + c}{3} \end{cases}$$

Soit continue au point $x_0 = 1$.

Exercice N° 95 :

Déterminer les deux réels a et b pour que la fonction f_n définie sur $\mathbb{R}$ par :

$$\forall n \in \mathbb{N}^* : \begin{cases} f_n(x) = \frac{(3-x)^n - a}{x - 2} & ; \quad x < 2 \\ f_n(x) = \frac{3x + b}{4} & ; \quad x \geq 2 \end{cases}$$

Soit continue au point $x_0 = 2$

Exercice N° 96 :

$$\text{Soit } \forall x \in \mathbb{R} : \begin{cases} f(x) = \frac{ax^2 + bx - 1}{x^2 - 2} & ; \quad x \geq 2 \\ f(x) = 3x + c & ; \quad x < 2 \end{cases}$$

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Déterminer les réels a, b, c pour que les conditions soient vérifiées dans chacun des cas suivants :

- 1) $\lim_{x \rightarrow +\infty} f(x) = 2$ et $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 3} f(x)$
- 2) f continue en $x_0 = 2$.

Exercice N° 97 :

Pour chacun des cas suivants, montrer que la fonction f admet un prolongement par continuité en x_0 puis donner ce prolongement :

$$1) f(x) = \frac{x\sqrt{x} - 1}{\sqrt{3x+1} - \sqrt{x+3}} ; \quad x_0 = 1$$

$$2) f(x) = \frac{\sqrt{x+6} + \sqrt{2x+5} - 3}{4 - x^2} ; \quad x_0 = -2$$

$$3) f(x) = \frac{\tan x - \sin x}{x + \sin x} ; \quad x_0 = 0$$

$$4) f(x) = \frac{\cos x - \sqrt{1 + \sin x}}{x} ; \quad x_0 = 0$$

Exercice N° 98 :

Soit f la fonction définie par :

$$\begin{cases} f(x) = \frac{(x-1)^2}{\sqrt{x^2-1}} & ; \quad |x| > 1 \\ f(x) = x^2 - 3x + 2 & ; \quad |x| < 1 \end{cases}$$

- 1) Calculer : $\lim_{x \rightarrow 1^-} f(x)$ et $\lim_{x \rightarrow 1^+} f(x)$
- 2) f admet-elle un prolongement par Continuité en 1 ?
- 3) f admet-elle un prolongement par Continuité en -1 ?

Exercice N° 99 :

Soit f la fonction définie sur $\left[0, \frac{\pi}{2}\right]$ par :

$$f(x) = \frac{(1 - \tan x)^2}{\cos(2x)}$$

Est-ce que la fonction f admet un prolongement par continuité en $\frac{\pi}{4}$.

Exercice N° 100 :

On considère la fonction f définie par :

$$\begin{cases} f(x) = \frac{x^2 - x + 1}{3x^2 + 6} & ; \quad x \geq 1 \\ f(x) = \frac{1}{x^2 - x} \cdot \sin\left(\frac{\pi x}{2}\right) & ; \quad x < 1 \end{cases}$$

1) Étudier la continuité de la fonction f au point d'abscisse 1.

2) f admet-elle un prolongement par continuité en zéro ?

Exercice N° 101 :

Montrer que chacune des équations suivantes admet au moins une solution dans l'intervalle I .

1) $x^3 - 3x^2 + 15x - 7 = 0$; $I =]0,1[$

2) $1 + \sin x - x^2 = 0$; $I = \left]0, \frac{\pi}{2}\right[$

3) $x^{17} = x^{11} + 1$; $I =]1,2[$

4) $\sqrt{x^3 + 5x + 4} = 100$; $I =]21,22[$

5) $\cos x = \frac{2}{(x+1)^2}$; $I =]0,1[$

6) $x^2 \cos x + x \sin x + 1 = 0$; $I =]0, \pi[$

Exercice N° 102 :

Résoudre chacune des équations :

1) (E) : $\sqrt[4]{\frac{2-x}{3+x}} + \sqrt[4]{\frac{3+x}{2-x}} = 2$

2) (E) : $\frac{\sqrt[3]{x+3}}{3} + \sqrt[3]{\frac{3}{x^3} + \frac{1}{x^2}} = \frac{\sqrt[3]{x}}{2}$

3) (E) : $2 \cdot \sqrt[3]{x^4} - \frac{3x}{\sqrt[3]{x}} = 20$

4) (E) : $\sqrt[3]{x^3 - 3x^2 + x + 1} = x - 1$

Exercice N° 103 :

Calculer chacune des limites suivantes :

1) $\lim_{x \rightarrow 0} \left(\frac{\sqrt[3]{x+8} - 2}{x} \right)$

2) $\lim_{x \rightarrow 0^+} \left(\frac{\sqrt[3]{x^2} - x}{x} \right)$

3) $\lim_{x \rightarrow +\infty} \left(\frac{\sqrt[3]{x} - 1}{\sqrt[3]{x^2} + 1} \right)$

4) $\lim_{x \rightarrow 2} \left(\frac{x - \sqrt[3]{x+6}}{3 - \sqrt{2x+5}} \right)$

5) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x} - x \right)$

6) $\lim_{x \rightarrow +\infty} \left(x - \sqrt[3]{x} - \sqrt{x} \right)$

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Exercice N° 104 :

Résoudre dans $\mathbb{R}$ les équations suivantes

- 1) $x^8 - 25 = 0$
- 2) $x^7 = \sqrt{3}$
- 3) $x^4 = 16$
- 4) $x^3 + 8 = 0$
- 5) $\sqrt[3]{x} = \sqrt[6]{7}$
- 6) $(3x - 4)^5 = 32$
- 7) $\sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 4 = 0$
- 8) $9x - 7 \cdot \sqrt[3]{x} - 2 = 0$
- 9) $x^4 - 5x^2 - 24 = 0$

Exercice N° 105 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow 0} \frac{\text{Arctan}(3x)}{x}$
- 2) $\lim_{|x| \rightarrow +\infty} \text{Arctan}(x^4 - x)$
- 3) $\lim_{x \rightarrow +\infty} \left(x \cdot \text{Arctan}(x) - \frac{\pi x}{2} \right)$
- 4) $\lim_{x \rightarrow 0} \frac{\text{Arctan}(x^2 + 4x)}{x}$
- 5) $\lim_{x \rightarrow -\infty} (x^2 + 1) \text{Arctan}\left(\frac{1}{x}\right)$
- 6) $\lim_{x \rightarrow 1^+} \left(\frac{\text{Arctan}\left(\frac{1}{1-x^2}\right) + \frac{\pi}{2}}{x-1} \right)$
- 7) $\lim_{x \rightarrow 1^+} \left(\frac{x - 2\sqrt{\text{Arctan}(x) - \frac{\pi}{4}} - 1}{x-1} \right)$
- 8) $\lim_{x \rightarrow 1^\pm} \left(\frac{\text{Arctan}(\sqrt[3]{x-1})}{x-1} \right)$

Exercice N° 106 :

Simplifier les expressions suivantes :

- 1) $\text{Arctan}\left(\tan\left(\frac{41\pi}{17}\right)\right)$
- 2) $\text{Arctan}\left(\tan\left(5(\text{Arctan}\sqrt{3})\right)\right)$
- 3) $\tan(\text{Arctan}(2016))$
- 4) $\tan(-\text{Arctan}(5))$
- 5) $\text{Arctan}\left(\tan\left(\frac{-79\pi}{3}\right)\right)$
- 6) $\text{Arctan}\left(\frac{1}{\tan\left(\frac{3\pi}{11}\right)}\right)$
- 7) $\tan\left(\text{Arctan}\left(\frac{2}{3}\right) - \text{Arctan}\left(\frac{3}{7}\right)\right)$
- 8) $\tan(2 \text{Arctan}(3))$

Exercice N° 107 :

Calculer les limites suivantes :

- 1) $\lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x+7} - 2}{\sqrt[4]{x} - 1} \right)$
- 2) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1} - 2x \right)$
- 3) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^2 - x} - x - 1 \right)$
- 4) $\lim_{x \rightarrow 4} \left(\frac{\sqrt[3]{5-x} - 1}{2 - \sqrt[3]{x+4}} \right)$
- 5) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1} - x \right)$
- 6) $\lim_{x \rightarrow -\infty} \left(\sqrt[3]{x^4 + 5} + 2x \right)$

Exercice N° 108 :

1) Résoudre les équations suivantes :

$$(E) : \sqrt[3]{x^2} - 3 \cdot \sqrt[3]{x(x-1)} + 2 \cdot \sqrt[3]{(x-1)^2} = 0$$

$$(F) : \sqrt[3]{(1+x)^2} - 4 \cdot \sqrt[3]{(1-x)^2} = 4 \cdot \sqrt[3]{1-x^2}$$

2) Calculer les limites suivantes :

$$\blacksquare \lim_{x \rightarrow +\infty} x^{\frac{3}{2}} \left(\sqrt[4]{x+1} - \sqrt[4]{x-1} \right)$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\sqrt[4]{x^4 + x} - x - 2 \right)$$

3) Étudier selon les valeurs du paramètre réel m la valeur de la limite :

$$\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1} + mx \right)$$

Exercice N° 109 :

1) Résoudre dans $\mathbb{R}$ l'équation suivante :

$$\sqrt{x+1} - \sqrt[3]{x} = 1$$

On pourra poser $t = \sqrt[3]{x}$

2) Soit h une fonction continue sur $[0,2]$ telle que $h(0) = h(2)$. Montrer que l'équation : $h(x+1) = h(x)$ admet au moins une solution dans $[0,1]$

3) Calculer les limites suivantes :

$$\blacksquare \lim_{x \rightarrow +\infty} x \left(\sqrt[3]{1 + \frac{1}{x}} - 1 \right)$$

$$\blacksquare \lim_{x \rightarrow -\infty} \left(\sqrt[4]{x^4 + x^2 + 1} + x - 2 \right)$$

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Exercice N° 110 :

Calculer chacune des limites suivantes :

$$1) \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\tan^2(2x)} \right)$$

$$2) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2 + x + 1} - 1}{4x} \right)$$

$$3) \lim_{x \rightarrow 0} \left(\frac{\cos x - \sqrt{1 + \sin x}}{x} \right)$$

$$4) \lim_{x \rightarrow \frac{2}{\pi}} \left(\frac{\pi x - 2}{1 - \sin\left(\frac{1}{x}\right)} \right)$$

$$5) \lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{1 - 2 \cos x}{\pi - 3x} \right)$$

$$6) \lim_{x \rightarrow 0} \left(\frac{2}{\sin^2 x} - \frac{1}{1 - \cos x} \right)$$

$$7) \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{1 + \sin x} - \cos x}{\sqrt{x}(2x - \pi)} \right)$$

$$8) \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + 3x + 2} + x + 1 \right)$$

$$9) \lim_{x \rightarrow 2} \left(\frac{x^2 + x - 6}{\sqrt{x} - \sqrt{2}} \right)$$

$$10) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{3x^2 + 1} - \sqrt{x^2 + x}}{x} \right)$$

$$11) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+4} - 2}{x - x^2} \right)$$

$$12) \lim_{x \rightarrow +\infty} \left(\sqrt{\frac{x^3}{x+1}} - x \right)$$

5 : Corrigés des Exercices

Solution N° 1 :

1) Montrons que $\lim_{x \rightarrow 0} (x^2 + x) = 0$

c-à-d on montre que :

$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) :$

$$|x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

Avec bien entendu $f(x) = x^2 + x$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que : si $|x| < \alpha$ alors on aurait $|f(x)| < \varepsilon$?

Soit $\varepsilon > 0$; si $|x| < \alpha$ Alors $-\alpha < x < \alpha$

$$\Rightarrow 1 - \alpha < x + 1 < \alpha + 1$$

$$\Rightarrow -1 - \alpha < 1 - \alpha < x + 1 < \alpha + 1$$

$$\Rightarrow -(1 + \alpha) < x + 1 < (\alpha + 1)$$

$$\Rightarrow |x + 1| < \alpha + 1$$

Donc si $|x| < \alpha$ Alors $|x + 1| < \alpha + 1$

Ainsi $|x| \cdot |x + 1| < \alpha(\alpha + 1)$

$$\Rightarrow |f(x)| < (\alpha^2 + \alpha)$$

On aimerait bien avoir $\alpha^2 + \alpha = \varepsilon$

C-à-d $(\alpha^2 + \alpha - \varepsilon) = 0$

$$\Rightarrow \alpha = \frac{-1 \pm \sqrt{1 + 4\varepsilon}}{2} ; \Delta = (1 + 4\varepsilon) > 0$$

$$\text{On prend alors } \alpha = \frac{-1 + \sqrt{1 + 4\varepsilon}}{2} > 0$$

You're not supposed to create new methods or new techniques. Just understand those that already exist. it's not about intelligence it's about hard work. It's about the amount of work per day dudes.

Voici une synthèse de notre travail :

$$|x| < \alpha \Rightarrow |x| < \frac{-1 + \sqrt{1 + 4\varepsilon}}{2}$$

$$\Rightarrow |x + 1| < \frac{1 + \sqrt{1 + 4\varepsilon}}{2}$$

$$\Rightarrow |x| \cdot |x + 1| < \frac{(\sqrt{1 + 4\varepsilon})^2 - 1^2}{4}$$

$$\Rightarrow |f(x)| < \varepsilon$$

$$D'où (\forall \varepsilon > 0) \left(\exists \alpha = \frac{-1 + \sqrt{1 + 4\varepsilon}}{2} \right)$$

$$(\forall x \in \mathbb{R}) : |x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$D'où \lim_{x \rightarrow 0} (x^2 + x) = 0$$

2) Montrons que $\lim_{x \rightarrow 0} \left(\frac{x}{2x + 1} \right) = 0$

C-à-d on montre que :

$(\forall \varepsilon > 0) (\exists \alpha > 0) \left(\forall x \in \mathbb{R} \setminus \left\{ \frac{-1}{2} \right\} \right) :$

$$|x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$\text{Avec bien entendu } f(x) = \frac{x}{2x + 1}$$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que : si $|x| < \alpha$ alors on aurait $|f(x)| < \varepsilon$?

Soit $\varepsilon > 0$ c-à-d un réel qui ressemble à 0,000001 > 0 (infinitement petit)

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$$\text{On a } \frac{x}{2x+1} = \frac{1}{2} \left(\frac{x}{x+\frac{1}{2}} \right) = \frac{1}{2} \left(\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}} \right)$$

$$= \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right)$$

On commence par $|f(x)| < \varepsilon$

$$C - \text{à} - d \quad -\varepsilon < \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < \varepsilon$$

$$\Rightarrow -2\varepsilon < \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < 2\varepsilon$$

$$\Rightarrow -1 - 2\varepsilon < \left(\frac{-\frac{1}{2}}{x+\frac{1}{2}} \right) < 2\varepsilon - 1$$

$$\Rightarrow -2(-1 - 2\varepsilon) < \left(\frac{1}{x+\frac{1}{2}} \right) < -2(2\varepsilon - 1)$$

$$\Rightarrow \frac{1}{2(1+2\varepsilon)} < \left(x + \frac{1}{2} \right) < \frac{1}{2(1-2\varepsilon)}$$

$$\Rightarrow \frac{1}{2(1+2\varepsilon)} - \frac{1}{2} < x < \frac{1}{2(1-2\varepsilon)} - \frac{1}{2}$$

$$\Rightarrow \frac{1}{2(1+2\varepsilon)} - \frac{1}{2} < x < \frac{1}{2} - \frac{1}{2(1+2\varepsilon)}$$

$$\text{Car } \frac{1}{2(1-2\varepsilon)} - \frac{1}{2} < \frac{1}{2} - \frac{1}{2(1+2\varepsilon)}$$

$$\Rightarrow |x| < \frac{1}{2} - \frac{1}{2(1+2\varepsilon)}$$

$$\text{On prend ainsi } \alpha = \frac{1}{2} - \frac{1}{2(1+2\varepsilon)} > 0$$

Récapitulation : pour $\alpha = \frac{\varepsilon}{1+2\varepsilon}$ on ait :

$$|x| < \alpha \quad \Rightarrow \quad |x| < \frac{1}{2} - \frac{1}{2(1+2\varepsilon)}$$

$$\Rightarrow \frac{1}{2(1+2\varepsilon)} - \frac{1}{2} < x < \frac{1}{2} - \frac{1}{2(1+2\varepsilon)}$$

$$\Rightarrow \frac{1}{2(1+2\varepsilon)} < \left(x + \frac{1}{2} \right) < 1 - \frac{1}{2(1+2\varepsilon)}$$

$$\Rightarrow \frac{2(1+2\varepsilon)}{1+4\varepsilon} < \left(\frac{1}{x+\frac{1}{2}} \right) < 2(2\varepsilon+1)$$

$$\Rightarrow -(1+2\varepsilon) < \left(\frac{-\frac{1}{2}}{x+\frac{1}{2}} \right) < \frac{-1}{2} \left(\frac{2(1+2\varepsilon)}{1+4\varepsilon} \right)$$

$$\Rightarrow -2\varepsilon < \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < 1 - \frac{1+2\varepsilon}{1+4\varepsilon}$$

$$\Rightarrow -\varepsilon < \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < \frac{2\varepsilon}{1+4\varepsilon} \cdot \frac{1}{2}$$

$$\Rightarrow -\varepsilon < \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < \frac{\varepsilon}{1+4\varepsilon}$$

$$\Rightarrow -\varepsilon < \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < \frac{\varepsilon}{1+4\varepsilon} < \varepsilon$$

$$\Rightarrow -\varepsilon < \frac{1}{2} \left(1 - \frac{\frac{1}{2}}{x+\frac{1}{2}} \right) < \varepsilon$$

$$\Rightarrow -\varepsilon < f(x) < \varepsilon$$

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$$\Rightarrow |f(x)| < \varepsilon$$

$$D'où (\forall \varepsilon > 0) \left(\exists \alpha = \frac{1}{2} - \frac{1}{2(1+2\varepsilon)} \right)$$

$$\forall x \in \mathbb{R} \setminus \left\{ \frac{-1}{2} \right\} : |x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$C - \text{à} - d \quad \lim_{x \rightarrow 0} \left(\frac{x}{2x+1} \right) = 0$$

$$3) \text{ Montrons que } \lim_{x \rightarrow 0} \left(\frac{3x^2}{x^2+1} \right) = 0$$

C-à-d on montre que :

$$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) : \\ |x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$\text{Avec bien entendu } f(x) = \frac{3x^2}{x^2+1}$$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que : si $|x| < \alpha$ alors on aurait $|f(x)| < \varepsilon$?

Soit $\varepsilon > 0$ c-à-d un réel qui ressemble à $0,000001 > 0$ (infinitement petit).

$$\text{On a } \frac{3x^2}{x^2+1} = 3 \left(1 - \frac{1}{x^2+1} \right)$$

On commence par $|f(x)| < \varepsilon$.

$$\Rightarrow -\varepsilon < 3 \left(1 - \frac{1}{x^2+1} \right) < \varepsilon$$

$$\Rightarrow \frac{-\varepsilon}{3} < \left(1 - \frac{1}{x^2+1} \right) < \frac{\varepsilon}{3}$$

$$\Rightarrow \frac{-\varepsilon}{3} - 1 < \left(\frac{-1}{x^2+1} \right) < \frac{\varepsilon}{3} - 1$$

$$\Rightarrow 1 - \frac{\varepsilon}{3} < \left(\frac{1}{x^2+1} \right) < 1 + \frac{\varepsilon}{3}$$

$$\Rightarrow \frac{3-\varepsilon}{3} < \left(\frac{1}{x^2+1} \right) < \frac{3+\varepsilon}{3}$$

$$\Rightarrow \frac{3}{3+\varepsilon} < x^2+1 < \frac{3}{3-\varepsilon}$$

$$\Rightarrow \frac{3}{3+\varepsilon} - 1 < x^2 < \frac{3}{3-\varepsilon} - 1$$

$$\Rightarrow \frac{-\varepsilon}{3+\varepsilon} < x^2 < \frac{\varepsilon}{3-\varepsilon}$$

$$\Rightarrow 0 < x^2 < \frac{\varepsilon}{3-\varepsilon}$$

$$\Rightarrow 0 < x < \sqrt{\frac{\varepsilon}{3-\varepsilon}}$$

$$\Rightarrow -\sqrt{\frac{\varepsilon}{3-\varepsilon}} < x < \sqrt{\frac{\varepsilon}{3-\varepsilon}}$$

$$\Rightarrow |x| < \sqrt{\frac{\varepsilon}{3-\varepsilon}}$$

$$\text{On prend alors } \alpha = \sqrt{\frac{\varepsilon}{3-\varepsilon}} > 0$$

Récapitulatif :

$$|x| < \alpha \Rightarrow |x| < \sqrt{\frac{\varepsilon}{3-\varepsilon}}$$

$$\Rightarrow |x^2| < \frac{\varepsilon}{3-\varepsilon}$$

$$\Rightarrow \frac{-\varepsilon}{3-\varepsilon} < x^2 < \frac{\varepsilon}{3-\varepsilon}$$

$$\Rightarrow 1 - \frac{\varepsilon}{3-\varepsilon} < x^2+1 < 1 + \frac{\varepsilon}{3-\varepsilon}$$

$$\Rightarrow \frac{3-\varepsilon}{3} < \frac{1}{x^2+1} < \frac{3-\varepsilon}{3-2\varepsilon}$$

$$\Rightarrow \frac{\varepsilon - 3}{3 - 2\varepsilon} < \frac{-1}{x^2 + 1} < \frac{\varepsilon - 3}{3}$$

$$\Rightarrow \frac{-\varepsilon}{3 - 2\varepsilon} < 1 - \frac{1}{x^2 + 1} < \frac{\varepsilon}{3}$$

$$\Rightarrow \frac{-3\varepsilon}{3 - 2\varepsilon} < 3 \left(1 - \frac{1}{x^2 + 1} \right) < \varepsilon$$

$$\Rightarrow -\varepsilon < \frac{-3\varepsilon}{3 - 2\varepsilon} < 3 \left(1 - \frac{1}{x^2 + 1} \right) < \varepsilon$$

$$\Rightarrow -\varepsilon < 3 \left(1 - \frac{1}{x^2 + 1} \right) < \varepsilon$$

$$\Rightarrow -\varepsilon < f(x) < \varepsilon$$

$$\Rightarrow |f(x)| < \varepsilon$$

$$D'où (\forall \varepsilon > 0) \left(\exists \alpha = \sqrt{\frac{\varepsilon}{3 - \varepsilon}} \right) (\forall x \in \mathbb{R}) :$$

$$|x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$C - \text{à} - d : \lim_{x \rightarrow 0} \left(\frac{3x^2}{x^2 + 1} \right) = 0$$

$$4) \text{ Montrons que } \lim_{x \rightarrow 0} (2x + x^2 - x^3) = 0$$

C-à-d on montre que :

$$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) :$$

$$|x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

$$\text{Avec bien entendu } f(x) = 2x + x^2 - x^3$$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que : si $|x| < \alpha$ alors on aurait $|f(x)| < \varepsilon$?

Soit $\varepsilon > 0$ c-à-d un réel qui ressemble à $0,0001 > 0$ (infinitement petit).

$$\text{Si } |x| < \alpha \text{ Alors } -\alpha < x < \alpha$$

$$\Rightarrow 1 - \alpha < x + 1 < \alpha + 1$$

$$\Rightarrow -(\alpha + 1) < 1 - \alpha < x + 1 < \alpha + 1$$

$$\Rightarrow -(\alpha + 1) < x + 1 < (\alpha + 1)$$

$$\Rightarrow |x + 1| < (\alpha + 1) \rightsquigarrow (1)$$

$$\text{Et on a aussi } -\alpha < x < \alpha$$

$$\Rightarrow -2 - \alpha < x - 2 < \alpha - 2$$

$$\Rightarrow -(2 + \alpha) < x - 2 < \alpha - 2 < (2 + \alpha)$$

$$\Rightarrow -(2 + \alpha) < x - 2 < (2 + \alpha)$$

$$\Rightarrow |x - 2| < (2 + \alpha) \rightsquigarrow (2)$$

$$\text{Et on a aussi } |x| < \alpha \rightsquigarrow (0)$$

$$(1) \times (2) \times (0) \Rightarrow$$

$$|x| \cdot |x + 1| \cdot |x - 2| < \alpha(\alpha + 1)(\alpha + 2)$$

$$\Rightarrow |x(x + 1)(x - 2)| < \varphi(\alpha)$$

$$\Rightarrow |-x(x + 1)(x - 2)| < \varphi(\alpha)$$

$$\Rightarrow |f(x)| < \varphi(\alpha)$$

On aimerait bien en avoir $\varphi(\alpha) = \varepsilon$

C'est possible car :

$$\varphi(x) = x^3 + 3x^2 + 2x = \alpha(\alpha + 1)(\alpha + 2)$$

Cette fonction est continue et étant strictement croissante sur $[0, +\infty[$ donc c'est une bijection de $[0, +\infty[$ vers $[0, +\infty[$

Donc : $(\forall y \geq 0) (\exists ! x \geq 0) : \varphi(x) = y$

(pour $\varepsilon > 0$) $(\exists ! \alpha = \varphi^{-1}(\varepsilon) > 0) : \varphi(\varepsilon) = \varepsilon$

Voici un récapitulatif :

$$\text{Soit } \varepsilon > 0 \text{ et } \alpha = \varphi^{-1}(\varepsilon)$$

$$|x| < \alpha \Rightarrow |x| < \varphi^{-1}(\varepsilon) \rightsquigarrow (0)$$

$$\Rightarrow |x + 1| < \varphi^{-1}(\varepsilon) + 1 \text{ selon (1)}$$

$$\Rightarrow |x + 2| < \varphi^{-1}(\varepsilon) + 2 \text{ selon (2)}$$

$$\Rightarrow |x(x + 1)(x + 2)| < \varphi(\varphi^{-1}(\varepsilon))$$

$$\Rightarrow |f(x)| < \varepsilon$$

D'où finalement :

$$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) :$$

$$|x| < \alpha \Rightarrow |f(x)| < \varepsilon$$

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Solution N° 2 :

1) Montrons que $\lim_{x \rightarrow 1} (3x^2 - 5x + 1) = -1$

C-à-d on montre que :

$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) :$

$$|x - 1| < \alpha \Rightarrow |f(x)| < \varepsilon$$

Avec bien entendu $f(x) = 3x^2 - 5x + 2$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que :

Si $|x - 1| < \alpha$ on aurait $|f(x)| < \varepsilon$?

Soit $\varepsilon > 0$ c-à-d un réel qui ressemble à $0,00000001 > 0$ (infiniment petit).

D'abord on a la chose suivante :

$$f(x) + 1 = 3x^2 - 5x + 2 = 3(x - 1)\left(x - \frac{2}{3}\right)$$

$$\begin{aligned} \text{Si } |x - 1| < \alpha \text{ Alors } -\alpha < x - 1 < \alpha \\ \Rightarrow 1 - \alpha < x < \alpha + 1 \\ \Rightarrow |x| < \alpha + 1 \rightsquigarrow (1) \end{aligned}$$

$$\begin{aligned} \text{Si } |x - 1| < \alpha \text{ Alors } -\alpha < x - 1 < \alpha \\ \Rightarrow 1 - \alpha < x < \alpha + 1 \\ \Rightarrow 1 - \alpha - \frac{2}{3} < x - \frac{2}{3} < \alpha + 1 - \frac{2}{3} \\ \Rightarrow \left(\frac{1}{3} - \alpha\right) < \left(x - \frac{2}{3}\right) < \left(\alpha + \frac{1}{3}\right) \\ \Rightarrow \left|x - \frac{2}{3}\right| < \alpha + \frac{1}{3} \rightsquigarrow (2) \end{aligned}$$

Et on a $|x - 1| < \alpha \rightsquigarrow (0)$

$$\begin{aligned} (0) \times (2) \Rightarrow \left|3(x - 1)\left(x - \frac{2}{3}\right)\right| < 3\alpha\left(\alpha + \frac{1}{3}\right) \\ \Rightarrow |f(x) + 1| < 3\alpha\left(\alpha + \frac{1}{3}\right) \end{aligned}$$

Soit $\varphi(\alpha) = 3\alpha\left(\alpha + \frac{1}{3}\right)$

$\varphi :]0, +\infty[\mapsto]0, +\infty[$ est une bijection

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Car φ est continue et strictement ↗

Donc $(\forall y > 0)(\exists! x > 0) : \varphi(x) = y$

(pour $\varepsilon > 0$)($\exists! \alpha = \varphi^{-1}(\varepsilon) > 0$) : $\varphi(\alpha) = \varepsilon$

Voici un récapitulatif :

$$|x - 1| < \alpha \Rightarrow |x - 1| < \varphi^{-1}(\varepsilon) \rightsquigarrow (0)$$

$$\Rightarrow \text{aussi } \left|x - \frac{2}{3}\right| < \varphi^{-1}(\varepsilon) + \frac{1}{3} \rightsquigarrow (2)$$

$$\begin{aligned} \left|3(x - 1)\left(x - \frac{2}{3}\right)\right| < 3\varphi^{-1}(\varepsilon)\left(\varphi^{-1}(\varepsilon) + \frac{1}{3}\right) \\ \Rightarrow |f(x) + 1| < \varphi(\varphi^{-1}(\varepsilon)) \\ \Rightarrow |f(x) + 1| < \varepsilon \end{aligned}$$

D'où : $(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R}) :$

$$|x - 1| < \alpha \Rightarrow |f(x) + 1| < \varepsilon$$

Donc Finalement $\lim_{x \rightarrow 1} f(x) = -1$

2) Montrons que $\lim_{x \rightarrow -1} \left(\frac{2x + 1}{x - 1}\right) = \frac{1}{2}$

C-à-d on montre que :

$(\forall \varepsilon > 0) (\exists \alpha > 0) (\forall x \in \mathbb{R} \setminus \{1\}) :$

$$|x + 1| < \alpha \Rightarrow \left|f(x) - \frac{1}{2}\right| < \varepsilon$$

Avec bien entendu $f(x) = \frac{2x + 1}{x - 1}$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que :

Si $|x + 1| < \alpha$ on aurait $\left|f(x) - \frac{1}{2}\right| < \varepsilon$?

Soit $\varepsilon > 0$ c-à-d un réel qui ressemble à $0,001 > 0$ (infiniment petit).

Soit $\varphi(x) = f(x) - \frac{1}{2} ; \forall x \neq 1$

$$\varphi(x) = f(x) - \frac{1}{2} = \frac{4x + 2 - x + 1}{2(x - 1)}$$

$$= \frac{3x + 3}{2(x - 1)} = \frac{3}{2} \left(\frac{x + 1}{x - 1} \right) = \frac{3}{2} \left(1 + \frac{2}{x - 1} \right)$$

La fonction φ réalise une bijection de l'intervalle $] -\infty, 1[$ vers l'intervalle $] -\infty, \frac{3}{2}[$ car continue et étant strictement $\searrow$.

Donc $\varphi^{-1} :] -\infty, \frac{3}{2}[\mapsto] -\infty, 1[$ existe et étant continue et décroissante aussi

Soit $1 > \varepsilon > 0$

Si $\left| f(x) - \frac{1}{2} \right| < \varepsilon$ Alors $|\varphi(x)| < \varepsilon$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow \varphi^{-1}(\varepsilon) < x < \varphi^{-1}(-\varepsilon) \text{ car } \varphi \searrow$$

$$\Rightarrow \underbrace{\varphi^{-1}(\varepsilon) + 1}_{\text{négatif}} < x + 1 < \underbrace{\varphi^{-1}(-\varepsilon) + 1}_{\text{positif}}$$

Soit $\alpha = \min(-\varphi^{-1}(\varepsilon) - 1 ; \varphi^{-1}(-\varepsilon) + 1)$

Voici un récapitulatif : soit $0 < \varepsilon < 1$

Si $-\varphi^{-1}(\varepsilon) - 1 < \varphi^{-1}(-\varepsilon) + 1 \rightsquigarrow (*)$

Alors $|x + 1| < \alpha \Rightarrow |x + 1| < -\varphi^{-1}(\varepsilon) - 1$

$$\Rightarrow \varphi^{-1}(\varepsilon) + 1 < x + 1 < -\varphi^{-1}(-\varepsilon) - 1$$

$$\Rightarrow \varphi^{-1}(\varepsilon) + 1 < x + 1 < \varphi^{-1}(-\varepsilon) + 1 (*)$$

$$\Rightarrow \varphi^{-1}(\varepsilon) < x < \varphi^{-1}(-\varepsilon)$$

$$\Rightarrow \varphi(\varphi^{-1}(-\varepsilon)) < \varphi(x) < \varphi(\varphi^{-1}(\varepsilon)) ; \varphi \searrow$$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow |\varphi(x)| < \varepsilon$$

$$\Rightarrow \left| f(x) - \frac{1}{2} \right| < \varepsilon$$

Si $-\varphi^{-1}(\varepsilon) - 1 > \varphi^{-1}(-\varepsilon) + 1 \rightsquigarrow (**)$

Alors $|x + 1| < \alpha \Rightarrow |x + 1| < \varphi^{-1}(-\varepsilon) + 1$

$$\Rightarrow -\varphi^{-1}(-\varepsilon) - 1 < x + 1 < \varphi^{-1}(-\varepsilon) + 1$$

$$\Rightarrow \varphi^{-1}(\varepsilon) + 1 < x + 1 < \varphi^{-1}(-\varepsilon) + 1 (**)$$

$$\Rightarrow \varphi^{-1}(\varepsilon) < x < \varphi^{-1}(-\varepsilon)$$

$$\Rightarrow \varphi(\varphi^{-1}(-\varepsilon)) < \varphi(x) < \varphi(\varphi^{-1}(\varepsilon)) ; \varphi \searrow$$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow |\varphi(x)| < \varepsilon$$

$$\Rightarrow \left| f(x) - \frac{1}{2} \right| < \varepsilon$$

D'où finalement : $(\forall \varepsilon > 0) :$

$$(\exists \alpha = \min(-\varphi^{-1}(\varepsilon) - 1 ; \varphi^{-1}(-\varepsilon) + 1))$$

$$(\forall x < 1) ; |x + 1| < \alpha \Rightarrow \left| f(x) - \frac{1}{2} \right| < \varepsilon$$

C-à-d $\lim_{x \rightarrow -1} f(x) = \frac{1}{2}$

3) Montrons que $\lim_{x \rightarrow 2} \sqrt{4x + 1} = 3$

C-à-d on montre que :

$$(\forall \varepsilon > 0) (\exists \alpha > 0) \left(\forall x \geq \frac{-1}{4} \right) :$$

$$|x - 2| < \alpha \Rightarrow \left| \sqrt{4x + 1} - 3 \right| < \varepsilon$$

Avec bien entendu $f(x) = \sqrt{4x + 1}$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $\alpha > 0$ tel que :

Si $|x - 2| < \alpha$ on aurait $|f(x) - 3| < \varepsilon$?

Soit $\varphi(x) = f(x) - 3 = \sqrt{4x + 1} - 3$

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Il est très facile de montrer que φ est une fonction continue et étant strictement croissante. Donc c'est une bijection de $\left] \frac{-1}{4}, +\infty \right[$ vers $] -3, +\infty[$.

Si $|f(x) - 3| < \varepsilon$ Alors $|\varphi(x)| < \varepsilon$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) < \varphi^{-1}(\varphi(x)) < \varphi^{-1}(\varepsilon) ; \varphi \nearrow$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) < x < \varphi^{-1}(\varepsilon)$$

$$\Rightarrow \underbrace{\varphi^{-1}(-\varepsilon) - 2}_{\text{négatif}} < x - 2 < \underbrace{\varphi^{-1}(\varepsilon) - 2}_{\text{positif}}$$

Il suffit de prendre alors :

$$\alpha = \min(\varphi^{-1}(\varepsilon) - 2 ; 2 - \varphi^{-1}(-\varepsilon))$$

Récapitulons : soit $1 > \varepsilon > 0$

Si $\varphi^{-1}(\varepsilon) - 2 < 2 - \varphi^{-1}(-\varepsilon)$

Alors $2 - \varphi^{-1}(\varepsilon) > \varphi^{-1}(-\varepsilon) - 2 \rightsquigarrow (*)$

On a donc $\alpha = \varphi^{-1}(\varepsilon) - 2$

$$|x - 2| < \alpha \Rightarrow |x - 2| < \varphi^{-1}(\varepsilon) - 2$$

$$\Rightarrow 2 - \varphi^{-1}(\varepsilon) < x - 2 < \varphi^{-1}(\varepsilon) - 2$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) - 2 < x - 2 < \varphi^{-1}(\varepsilon) - 2 *$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) < x < \varphi^{-1}(\varepsilon)$$

$$\Rightarrow \varphi(\varphi^{-1}(-\varepsilon)) < \varphi(x) < \varphi(\varphi^{-1}(\varepsilon)) ; \varphi \nearrow$$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow |\varphi(x)| < \varepsilon$$

$$\Rightarrow |f(x) - 3| < \varepsilon$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = 3$$

Si $\varphi^{-1}(\varepsilon) - 2 > 2 - \varphi^{-1}(-\varepsilon) \rightsquigarrow (**)$

On a donc $\alpha = 2 - \varphi^{-1}(-\varepsilon)$

$$|x - 2| < \alpha \Rightarrow |x - 2| < 2 - \varphi^{-1}(-\varepsilon)$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) - 2 < x - 2 < 2 - \varphi^{-1}(-\varepsilon)$$

$$\Rightarrow \varphi^{-1}(-\varepsilon) < x < \varphi^{-1}(\varepsilon)$$

$$\Rightarrow \varphi(\varphi^{-1}(-\varepsilon)) < \varphi(x) < \varphi(\varphi^{-1}(\varepsilon)) ; \varphi \nearrow$$

$$\Rightarrow -\varepsilon < \varphi(x) < \varepsilon$$

$$\Rightarrow |\varphi(x)| < \varepsilon$$

$$\Rightarrow |f(x) - 3| < \varepsilon$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = 3$$

4) Montrons que $\lim_{x \rightarrow 2} \left(-1 + \frac{1}{3\sqrt{x}} \right) = -1$

C-à-d on montre que :

$(\forall \varepsilon > 0) (\exists B > 0) (\forall x \in \mathbb{R}_+^*) :$

$$x > B \Rightarrow |f(x) + 1| < \varepsilon$$

Avec bien entendu $f(x) = -1 + \frac{1}{3\sqrt{x}}$

Autrement-dit : étant donnée un $\varepsilon > 0$ existe-t-il un $B > 0$ tel que :

Si $x > B$ on aurait $|f(x) + 1| < \varepsilon$?

Si $|f(x) + 1| < \varepsilon$ Alors $\left| \frac{1}{3\sqrt{x}} \right| < \varepsilon$

$$\Rightarrow 3|\sqrt{x}| > \frac{1}{\varepsilon} \Rightarrow |\sqrt{x}| > \frac{1}{3\varepsilon}$$

$$\Rightarrow x > \frac{1}{9\varepsilon^2}$$

On prend alors $B = \frac{1}{9\varepsilon^2}$

Voici un récapitulatif : soit $(\varepsilon > 0)$

$$x > B \Rightarrow x > \frac{1}{9\varepsilon^2} \Rightarrow \sqrt{x} > \frac{1}{3\varepsilon}$$

$$\Rightarrow \frac{1}{\sqrt{x}} < 3\varepsilon \Rightarrow \frac{1}{3\sqrt{x}} < \varepsilon$$

$$\Rightarrow 0 < \frac{1}{3\sqrt{x}} < \varepsilon \Rightarrow -\varepsilon < \frac{1}{3\sqrt{x}} < \varepsilon$$

$$\Rightarrow \left| \frac{1}{3\sqrt{x}} \right| < \varepsilon \Rightarrow \left| -1 + \frac{1}{3\sqrt{x}} + 1 \right| < \varepsilon$$

$$\Rightarrow |f(x) + 1| < \varepsilon$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = -1$$

Solution N° 3 :

1) on montre que $\forall x \in \mathbb{R}$ on ait :

$$|f(x) - 1| \leq (x - 1)^2$$

On a $f(x) = \frac{2x}{1+x^2}$

D'abord $|f(x) - 1| = \left| \frac{2x - 1 - x^2}{x^2 + 1} \right|$

$$= \left| \frac{-(x-1)^2}{x^2+1} \right| = \left| \frac{(x-1)^2}{x^2+1} \right| = \left| \frac{1}{x^2+1} \right| \cdot (x-1)^2$$

Or, on a $x^2 + 1 \geq 1 \Rightarrow \frac{1}{x^2 + 1} \leq 1$

$$\Rightarrow 0 < \frac{1}{x^2 + 1} \leq 1$$

$$\Rightarrow \left| \frac{1}{x^2 + 1} \right| \leq 1$$

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$$\Rightarrow \left| \frac{1}{x^2 + 1} \right| \cdot (x-1)^2 \leq (x-1)^2$$

$$\Rightarrow |f(x) - 1| \leq (x-1)^2$$

2) Montrons que $\lim_{x \rightarrow 1} f(x) = 1$

Soit $\varepsilon > 0$; pour que $|f(x) - 1| \leq \varepsilon$

On pourrait choisir le cas où $(x-1)^2 \leq \varepsilon$

Ainsi : $|f(x) - 1| \leq (x-1)^2 < \varepsilon$

$$\Rightarrow \sqrt{(x-1)^2} \leq \sqrt{\varepsilon} \Rightarrow |x-1| \leq \sqrt{\varepsilon}$$

On peut prendre alors $\alpha = \sqrt{\varepsilon} > 0$

Et voici un récapitulatif : soit $\varepsilon > 0$

$$|x-1| \leq \alpha \Rightarrow |x-1| \leq \sqrt{\varepsilon}$$

$$\Rightarrow (x-1)^2 < \varepsilon$$

$$\Rightarrow |f(x) - 1| \leq (x-1)^2 < \varepsilon$$

$$\Rightarrow |f(x) - 1| < \varepsilon$$

Ainsi : $(\forall \varepsilon > 0) (\exists \alpha = \sqrt{\varepsilon}) (\forall x \in \mathbb{R}) :$

$$|x-1| < \alpha \Rightarrow |f(x) - 1| < \varepsilon$$

D'où finalement $\lim_{x \rightarrow 1} f(x) = 1$

Solution N° 4 :

1) Montrons que $(\forall x \in \mathbb{R}^+)$ on ait :

$$\left| g(x) - \frac{1}{2} \right| \leq \frac{1}{2} |x-1|$$

D'abord on a : $\left| g(x) - \frac{1}{2} \right| = \left| \frac{\sqrt{x}}{\sqrt{x}+1} - \frac{1}{2} \right|$

$$= \left| \frac{2\sqrt{x} - \sqrt{x} - 1}{2(\sqrt{x} + 1)} \right| = \frac{1}{2} \left| \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right|$$

$$\text{On a } \sqrt{x} \geq 0 \Rightarrow \sqrt{x} + 1 \geq 1$$

$$\Rightarrow \frac{1}{\sqrt{x} + 1} \leq 1 \Rightarrow 0 < \frac{1}{\sqrt{x} + 1} \leq 1$$

$$\Rightarrow \left| \frac{1}{\sqrt{x} + 1} \right| \leq 1$$

$$\Rightarrow \frac{1}{2} \cdot \left| \frac{1}{\sqrt{x} + 1} \right| \cdot |\sqrt{x} - 1| \leq \frac{1}{2} |\sqrt{x} - 1|$$

$$\Rightarrow \left| g(x) - \frac{1}{2} \right| \leq \frac{1}{2} |\sqrt{x} - 1| \rightsquigarrow (*)$$

$$\text{Or ; } x \geq 1 \Rightarrow \sqrt{x} \geq 1 \Rightarrow x \geq \sqrt{x}$$

$$\Rightarrow x - 1 \geq \sqrt{x} - 1 \geq 0$$

$$\Rightarrow \frac{1}{2} |x - 1| \geq \frac{1}{2} |\sqrt{x} - 1|$$

$$\text{On a aussi } 0 \leq x \leq 1 \Rightarrow \sqrt{x} \leq 1$$

$$\Rightarrow x \leq \sqrt{x}$$

$$\Rightarrow x - 1 \leq \sqrt{x} - 1 \leq 0$$

$$\Rightarrow |x - 1| \leq |\sqrt{x} - 1|$$

$$\Rightarrow \frac{1}{2} |x - 1| \leq \frac{1}{2} |\sqrt{x} - 1|$$

D'où l'on tire la chose suivante :

$$(\forall x \in \mathbb{R}^+) ; \frac{1}{2} |\sqrt{x} - 1| \leq \frac{1}{2} |x - 1| \rightsquigarrow (**)$$

Et d'après les résultats (*) et (**) :

$$(\forall x \in \mathbb{R}^+) ; \left| g(x) - \frac{1}{2} \right| \leq \frac{1}{2} |x - 1|$$

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$$2) \text{ Montrons que : } \lim_{x \rightarrow 1} g(x) = \frac{1}{2}$$

Soient $(\varepsilon > 0)$ et $x \in \mathbb{R}^+$:

$$\text{pour que } \left| g(x) - \frac{1}{2} \right| < \varepsilon$$

$$\text{on peut choisir le cas } \frac{1}{2} |x - 1| < \varepsilon$$

$$C - \text{à} - d \quad |x - 1| < 2\varepsilon$$

On prend ainsi $\alpha = 2\varepsilon$

Et voici un récapitulatif : soit $\varepsilon > 0$

$$|x - 1| < \alpha \Rightarrow |x - 1| < 2\varepsilon$$

$$\Rightarrow \left| g(x) - \frac{1}{2} \right| \leq \frac{1}{2} |x - 1| \leq \frac{1}{2} \cdot 2\varepsilon$$

$$\Rightarrow \left| g(x) - \frac{1}{2} \right| \leq \varepsilon$$

Donc $(\forall \varepsilon > 0) (\exists \alpha = 2\varepsilon > 0) (\forall x \in \mathbb{R}^+) :$

$$|x - 1| < \alpha \Rightarrow \left| f(x) - \frac{1}{2} \right| < \varepsilon$$

Solution N° 5 :

$$1) \text{ on a } \left(\frac{h(x) - 2}{x + 1} \right) = \frac{1}{x + 1} \left(\frac{x - 1}{2x + 1} - \frac{2(2x + 1)}{2x + 1} \right)$$

$$= \frac{1}{x + 1} \left(\frac{-3x - 3}{2x + 1} \right)$$

$$= -3 \left(\frac{1}{x + 1} \right) \left(\frac{x + 1}{2x + 1} \right)$$

$$= \frac{-3}{2x + 1}$$

pour que $|x + 1| < \frac{1}{3}$ On aurait :

$$-\frac{1}{3} < x + 1 < \frac{1}{3}$$

$$\Rightarrow -\frac{4}{3} < x < -\frac{2}{3}$$

$$\Rightarrow -\frac{8}{3} < 2x < -\frac{4}{3}$$

$$\Rightarrow -\frac{5}{3} < 2x + 1 < -\frac{1}{3}$$

$$\Rightarrow \frac{1}{3} < -(2x + 1) < \frac{5}{3}$$

$$\Rightarrow \frac{3}{5} < \frac{-1}{2x + 1} < 3$$

$$\Rightarrow \frac{9}{5} < \frac{-3}{2x + 1} < 9$$

$$\Rightarrow \left| \frac{-3}{2x + 1} \right| < 9$$

$$\Rightarrow \left| \frac{h(x) - 2}{x + 1} \right| = \left| \frac{-3}{2x + 1} \right| < 9$$

$$\Rightarrow \frac{|h(x) - 2|}{|x + 1|} < 9$$

$$\Rightarrow |h(x) - 2| \leq 9 |x + 1|$$

2) Soient $\varepsilon > 0$ et $x \neq \frac{-1}{2}$

Soit $\alpha = \frac{\varepsilon}{9} > 0$; On a $|x + 1| \leq \frac{\varepsilon}{9}$

$$\Rightarrow |h(x) - 2| \leq 9 |x + 1| \leq \frac{9\varepsilon}{9}$$

$$\Rightarrow |h(x) - 2| \leq \varepsilon$$

$$\Rightarrow \lim_{x \rightarrow -1} h(x) = 2$$

Solution N° 6 :

$$1) \text{ On a : } \left(\frac{f(x) - 1}{x} \right) = x^2 - 2x + 2$$

pour que $-1 \leq x \leq 1$ on aurait :

$$\begin{cases} 0 \leq x^2 \leq 1 \\ -2 \leq -2x \leq 2 \end{cases}$$

$$\Rightarrow 0 \leq x^2 - 2x + 2 \leq 5$$

$$\Rightarrow 0 \leq \frac{f(x) - 1}{x} \leq 5$$

$$\Rightarrow \left| \frac{f(x) - 1}{x} \right| \leq 5$$

$$\Rightarrow \frac{|f(x) - 1|}{|x|} \leq 5$$

$$\Rightarrow |f(x) - 1| \leq 5 |x|$$

2) soient $\varepsilon > 0$ et $|x| \leq 1$ et $\alpha = \frac{\varepsilon}{5} > 0$

$$|x| < \alpha \Rightarrow |x| < \frac{\varepsilon}{5} \Rightarrow 5 |x| < \frac{5\varepsilon}{5}$$

$$\Rightarrow |f(x) - 1| \leq 5 |x| < \varepsilon$$

$$\Rightarrow |f(x) - 1| < \varepsilon$$

D'où $(\forall \varepsilon > 0) \left(\exists \alpha = \frac{\varepsilon}{5} > 0 \right) (\forall |x| \leq 1) :$

$$|x| < \alpha \Rightarrow |f(x) - 1| < \varepsilon$$

Donc on a pu montrer la limite suivante

$$\lim_{x \rightarrow 0} f(x) = 1$$

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Solution N° 7 :

$$1) \lim_{x \rightarrow -\infty} \left(\frac{5x^2 + x}{(x-2)^2} \right) = \lim_{x \rightarrow -\infty} \left(\frac{5x^2}{x^2} \right) = 5$$

$$2) \lim_{x \rightarrow 3^+} \left(\frac{2x}{3-x} \right) = \left(\frac{6}{0^-} \right) = -\infty$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{x^{2018}}{x^{2019} + 1} \right) = \lim_{x \rightarrow +\infty} \left(\frac{x^{2018}}{x^{2019}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{x} \right) = 0^+ = 0$$

$$4) \lim_{x \rightarrow +\infty} \left(\frac{3 - \sqrt{x}}{x} \right) = \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x}}{\sqrt{x}} \right) \left(\frac{\frac{3}{\sqrt{x}} - 1}{\sqrt{x}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{\frac{3}{\sqrt{x}} - 1}{\sqrt{x}} \right) = \left(\frac{-1}{+\infty} \right) = 0^- = 0$$

$$5) \lim_{x \rightarrow 1^+} \left(\frac{x}{(1-x)^2} \right) = \frac{1}{(0^-)^2} = \frac{1}{0^+} = +\infty$$

$$6) \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x}}{x^2 + 2} \right) = \left(\frac{1 + 0^+}{+\infty} \right) = 0^+ = 0$$

Solution N° 8 :

$$1) \lim_{x \rightarrow +\infty} (x^2 - x) = \lim_{x \rightarrow +\infty} x^2 = +\infty$$

$$2) \lim_{x \rightarrow -\infty} (2 - x + x^3) = \lim_{x \rightarrow -\infty} x^3 = -\infty$$

$$3) \lim_{x \rightarrow +\infty} (2x^3 + (x^2 - 1)(1 - 3x))$$

$$= \lim_{x \rightarrow -\infty} -x^3 = (-1)(-\infty) = +\infty$$

$$4) \lim_{x \rightarrow -\infty} x^3(1 - 2x)^5 = \lim_{x \rightarrow -\infty} (x^3)(-2x)^5$$

$$= \lim_{x \rightarrow -\infty} (-32)(x^8) = (-32)(x^8) = -\infty$$

$$5) \lim_{x \rightarrow -\infty} (-2x^4 - x^2 + x + 1) = \lim_{x \rightarrow -\infty} -2x^4$$

$$= (-2)(+\infty) = -\infty$$

$$6) \lim_{x \rightarrow +\infty} (1 - 2x^2)(1 + 3x) = \lim_{x \rightarrow -\infty} -6x^3$$

$$= (-6)(+\infty) = -\infty$$

Solution N° 9 :

$$1) \lim_{x \rightarrow 0} \left(\frac{\sin(\pi x)}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{\sin(\pi x)}{\pi x} \right) \left(\frac{\pi}{1} \right) = \pi$$

$$2) \lim_{x \rightarrow 0} \left(\frac{1 - \cos(2x)}{x^2} \right) = \lim_{\substack{t \rightarrow 0 \\ t=2x}} \left(\frac{1 - \cos t}{\left(\frac{t}{2} \right)^2} \right)$$

$$= \lim_{t \rightarrow 0} 4 \left(\frac{1 - \cos t}{t^2} \right) = 4 \times \frac{1}{2} = 2$$

$$3) \lim_{x \rightarrow 0} \left(\frac{\sin^2(x)}{3x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{3} \right) \left(\frac{\sin x}{x} \right)^2 = \frac{1}{3}$$

$$4) \lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{\tan(3x)} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{5x} \right) \left(\frac{1}{\frac{\tan(3x)}{3x}} \right) \left(\frac{5x}{3x} \right) = \frac{5}{3}$$

$$5) \lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{\tan(3x)} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin(7x)}{7x} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) \left(\frac{7x}{x} \right) = 7$$

$$\begin{aligned}
 6) \quad \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{\sin x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{x} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}{x(\sqrt{x+1} + 1)} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{x}{x(\sqrt{x+1} + 1)} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{x+1} + 1} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) = \frac{1}{2}
 \end{aligned}$$

Solution N° 10 :

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} \left(\frac{\sin(x-1)}{2(x^2-x)} \right) \\
 &= \lim_{x \rightarrow 1^+} \left(\frac{\sin(x-1)}{x-1} \right) \left(\frac{1}{2x} \right) \\
 &= \lim_{\substack{t \rightarrow 0^+ \\ t=x-1}} \left(\frac{\sin(t)}{t} \right) \left(\frac{1}{2(t+1)} \right) = \frac{1}{2}
 \end{aligned}$$

$$\blacksquare \quad \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left(\frac{x-1}{|2x-1|-1} \right) = \frac{1}{2}$$

$$\text{car : } \left(\frac{x-1}{|2x-1|-1} \right) = \begin{cases} \frac{1}{2} & ; x \geq \frac{1}{2} \\ \frac{1-x}{2x} & ; x \leq \frac{1}{2} \end{cases}$$

quand $x \rightarrow 1^-$ Alors $x \geq \frac{1}{2}$

Donc f est bien continue en 1

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Solution N° 11 :

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow -1} f(x) &= \lim_{x \rightarrow -1} \left(\frac{x^5 - x^4 + x^3 + 3}{x+1} \right) \\
 &= \lim_{x \rightarrow -1} \frac{(x+1)(x^4 - 2x^3 + 3x^2 - 3x + 3)}{(x+1)} \\
 &= \lim_{x \rightarrow -1} (x^4 - 2x^3 + 3x^2 - 3x + 3) = 12
 \end{aligned}$$

Donc f est continue en -1 .

Solution N° 12 :

$$1) \quad \text{Arctan}(3x) = \frac{\pi}{8}$$

$$\Leftrightarrow \tan(\text{Arctan}(3x)) = \tan\left(\frac{\pi}{8}\right) ; \frac{\pi}{8} \notin \frac{\pi}{2}[\pi]$$

$$\Leftrightarrow 3x = \sqrt{2} - 1 \Leftrightarrow x = \frac{\sqrt{2} - 1}{3}$$

pour calculer $\tan\left(\frac{\pi}{8}\right)$ remarquer que :

$$1 = \tan\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{8} + \frac{\pi}{8}\right) = \frac{\tan\left(\frac{\pi}{8}\right) + \tan\left(\frac{\pi}{8}\right)}{1 - \tan^2\left(\frac{\pi}{8}\right)}$$

$$2) \quad \text{Arctan}(x^2 + 2) = \text{Arctan}(3x)$$

$$\Leftrightarrow \tan(\text{Arctan}(x^2 + 2)) = \tan(\text{Arctan}(3x))$$

$$\Leftrightarrow x^2 + 2 = 3x$$

$$\Leftrightarrow x^2 - 3x + 2 = 0$$

$$\Leftrightarrow x \in \{1; 2\}$$

$$3) \quad \text{Arctan}(x^2 - x) = \frac{3\pi}{4}$$

$$\Leftrightarrow \tan(\text{Arctan}(x^2 - x)) = \tan\left(\frac{3\pi}{4}\right)$$

$$\Leftrightarrow x^2 - x = \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$$

$$\Leftrightarrow x^2 - x = -\tan\left(\frac{\pi}{4}\right) = -1$$

$$\Leftrightarrow x^2 - x + 1 = 0$$

$$\Leftrightarrow x = \left(\frac{1 \pm i\sqrt{3}}{2}\right) \in \mathbb{C}$$

$\Leftrightarrow$ Elle n'admet aucune solution dans $\mathbb{R}$

$$4) \operatorname{Arctan}(x) = \frac{\pi}{4} + 2 \operatorname{Arctan}\left(\frac{1}{4}\right)$$

$$\Leftrightarrow \tan(\operatorname{arctan}(x)) = \tan\left(\frac{\pi}{4} + 2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right)$$

$$\Leftrightarrow x = \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right)}{1 - \tan\left(\frac{\pi}{4}\right) \cdot \tan\left(2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right)}$$

$$\Leftrightarrow x = \frac{1 + \tan\left(2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right)}{1 - \tan\left(2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right)}$$

$$\Leftrightarrow x = \frac{1 + \frac{8}{15}}{1 - \frac{8}{15}} = \frac{23}{7}$$

$$\text{Car } \tan\left(2 \operatorname{Arctan}\left(\frac{1}{4}\right)\right) = \frac{\frac{1}{4} + \frac{1}{4}}{1 - \frac{1}{4} \times \frac{1}{4}} = \frac{8}{15}$$

$$5) \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) = \frac{\pi}{3}$$

$$\Leftrightarrow \frac{x + 2x}{1 - x \cdot 2x} = \sqrt{3}$$

$$\Leftrightarrow \sqrt{3}(1 - 2x^2) = 3x$$

$$\Leftrightarrow 2x^2 + \sqrt{3}x - 1 = 0$$

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$$\Leftrightarrow x = \frac{-\sqrt{3} \pm \sqrt{11}}{4}$$

$$\Leftrightarrow x = \frac{-\sqrt{3} + \sqrt{11}}{4} ; \text{ car } x \geq 0$$

Sinon on aurait : $\operatorname{Arctan}(x) + \operatorname{Arctan}(2x) \leq 0$

$$6) \operatorname{Arctan}(\sqrt{x}) = \frac{-\pi}{4}$$

On a : $\sqrt{x} \geq 0$ Alors $\operatorname{Arctan}(\sqrt{x}) \geq 0$

Ainsi : $\frac{-\pi}{4} \geq 0$ (Absurde)

D'où l'équation n'admet aucune solution

Solution N° 13 :

$$\blacksquare f(x) = \tan\left(\frac{\pi}{2x-1}\right)$$

$$D_f = \left\{ x \in \mathbb{R} ; 2x - 1 \neq 0 \text{ et } \frac{\pi}{2x-1} \neq \frac{\pi}{2}[\pi] \right\}$$

$$= \left\{ x \in \mathbb{R} ; x \neq \frac{1}{2} \text{ et } x \neq \frac{2k+3}{2(2k+1)} ; k \in \mathbb{Z} \right\}$$

$$= \mathbb{R} \setminus \left\{ \frac{1}{2} ; \frac{2k+3}{2(2k+1)} ; k \in \mathbb{Z} \right\}$$

La fonction f est bien évidemment continue sur chaque intervalle de l'ensemble D_f car c'est une composition bien définie de deux fonctions continues. Étudions maintenant la continuité en $\frac{1}{2}$ et en c.

$$\text{Soit } c = \frac{2k+3}{2(2k+1)}$$

$$\lim_{x \rightarrow \left(\frac{1}{2}\right)^{\pm}} f(x) = \lim_{t \rightarrow \pm\infty} \tan(t) = n' \text{ existe pas}$$

$$\text{On a } \lim_{x \rightarrow c^+} f(x) = \lim_{t \rightarrow (\frac{\pi}{2})^+} \tan(t) = +\infty$$

$$\text{Et On a } \lim_{x \rightarrow c^-} f(x) = \lim_{t \rightarrow (\frac{\pi}{2})^-} \tan(t) = -\infty$$

Donc les points où la fonction présente des discontinuités sont $\frac{1}{2}$ et les points c

$$\blacksquare \quad g(x) = \sin\left(\cos\left(\frac{\pi}{x}\right)\right)$$

$$D_g = \{x \in \mathbb{R} ; x \neq 0\} = \mathbb{R}^*$$

La fonction g est bien évidemment continue sur chacun des intervalles $]0, +\infty[$ et $]-\infty, 0[$ de l'ensemble D_g car c'est une composition bien définie de trois fonctions continues. Étudions maintenant la continuité en 0

$$\lim_{x \rightarrow 0^\pm} g(x) = \lim_{t \rightarrow \pm\infty} \sin(\cos t) = n' \text{ existe pas}$$

Donc g n'est pas continue en 0.

$$\blacksquare \quad h(x) = \frac{x^2 - \sqrt{2-x}}{|x+1| - 2}$$

$$D_h = \{x \in \mathbb{R} ; 2 - x \geq 0 \text{ et } |x+1| - 2 \neq 0\}$$

$$= \{x \in \mathbb{R} ; x \leq 2 \text{ et } x+1 \neq \pm 2\}$$

$$= \{x \in \mathbb{R} ; x \leq 2 \text{ et } x \neq 1 \text{ et } x \neq -3\}$$

$$=]-\infty, -3[\cup]-3, 1[\cup]1, 2]$$

$$\text{On a : } h(x) = \begin{cases} \frac{x^2 - \sqrt{2-x}}{x-1} ; & |x > -1 \\ & x \neq 1 \\ \frac{x^2 - \sqrt{2-x}}{-x-3} ; & |x < -1 \\ & x \neq -3 \end{cases}$$

D'abord la continuité sur les intervalles :

$$]-\infty, -3[\text{ et }]-3, 1[\text{ et }]1, 2]$$

La fonction h est trivialement continue sur chacun de ces intervalles comme étant quotient de deux fonctions toutes continues et bien définies (*dénominateur* $\neq 0$). Étudions maintenant la continuité aux bornes de ces intervalles :

$$\begin{aligned} \lim_{x \rightarrow (-3)^+} h(x) &= \lim_{x \rightarrow (-3)^+} \left(\frac{x^2 - \sqrt{2-x}}{-x-3} \right) \\ &= \lim_{x \rightarrow (-3)^+} \left(\frac{-1}{x+3} \right) \left(\frac{x^2 - \sqrt{2-x}}{1} \right) \\ &= \left(\frac{-1}{0^-} \right) \left(\frac{9 - \sqrt{5}}{1} \right) = +\infty \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow (-3)^-} h(x) &= \lim_{x \rightarrow (-3)^-} \left(\frac{x^2 - \sqrt{2-x}}{-x-3} \right) \\ &= \lim_{x \rightarrow (-3)^-} \left(\frac{-1}{x+3} \right) \left(\frac{x^2 - \sqrt{2-x}}{1} \right) \\ &= \left(\frac{-1}{0^+} \right) \left(\frac{9 - \sqrt{5}}{1} \right) = -\infty \end{aligned}$$

Donc h est discontinue en -3 .

$$\begin{aligned} \lim_{x \rightarrow 1^\pm} h(x) &= \lim_{x \rightarrow 1^\pm} \left(\frac{x^2 - \sqrt{2-x}}{x-1} \right) \\ &= \lim_{x \rightarrow 1^\pm} \left(\frac{x^2 - \sqrt{2-x}}{x-1} \right) \left(\frac{x^2 + \sqrt{2-x}}{x^2 + \sqrt{2-x}} \right) \\ &= \lim_{x \rightarrow 1^\pm} \left(\frac{x^4 - 2 + x}{x-1} \right) \left(\frac{1}{x^2 + \sqrt{2-x}} \right) \\ &= \lim_{x \rightarrow 1^\pm} \frac{(x-1)(x^3 + x^2 + x + 2)}{(x-1)} \left(\frac{1}{x^2 + \sqrt{2-x}} \right) \end{aligned}$$

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$$= \lim_{x \rightarrow 1^\pm} \left(\frac{x^3 + x^2 + x + 2}{x^2 + \sqrt{2-x}} \right) = \frac{5}{2}$$

$$\text{Alors : } \tilde{h}(x) = \begin{cases} \frac{x^2 - \sqrt{2-x}}{x-1} ; & | \begin{array}{l} x > -1 \\ x \neq 1 \end{array} \\ \frac{x^2 - \sqrt{2-x}}{-x-3} ; & | \begin{array}{l} x < -1 \\ x \neq -3 \end{array} \\ \frac{5}{2} ; & x = 1 \end{cases}$$

$$\text{ou : } \tilde{h}(x) = \begin{cases} \frac{x^2 - \sqrt{2-x}}{-x-3} ; & | \begin{array}{l} x < -1 \\ x \neq -3 \end{array} \\ \frac{x^2 - \sqrt{2-x}}{x-1} ; & | \begin{array}{l} -1 < x < 2 \\ x \neq 1 \end{array} \\ \frac{5}{2} ; & x = 1 \end{cases}$$

$$\blacksquare \quad k(x) = \frac{1 - \cos(2\pi x)}{x(x-1)}$$

$$\begin{aligned} D_k &= \{x \in \mathbb{R} ; x(x-1) \neq 0\} \\ &= \{x \in \mathbb{R} ; x \neq 0 \text{ et } x \neq 1\} \\ &= \mathbb{R} \setminus \{0, 1\} \\ &=]-\infty, 0[\cup]0, 1[\cup]1, +\infty[\end{aligned}$$

Sur chacun de ces intervalles, la fonction k est continue comme étant quotient de deux fonctions continues et bien définies (*dénominateur est non nul*)

Étudions la continuité de f en 0 et 1 :

$$\begin{aligned} \lim_{x \rightarrow 0^\pm} k(x) &= \lim_{x \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi x)}{x(x-1)} \right) \\ &= \lim_{x \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi x)}{(2\pi x)^2} \right) \left(\frac{(2\pi x)^2}{x(x-1)} \right) \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi x)}{(2\pi x)^2} \right) \left(\frac{x^2}{x^2} \right) \left(\frac{4\pi^2}{1 - \frac{1}{x}} \right) \\ &= \lim_{x \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi x)}{(2\pi x)^2} \right) \left(\frac{4\pi^2}{1 - \frac{1}{x}} \right) \\ &= \left(\frac{1}{2} \right) \left(\frac{4\pi^2}{1 - (\pm\infty)} \right) = 0^\pm = 0 \end{aligned}$$

Donc la fonction f est continue en zéro.

$$\begin{aligned} \lim_{x \rightarrow 1^\pm} k(x) &= \lim_{x \rightarrow 1^\pm} \left(\frac{1 - \cos(2\pi x)}{x(x-1)} \right) \\ &= \lim_{\substack{t \rightarrow 0^\pm \\ t=x-1}} \left(\frac{1 - \cos(2\pi t + 2\pi)}{t(t+1)} \right) \\ &= \lim_{t \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi t)}{t(t+1)} \right) \\ &= \lim_{t \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi t)}{(2\pi t)^2} \right) \left(\frac{(2\pi t)^2}{t(t+1)} \right) \\ &= \lim_{t \rightarrow 0^\pm} \left(\frac{1 - \cos(2\pi t)}{(2\pi t)^2} \right) \left(\frac{4\pi^2}{1 + \frac{1}{t}} \right) \\ &= \left(\frac{1}{2} \right) \left(\frac{4\pi^2}{1 + (\pm\infty)} \right) = 0^\pm = 0 \end{aligned}$$

Donc la fonction k est continue en 1.

$$\text{Alors : } \tilde{k}(x) = \begin{cases} \frac{1 - \cos(2\pi x)}{x(x-1)} ; & | \begin{array}{l} x \neq 0 \\ x \neq 1 \end{array} \\ 0 ; & | \text{ou bien } \begin{array}{l} x = 0 \\ x = 1 \end{array} \end{cases}$$

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$$\blacksquare u(x) = \begin{cases} \frac{1 - \cos \sqrt{|x|}}{|x|} & ; x \neq 0 \\ \frac{1}{2} & ; x = 0 \end{cases} ; D_u = \mathbb{R}$$

La fonction u est continue sur chacun des intervalles de $\mathbb{R}^*$ puisque c'est un quotient de deux fonctions toutes continues et bien définies.

Étudions maintenant la continuité en 0 :

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos \sqrt{|x|}}{|x|} \right) = \lim_{\substack{t \rightarrow 0 \\ t = \sqrt{|x|}}} \left(\frac{1 - \cos t}{t^2} \right) = \frac{1}{2}$$

Donc la fonction u est continue sur l'ensemble $\mathbb{R}$ tout entier.

$$\blacksquare v(x) = \frac{x}{\tan(\pi x)}$$

$$\begin{aligned} D_v &= \left\{ x \in \mathbb{R} ; \tan(\pi x) \neq 0 \text{ et } \pi x \not\equiv \frac{\pi}{2} [\pi] \right\} \\ &= \left\{ x \in \mathbb{R} ; \pi x \equiv 0 [\pi] \text{ et } \pi x \not\equiv \frac{\pi}{2} [\pi] \right\} \\ &= \left\{ x \in \mathbb{R} ; x \neq k \text{ et } x \neq k + \frac{1}{2} ; k \in \mathbb{Z} \right\} \\ &= \mathbb{R} \setminus \left\{ k ; k + \frac{1}{2} ; k \in \mathbb{Z} \right\} \end{aligned}$$

La fonction v est continue sur cet ensemble car v est un quotient de deux fonctions continues et bien définies.

Étudions maintenant la continuité en chaque k et en chaque point $\left(k + \frac{1}{2}\right)$:

$$\lim_{\substack{x \rightarrow k \\ k \in \mathbb{Z}}} v(x) = \lim_{x \rightarrow k} \left(\frac{x}{\tan(\pi x)} \right) = \frac{k}{\tan(k\pi)} = \pm \infty$$

Ainsi la fonction v est clairement discontinue en chaque point k de $\mathbb{Z}^*$

$$\begin{aligned} \lim_{x \rightarrow 0} v(x) &= \lim_{x \rightarrow 0} \left(\frac{x}{\tan(\pi x)} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{\frac{\tan(\pi x)}{x}} \right) \left(\frac{1}{\pi} \right) \\ &= \frac{1}{1} \times \frac{1}{\pi} = \frac{1}{\pi} \end{aligned}$$

Ainsi la fonction v est continue en 0.

$$\begin{aligned} \lim_{\substack{x \rightarrow \left(\frac{1}{2} + k\right)^\pm \\ k \in \mathbb{Z}}} v(x) &= \lim_{x \rightarrow k} \left(\frac{x}{\tan(\pi x)} \right) \\ &= \lim_{x \rightarrow k} \left(\frac{\frac{1}{2} + k}{\tan\left(\frac{\pi}{2} + k\pi\right)} \right) = \left(\frac{\frac{1}{2} + k}{\pm \infty} \right) = 0 \end{aligned}$$

Donc la fonction v est continue en chaque point $(k+1/2)$. Et on redéfinit la fonction $\tilde{v}$ ainsi :

$$\forall x \in \mathbb{R} \setminus \mathbb{Z} ; \tilde{v}(x) = \begin{cases} \frac{x}{\tan(\pi x)} & ; x \neq \left(\frac{1}{2} + k\right) \\ 0 & ; x = \left(\frac{1}{2} + k\right) \\ \frac{1}{\pi} & ; x = 0 \end{cases}$$

Solution N° 14 :

- $\lim_{x \rightarrow -2} (x^2 - 7x - 1) = 17$
- $\lim_{x \rightarrow -1} (x^{2018} - x^{2017} + 2) = 4$
- $\lim_{x \rightarrow 3} \left(\frac{2x^3 - 3x - 9}{x - 1} \right) = 18$
- $\lim_{x \rightarrow \frac{3}{2}} \left(\frac{x^2 + x}{2x - 1} \right) = \frac{15}{8}$
- $\lim_{x \rightarrow \frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2}$
- $\lim_{x \rightarrow \frac{-\pi}{4}} \tan x = -1$

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Solution N° 15 :

$$\blacksquare \lim_{x \rightarrow 3} \left(\frac{-x^2 + x + 6}{x^2 - 4x + 3} \right)$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(-x-2)}{(x-3)(x-1)} = \lim_{x \rightarrow 3} \left(\frac{-x-2}{x-1} \right) = \frac{-5}{2}$$

$$\blacksquare \lim_{x \rightarrow 2} \left(\frac{x^3 - 6x^2 + 11x - 6}{x^5 - 32} \right)$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 - 4x + 3)}{(x-2)(x^4 + 2x^3 + 4x^2 + 8x + 16)}$$

$$= \lim_{x \rightarrow 2} \left(\frac{x^2 - 4x + 3}{x^4 + 2x^3 + 4x^2 + 8x + 16} \right) = \frac{-1}{80}$$

$$\blacksquare \lim_{x \rightarrow -1} \left(\frac{2 - \sqrt{1-3x}}{x^2 - 1} \right)$$

$$= \lim_{x \rightarrow -1} \left(\frac{2 - \sqrt{1-3x}}{x^2 - 1} \right) \left(\frac{2 + \sqrt{1-3x}}{2 + \sqrt{1-3x}} \right)$$

$$= \lim_{x \rightarrow -1} \left(\frac{4 - (1-3x)}{x^2 - 1} \right) \left(\frac{1}{2 + \sqrt{1-3x}} \right)$$

$$= \lim_{x \rightarrow -1} \left(\frac{x+1}{(x-1)(x+1)} \right) \left(\frac{3}{2 + \sqrt{1-3x}} \right)$$

$$= \lim_{x \rightarrow -1} \left(\frac{1}{x-1} \right) \left(\frac{3}{2 + \sqrt{1-3x}} \right)$$

$$= \left(\frac{1}{-1-1} \right) \left(\frac{3}{2 + \sqrt{1+3}} \right) = \frac{-3}{8}$$

$$\blacksquare \lim_{x \rightarrow 3^+} \left(\frac{\sqrt{x^2-9}}{x-3} \right) = \lim_{x \rightarrow 3^+} \frac{(\sqrt{x-3})(\sqrt{x+3})}{(\sqrt{x-3})(\sqrt{x-3})}$$

$$= \lim_{x \rightarrow 3^+} \sqrt{\frac{x+3}{x-3}} = \sqrt{\frac{6}{0^+}} = +\infty$$

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$$\blacksquare \lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x^2-x}}{x} \right) = \lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x^2} \times \sqrt{1-\frac{1}{x}}}{x} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{|x| \times \sqrt{1-\frac{1}{x}}}{x} \right) = \lim_{x \rightarrow 0^-} \left(\frac{-x \sqrt{1-\frac{1}{x}}}{x} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(-\sqrt{1-\frac{1}{x}} \right) = -\sqrt{1-\frac{1}{0^-}} = -\infty$$

$$\blacksquare \lim_{x \rightarrow 1^-} \left(\frac{\sqrt{1-\sqrt{x}}}{x-1} \right) = \lim_{x \rightarrow 1^-} \frac{-\sqrt{1-\sqrt{x}}}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \lim_{x \rightarrow 1^-} \frac{-\sqrt{1-\sqrt{x}}}{(\sqrt{1-\sqrt{x}})(\sqrt{1-\sqrt{x}})(1+\sqrt{x})}$$

$$= \lim_{x \rightarrow 1^-} \left(\sqrt{\frac{1}{1-\sqrt{x}}} \right) \left(\frac{-1}{1+\sqrt{x}} \right)$$

$$= \left(\sqrt{\frac{1}{1-1^-}} \right) \left(\frac{-1}{1+\sqrt{1}} \right)$$

$$= \left(\sqrt{\frac{1}{0^+}} \right) \left(\frac{-1}{2} \right) = -\infty$$

Solution N° 16 :

$$\blacksquare \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \left(\frac{2x^2 - x + 1}{x - 2} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{2x^2}{x} \right) = \lim_{x \rightarrow +\infty} (2x) = +\infty$$

$$\blacksquare \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \left(\frac{x^3 - 1}{x^2 - 1} \right) = \lim_{x \rightarrow -\infty} \left(\frac{x^3}{x^2} \right)$$

$$= \lim_{x \rightarrow -\infty} x = -\infty$$

$$\blacksquare \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \left(\frac{2x^2 - x + 1}{x - 2} \right) = -2$$

$$\blacksquare \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left(\frac{x^3 - 1}{x^2 - 1} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{x - 1}{x - 1} \right) \left(\frac{x^2 + x + 1}{x + 1} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{x^2 + x + 1}{x + 1} \right) = \frac{3}{2}$$

$$\blacksquare \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \left(\frac{2x^2 - x + 1}{x - 2} \right) = \frac{7}{0^+}$$

$$= +\infty$$

$$\blacksquare \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \left(\frac{2x^2 - x + 1}{x - 2} \right) = \frac{7}{0^-}$$

$$= -\infty$$

Solution N° 17 :

$$\blacksquare f(x) = \begin{cases} 2x - 3 & ; x < 2 \\ x^2 - 3 & ; x \geq 2 \end{cases}$$

Sur chacun des intervalles $]-\infty, 2[$ et $[2, +\infty[$ la fonction f est continue car c'est un polynôme. Etudions la continuité de la fonction f en 2 :

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^2 - 3) = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2x - 3) = 1$$

Donc la fonction f est continue en 2. Finalement on conclut que la fonction f est continue sur $\mathbb{R}$ tout entier.

$$f([-2, 4]) = [f(-2); f(4)] = [-7; 13]$$

$$f(]-\infty, 1]) = \left] \lim_{x \rightarrow -\infty} f(x); f(1) \right] =]-\infty, 1]$$

Solution N° 18 :

$$\blacksquare \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{x} - \sqrt{1 + x^2}}{x + 1} \right) = -1$$

$$\blacksquare \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left(\frac{\cos x - \sqrt{1 + \sin x}}{x} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{\cos x - \sqrt{1 + \sin x}}{x} \right) \left(\frac{\cos x + \sqrt{1 + \sin x}}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{\cos^2(x) - (1 + \sin x)}{x} \right) \left(\frac{1}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{1 - \sin^2 x - (1 + \sin x)}{x} \right) \left(\frac{1}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{-\sin^2 x - \sin x}{x} \right) \left(\frac{1}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\frac{\sin^2 x}{x} + \frac{\sin x}{x} \right) \left(\frac{-1}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$= \lim_{x \rightarrow 0^-} \left(\sin x \cdot \left(\frac{\sin x}{x} \right) + \frac{\sin x}{x} \right) \left(\frac{-1}{\cos x + \sqrt{1 + \sin x}} \right)$$

$$(\sin 0 \times (1) + 1) \left(\frac{-1}{\cos 0 + \sqrt{1 + \sin 0}} \right) = \frac{-1}{2}$$

Donc on en déduit que la fonction f n'est pas du tout continue en zéro.

$$\blacksquare \lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} \left(\frac{\sin x - \tan x}{\sqrt{x}} \right)$$

$$= \lim_{x \rightarrow 0^+} \left(\sqrt{x} \left(\frac{\sin x}{x} \right) - \sqrt{x} \left(\frac{\tan x}{x} \right) \right)$$

$$= (\sqrt{0} \times 1 - \sqrt{0} \times 1) = 0$$

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$$\blacksquare \lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} \left(x \cdot \sin\left(\frac{1}{x}\right) \right) = \lim_{\substack{t \rightarrow -\infty \\ t = \frac{1}{x}}} \frac{\sin t}{t}$$

$$= 0 ; \text{ car } \left| \frac{\sin t}{t} \right| \leq \frac{1}{t} \rightarrow 0$$

Donc on en déduit que la fonction f est continue en zéro.

Solution N° 19 :

$$\blacksquare \lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^-} \left(\frac{x^3 - 1}{\sqrt[3]{x} + x - 2} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{x - 1}{\sqrt[3]{x} + x - 2} \right) \left(\frac{x^2 + x + 1}{1} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{x - 1}{(\sqrt[3]{x} - 1) + (x - 1)} \right) \left(\frac{x^2 + x + 1}{1} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{(x - 1)}{(x - 1) \left(\left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) + 1 \right)} \right) \left(\frac{x^2 + x + 1}{1} \right)$$

$$= \lim_{x \rightarrow 1^-} \left(\frac{1}{\left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) + 1} \right) \left(\frac{x^2 + x + 1}{1} \right)$$

$$= \left(\frac{1}{\frac{1}{3} + 1} \right) \left(\frac{1^2 + 1 + 1}{1} \right) = \frac{9}{4}$$

$$\text{Car : } \lim_{x \rightarrow 1^+} \left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) \left(\frac{\sqrt[3]{x^2} + \sqrt[3]{x} + 1}{\sqrt[3]{x^2} + \sqrt[3]{x} + 1} \right)$$

$$= \lim_{x \rightarrow 1^+} \frac{(x - 1)}{(x - 1)(\sqrt[3]{x^2} + \sqrt[3]{x} + 1)}$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{1}{\sqrt[3]{x^2} + \sqrt[3]{x} + 1} \right)$$

$$= \left(\frac{1}{\sqrt[3]{1^2} + \sqrt[3]{1} + 1} \right) = \frac{1}{3}$$

Ou Encore : $\lim_{x \rightarrow 1^+} \left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) = \left(x^{\frac{1}{3}} \right)'_{x=1} = \frac{1}{3}$

$$\blacksquare \lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^+} \left(\frac{ax^2 - ax}{x^2 - 5x + 4} \right)$$

$$= \lim_{x \rightarrow 1^+} \frac{(ax)(x - 1)}{(x - 1)(x - 4)} = \lim_{x \rightarrow 1^+} \left(\frac{ax}{x - 4} \right) = \frac{-a}{3}$$

Pour que la fonction f soit continue en 1, il suffirait de vérifier la condition :

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$$

$$C - \text{à} - d \quad \frac{9}{4} = \frac{-a}{3} \quad \text{donc} \quad a = \frac{-27}{4}$$

Solution N° 20 :

$$\blacksquare \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{\sqrt{3 + \cos x} - 2}{x^2} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sqrt{3 + \cos x} - 2}{x^2} \right) \left(\frac{\sqrt{3 + \cos x} + 2}{\sqrt{3 + \cos x} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{3 + \cos x - 4}{x^2} \right) \left(\frac{1}{\sqrt{3 + \cos x} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{-1}{\sqrt{3 + \cos x} + 2} \right)$$

$$= \left(\frac{1}{2} \right) \left(\frac{-1}{\sqrt{3 + \cos 0} + 2} \right) = \frac{-1}{8} = f(0)$$

Donc la fonction f est continue en zéro

Solution N° 21 :

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \left(\frac{\cos^3 x - 1}{\sin^2 x} \right) \\
 &= \lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos^2 x + \cos x + 1)}{1 - \cos^2 x} \\
 &= \lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos^2 x + \cos x + 1)}{(1 - \cos x)(1 + \cos x)} \\
 &= \lim_{x \rightarrow 0} - \left(\frac{\cos^2 x + \cos x + 1}{1 + \cos x} \right) = \frac{-3}{2}
 \end{aligned}$$

Donc pour que la fonction f soit continue en zéro, il suffirait de prendre.

$$a = \frac{-3}{2}$$

Solution N° 22 :

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow \frac{\pi}{2}} f(x) &= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{\sin x} - 1}{x - \frac{\pi}{2}} \right) \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{\sin x} - 1}{x - \frac{\pi}{2}} \right) \left(\frac{\sqrt{\sin x} + 1}{\sqrt{\sin x} + 1} \right) \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin x - 1}{x - \frac{\pi}{2}} \right) \left(\frac{1}{\sqrt{\sin x} + 1} \right) \\
 &= \lim_{\substack{t \rightarrow 0 \\ t = x - \frac{\pi}{2}}} \left(\frac{\sin \left(t + \frac{\pi}{2} \right) - 1}{t} \right) \left(\frac{1}{\sqrt{\sin \left(t + \frac{\pi}{2} \right)} + 1} \right) \\
 &= \lim_{t \rightarrow 0} \left(\frac{\cos t - 1}{t} \right) \left(\frac{1}{\sqrt{\cos t} + 1} \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{t \rightarrow 0} \left(\frac{\cos t - 1}{t} \right) \left(\frac{1}{\sqrt{\cos t} + 1} \right) \left(\frac{t}{t} \right) \\
 &= \lim_{t \rightarrow 0} \left(\frac{1 - \cos t}{t^2} \right) \left(\frac{-t}{\sqrt{\cos t} + 1} \right) \\
 &= \left(\frac{1}{2} \right) \left(\frac{-0}{\sqrt{\cos 0} + 1} \right) = 0
 \end{aligned}$$

Donc pour que la fonction f soit continue en zéro, il suffirait de prendre pour a la valeur 0.

Solution N° 23 :

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow 0} g(x) &= \lim_{x \rightarrow 0} \left(\frac{x^2 + \sqrt{x+a} - \sqrt{a}}{x} \right) \\
 &= \lim_{x \rightarrow 0} \left(x + \frac{\sqrt{x+a} - \sqrt{a}}{x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+a} - \sqrt{a}}{x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+a} - \sqrt{a}}{x} \right) \left(\frac{\sqrt{x+a} + \sqrt{a}}{\sqrt{x+a} + \sqrt{a}} \right) \\
 &= \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{x + a - a}{\sqrt{x+a} + \sqrt{a}} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{x+a} + \sqrt{a}} \right) \\
 &= \left(\frac{1}{\sqrt{0+a} + \sqrt{a}} \right) = \frac{1}{2\sqrt{a}} = f(0)
 \end{aligned}$$

On peut suivre une deuxième méthode pour le calcul de cette limite en utilisant la technique du nombre dérivé.

$$\begin{aligned}
 \blacksquare \quad \lim_{x \rightarrow +\infty} g(x) &= \lim_{x \rightarrow +\infty} \left(\frac{x^2 + \sqrt{x+a} - \sqrt{a}}{x} \right) \\
 &= \lim_{x \rightarrow +\infty} \left(x + \frac{\sqrt{x+a} - \sqrt{a}}{x} \right)
 \end{aligned}$$

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$$= \lim_{x \rightarrow +\infty} \left(x + \sqrt{\frac{1}{x} + \frac{a}{x^2}} - \sqrt{\frac{a}{x^2}} \right)$$

$$= \left(+\infty + \sqrt{0^+ + 0^+} - \sqrt{0^+} \right) = +\infty$$

Solution N° 24 :

$$\begin{aligned} \blacksquare \quad \lim_{x \rightarrow 0} g(x) &= \lim_{x \rightarrow 0} \left(\frac{x + \tan(2x)}{\sin(3x)} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{x}{3x} \right) \left(\frac{1 + \frac{2 \tan(2x)}{2x}}{\frac{\sin(3x)}{3x}} \right) \\ &= \left(\frac{1}{3} \right) \left(\frac{1 + 2 \times 1}{1} \right) = 1 = f(0) \end{aligned}$$

Donc g est bien continue en zéro.

Solution N° 25 :

$$\blacksquare \quad \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left(\frac{x^2 - 3}{2x - 1} \right) = 3$$

$$\blacksquare \quad \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (3 - x^2) = 3$$

Donc f est continue en zéro.

Solution N° 26 :

$$\begin{aligned} \blacksquare \quad \lim_{x \rightarrow 1^\pm} g(x) &= \lim_{x \rightarrow 1^\pm} \left(\frac{x^2 - 1}{|x - 1|} \right) \\ &= \lim_{x \rightarrow 1^\pm} \frac{(x - 1)(x + 1)}{|x - 1|} = \lim_{x \rightarrow 1^\pm} \pm(x + 1) = \pm 2 \end{aligned}$$

Ainsi la fonction g n'est pas continue en 1 car les limites à droite et à gauche en 1 sont différentes.

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Solution N° 27 :

$$\blacksquare \quad f(x) = \frac{|x^2 - 5| - 4}{\sqrt{x} - 1} \quad ; \quad a = 1$$

On peut réécrire la fonction f ainsi :

$$f(x) = \begin{cases} \frac{x^2 - 9}{\sqrt{x} - 1} & ; \quad x \geq \sqrt{5} \\ \frac{-x^2 + 1}{\sqrt{x} - 1} & ; \quad x \leq \sqrt{5} \end{cases}$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \left(\frac{-x^2 + 1}{\sqrt{x} - 1} \right)$$

$$= \lim_{x \rightarrow 1^+} \frac{-(\sqrt{x} - 1)(\sqrt{x} + 1)}{(\sqrt{x} - 1)} = -2$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left(\frac{-x^2 + 1}{\sqrt{x} - 1} \right) = -2$$

Ainsi la fonction f est continue en $a = 1$

$$\blacksquare \quad f(x) = \frac{|x^2 - 5| - 4}{\sqrt{x} - 1} \quad ; \quad a = \sqrt{5}$$

On peut réécrire la fonction f ainsi :

$$f(x) = \begin{cases} \frac{x^2 - 9}{\sqrt{x} - 1} & ; \quad x \geq \sqrt{5} \\ \frac{-x^2 + 1}{\sqrt{x} - 1} & ; \quad x \leq \sqrt{5} \end{cases}$$

$$\lim_{x \rightarrow \sqrt{5}^+} f(x) = \lim_{x \rightarrow \sqrt{5}^+} \left(\frac{x^2 - 9}{\sqrt{x} - 1} \right) = \frac{-4}{\sqrt{5} - 1}$$

$$\lim_{x \rightarrow \sqrt{5}^-} f(x) = \lim_{x \rightarrow \sqrt{5}^-} \left(\frac{-x^2 + 1}{\sqrt{x} - 1} \right) = \frac{-4}{\sqrt{5} - 1}$$

Remarque : On peut utiliser l'expression de f sans distinguer les cas à droite ou à gauche :

$$\lim_{x \rightarrow \sqrt{5}} f(x) = \lim_{x \rightarrow \sqrt{5}} \left(\frac{|x^2 - 5| - 4}{\sqrt{5} - 1} \right) = \frac{-4}{\sqrt{5} - 1}$$

D'où f est continue en $\sqrt{5}$.

$$\blacksquare \quad f(x) = \begin{cases} \frac{|x^2 - 5| - 4}{\sqrt{x} - 1} & ; \quad x \neq 2 \\ 18 & ; \quad x = 2 \end{cases} ; \quad a = 2$$

$$\begin{aligned} \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{\sqrt{x^2 + 5} - 3} \right) \\ &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)(\sqrt{x^2 + 5} + 3)}{(\sqrt{x^2 + 5} - 3)(\sqrt{x^2 + 5} + 3)} \\ &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)(\sqrt{x^2 + 5} + 3)}{(x^2 + 5) - 9} \\ &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)(\sqrt{x^2 + 5} + 3)}{x^2 - 4} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x^2 + 2x + 4)(\sqrt{x^2 + 5} + 3)}{(x - 2)(x + 2)} \\ &= \lim_{x \rightarrow 2} \frac{(x^2 + 2x + 4)(\sqrt{x^2 + 5} + 3)}{(x + 2)} \\ &= \frac{(2^2 + 4 + 4)(\sqrt{2^2 + 5} + 3)}{(2 + 2)} = 18 = f(2) \end{aligned}$$

Ainsi la fonction f est continue en $a = 2$

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Solution N° 28 :

$$\blacksquare \quad g(x) = \begin{cases} (x^2 - 9) \cdot \sin\left(\frac{1}{x-3}\right) & ; \quad x \neq 3 \\ 0 & ; \quad x = 3 \end{cases}$$

$$\lim_{x \rightarrow 3^\pm} g(x) = \lim_{x \rightarrow 3^\pm} \left((x^2 - 9) \cdot \sin\left(\frac{1}{x-3}\right) \right)$$

$$= \lim_{x \rightarrow 3^\pm} \left(\frac{\sin\left(\frac{1}{x-3}\right)}{\left(\frac{1}{x-3}\right)} \right)$$

$$= \lim_{\substack{t \rightarrow 0^\pm \\ t = \frac{1}{x-3}}} \left(\frac{\sin t}{t} \right) \times t = 1 \times 0 = 0 = g(3)$$

Donc la fonction g est continue en $a = 3$

$$\blacksquare \quad g(x) = \begin{cases} \frac{(1 - \tan x)^2}{1 + \cos(4x)} & ; \quad x \neq \frac{\pi}{4} \\ \frac{1}{2} & ; \quad x = \frac{\pi}{4} \end{cases}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} g(x) = \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{(1 - \tan x)^2}{1 + \cos(4x)} \right)$$

$$= \lim_{\substack{t \rightarrow 0 \\ t = (x - \frac{\pi}{4})}} \left(\frac{\left(1 - \tan\left(t + \frac{\pi}{4}\right)\right)^2}{1 + \cos(4t + \pi)} \right)$$

$$= \lim_{t \rightarrow 0} \left(\frac{\left(1 - \frac{1 + \tan t}{1 - \tan t}\right)^2}{1 - \cos(4t)} \right)$$

$$= \lim_{t \rightarrow 0} \left(\frac{-2 \tan t}{1 - \tan t} \right)^2 \left(\frac{1}{1 - \cos(4t)} \right)$$

$$= \lim_{t \rightarrow 0} \left(\frac{-2 \tan t}{1 - \tan t} \right)^2 \left(\frac{1}{1 - \cos(4t)} \right) \left(\frac{16t^2}{16t^2} \right)$$

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$$= \lim_{t \rightarrow 0} \left(\frac{1}{1 - \tan t} \right)^2 \left(\frac{1}{\frac{(1 - \cos(4t))}{(4t)^2}} \right) \frac{1}{4} \left(\frac{\tan t}{t} \right)^2$$

$$= \left(\frac{1}{1 - \tan 0} \right)^2 \times \left(\frac{1}{\left(\frac{1}{2} \right)} \right) \times \frac{1}{4} \times (1)^2 = \frac{1}{2} = g\left(\frac{\pi}{4}\right)$$

Donc la fonction g est continue en $\frac{\pi}{4}$

Solution N° 29 :

$$1) \lim_{\substack{x \rightarrow 1 \\ x > 1}} \frac{|x - 1|x}{x^2 - 1} = \lim_{\substack{x \rightarrow 1 \\ x > 1}} \frac{(x - 1)x}{x^2 - 1}$$

$$= \lim_{\substack{x \rightarrow 1 \\ x > 1}} \left(\frac{x}{x + 1} \right) = \frac{1}{2}$$

$$\lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{|x - 1|x}{x^2 - 1} = \lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{-(x - 1)x}{x^2 - 1}$$

$$= \lim_{\substack{x \rightarrow 1 \\ x < 1}} \left(\frac{-x}{x + 1} \right) = \frac{-1}{2}$$

$$\text{Donc } \lim_{x \rightarrow 1} \frac{|x - 1|x}{x^2 - 1} = n' \text{ existe pas}$$

C'est-à-dire que la fonction n'est pas continue en 1.

2) On remplace directement x par -1 parce que l'image de -1 est bien définie

$$\lim_{x \rightarrow -1} \frac{(x + 1)^2}{|x^2 - 1|} = \frac{(-1 + 1)^2}{|(-1)^2 - 1|} = 0$$

Donc f est continue en -1

$$3) \lim_{\substack{x \rightarrow \pi \\ x > \pi}} f(x) = \lim_{\substack{x \rightarrow \pi \\ x > \pi}} \sin x = 0$$

$$\text{Et } \lim_{\substack{x \rightarrow \pi \\ x < \pi}} f(x) = \lim_{\substack{x \rightarrow \pi \\ x < \pi}} \cos x = -1$$

$$\text{Comme } \lim_{x \rightarrow \pi^+} f(x) \neq \lim_{x \rightarrow \pi^-} f(x)$$

Alors f n'est pas continue en π

$$4) \lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x) = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{x^2 - 2x}{x + 2} \right) = 0$$

$$\text{Et } \lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x) = \lim_{\substack{x \rightarrow 0 \\ x < 0}} E(x) = 0$$

$$\text{Comme } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = 0 = f(0)$$

Donc f est bien continue en 0

Solution N° 30 :

$$1) \lim_{x \rightarrow 0} \frac{\sqrt{1 + x^2}}{x^2} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2 \left(1 + \frac{1}{x^2} \right)}}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{|x| \sqrt{\left(1 + \frac{1}{x^2} \right)}}{x^2}$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{\sqrt{1 + x^2}}{x^2} = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{1}{x} \sqrt{1 + \frac{1}{x^2}} = +\infty$$

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{\sqrt{1 + x^2}}{x^2} = \lim_{\substack{x \rightarrow 0 \\ x < 0}} \frac{-1}{x} \sqrt{1 + \frac{1}{x^2}} = +\infty$$

$$2) \lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{x^2 + 1} \right) = \frac{1}{0^-} - 1 = -\infty$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{1}{x} + 2 + \sin \frac{1}{x} \right) = 0 + 2 + \sin 0 = 2$$

$$\begin{aligned}
 4) \quad \lim_{x \rightarrow 0^-} \frac{\sqrt{1-x+x^2}}{x^3} &= \lim_{x \rightarrow 0^-} \sqrt{1-x+x^2} \times \lim_{x \rightarrow 0^-} \left(\frac{1}{x^3}\right) \\
 &= 1 \times \left(\frac{1}{0^-}\right)^3 = 1 \times (-\infty) = -\infty
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \lim_{x \rightarrow 0^+} \left(\frac{x^2+1}{\sqrt{x}}\right) &= \lim_{x \rightarrow 0^+} (x^2+1) \times \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{x}} \\
 &= 1 \times \frac{1}{\sqrt{0^+}} = 1 \times (+\infty) = +\infty
 \end{aligned}$$

$$6) \quad \lim_{x \rightarrow 0^-} \left(\frac{2}{x} - 1 + \cos\left(\frac{2}{x}\right)\right)$$

$$\left|\cos\left(\frac{2}{x}\right)\right| < 1 \Rightarrow -1 < \cos\left(\frac{2}{x}\right) < 1$$

$$\Rightarrow -2 < \cos\left(\frac{2}{x}\right) - 1 < 0$$

$$\Rightarrow -1 + \cos\left(\frac{2}{x}\right) < 0$$

$$\Rightarrow \left(\frac{2}{x} - 1 + \cos\left(\frac{2}{x}\right)\right) < \frac{2}{x} \xrightarrow[x \rightarrow 0^-]{\omega} -\infty$$

$$\Rightarrow \lim_{x \rightarrow 0^-} \left(\frac{2}{x} - 1 + \cos\left(\frac{2}{x}\right)\right) = -\infty$$

Solution N° 31 :

$$1) \quad \lim_{x \rightarrow -1} \frac{3}{(x+1)^2} = \lim_{\substack{t \rightarrow 0^\pm \\ t=x+1}} \left(\frac{3}{t^2}\right) = \frac{3}{(0^\pm)^2} = +\infty$$

$$2) \quad \lim_{x \rightarrow -1} \left(\frac{\tan^2 x + 1}{(x+1)^2}\right) = \lim_{\substack{t \rightarrow 0 \\ t=x+1}} \frac{\tan^2(t-1) + 1}{t^2}$$

$$\begin{aligned}
 &= (\tan^2(-1)) \times \frac{1}{(0^\pm)^2} = \text{positif} \times (+\infty) \\
 &= +\infty
 \end{aligned}$$

$$3) \quad \lim_{x \rightarrow 1^+} \frac{x^3 - 1}{(x-1)^3} = \lim_{x \rightarrow 1^+} \frac{(x-1)(x^2+x+1)}{(x-1)^3}$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{x^2+x+1}{(x-1)^2}\right)$$

$$= \lim_{x \rightarrow 1^+} (x^2+x+1) \times \lim_{x \rightarrow 1^+} \frac{1}{(x-1)^2}$$

$$= 3 \times \lim_{\substack{t \rightarrow 0^- \\ t=x-1}} \left(\frac{1}{t^2}\right)$$

$$= 3 \times \left(\frac{1}{0^-}\right)^2 = 3 \times (+\infty) = +\infty$$

$$4) \quad \lim_{x \rightarrow 2^-} \left(\frac{1}{x-2} - \left|\sin\left(\frac{2}{(x-2)^2}\right)\right|\right)$$

$$= \lim_{\substack{t \rightarrow 0^- \\ t=x-2}} \left(\frac{1}{t} - \left|\sin\left(\frac{2}{t^2}\right)\right|\right)$$

$$\text{On a } -\left|\sin\left(\frac{2}{t^2}\right)\right| < 0$$

$$\text{Alors } \frac{1}{t} - \left|\sin\left(\frac{2}{t^2}\right)\right| < \frac{1}{t} \xrightarrow[t \rightarrow 0^-]{\omega} -\infty$$

$$D'où \quad \lim_{t \rightarrow 0^-} \left(\frac{1}{t} - \left|\sin\left(\frac{2}{t^2}\right)\right|\right) = -\infty$$

$$5) \quad \lim_{x \rightarrow 1^-} \left(1 + \frac{1}{\sqrt{x}}\right) \left(\frac{1}{(x-1)^{2019}}\right)$$

$$= \lim_{x \rightarrow 1^-} \left(1 + \frac{1}{\sqrt{x}}\right) \times \lim_{x \rightarrow 1^-} \frac{1}{(x-1)^{2019}}$$

$$= 2 \times \lim_{\substack{t \rightarrow 0^- \\ t=x-1}} \frac{1}{t^{2019}} = 2 \times (-\infty) = -\infty$$

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$$6) \lim_{x \rightarrow (-4)^-} \frac{E(x)}{(x+4)^3} = \lim_{\substack{t \rightarrow 0^- \\ t=x+4}} \frac{E(t-4)}{t^3}$$

$$= \frac{-4}{0^-} = +\infty$$

Solution N° 32 :

$$1) \lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x}\right) \left(\frac{3}{\sqrt{x}} - 1\right)$$

$$= \lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x}\right) \times \lim_{x \rightarrow +\infty} \left(\frac{3}{\sqrt{x}} - 1\right) = 2(-1)$$

$$= -2$$

2) Première méthode : l'utilisation du nombre dérivé.

$$\lim_{x \rightarrow 0} \left(\frac{\cos x - 1}{x}\right) = \lim_{x \rightarrow 0} \left(\frac{\cos x - \cos 0}{x - 0}\right)$$

$$= (\cos x)'_{/x=0} = (-\sin x)_{/x=0} = 0$$

Deuxième Méthode :

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x}\right) = \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x}\right) \times \frac{x}{x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2}\right) \times \left(\frac{x}{1}\right) = \frac{1}{2} \times 0 = 0$$

$$3) \lim_{x \rightarrow 1} \left(\frac{\sqrt{x} - 1}{x - 1}\right) = \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)}{(\sqrt{x})^2 - 1^2}$$

$$= \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)}$$

$$= \lim_{x \rightarrow 1} \left(\frac{1}{\sqrt{x} + 1}\right) = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

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$$4) \lim_{x \rightarrow -\infty} x^2(2 + \sin x) = +\infty$$

Car on a : $-1 \leq \sin x \leq 1$

$$\Rightarrow x^2 \leq x^2(2 + \sin x) \leq 3x^2$$

$$\Rightarrow \underbrace{x^2}_{\substack{\text{tend vers } +\infty \\ \text{quand } x \rightarrow -\infty}} \leq x^2(2 + \sin x)$$

$$\Rightarrow \lim_{x \rightarrow -\infty} x^2(2 + \sin x) = +\infty$$

$$5) \lim_{x \rightarrow +\infty} \left(\frac{5x^2 - 1}{3x^2 + 4}\right) = \lim_{x \rightarrow +\infty} \left(\frac{5x^2}{3x^2}\right) = \frac{5}{3}$$

$$6) \lim_{x \rightarrow +\infty} (\sqrt{x} - 1 + \cos x) = +\infty$$

car on a : $-1 \leq \cos x \leq 1$

$$\Rightarrow \sqrt{x} - 2 \leq (\sqrt{x} - 1 + \cos x) \leq \sqrt{x}$$

$$\Rightarrow \underbrace{\sqrt{x} - 2}_{\substack{\text{tend vers } +\infty \\ \text{quand } x \rightarrow +\infty}} \leq (\sqrt{x} - 1 + \cos x)$$

$$\Rightarrow \lim_{x \rightarrow +\infty} (\sqrt{x} - 1 + \cos x) = +\infty$$

Solution N° 33 :

$$1) \text{ On a : } -1 \leq \sin\left(\frac{1}{x}\right) \leq 1 ; \forall x \neq 0$$

$$\Rightarrow -\underbrace{(x^2 + x^4)}_{\rightarrow 0} \leq (x^2 + x^4) \sin\left(\frac{1}{x}\right) \leq \underbrace{(x^2 + x^4)}_{\rightarrow 0}$$

$$\Rightarrow \lim_{x \rightarrow 0} (x^2 + x^4) \sin\left(\frac{1}{x}\right) = 0$$

$$2) \text{ On a : } x - 1 \leq E(x) < x$$

$$\Rightarrow 1 - \frac{1}{x} \leq \frac{E(x)}{x} \leq 1 ; \forall x > 0$$

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$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{E(x)}{x} = 1$$

$$3) \lim_{x \rightarrow +\infty} \frac{x+1}{\sqrt{1+x^2}} = \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{1}{x}\right)}{\sqrt{x^2} \cdot \sqrt{1 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{1}{x}\right)}{|x| \cdot \sqrt{1 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{1}{x}\right)}{x \cdot \sqrt{1 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\left(1 + \frac{1}{x}\right)}{\sqrt{1 + \frac{1}{x^2}}}$$

$$= \frac{(1 + 0^+)}{\sqrt{1 + 0^+}} = 1^+ = 1$$

$$4) \lim_{x \rightarrow +\infty} \frac{|\sin x|}{\sqrt{1+x^2}} = 0$$

car on a : $|\sin x| \leq 1$

$$\Rightarrow \frac{|\sin x|}{\sqrt{1+x^2}} \leq \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \underbrace{0}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq \frac{|\sin x|}{\sqrt{1+x^2}} \leq \underbrace{\frac{1}{\sqrt{1+x^2}}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{|\sin x|}{\sqrt{1+x^2}} = 0$$

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Solution N° 34 :

1) On a : $-1 \leq \cos x \leq 1$

$$\Rightarrow \underbrace{\frac{-1}{x^2+1}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq \frac{\cos x}{x^2+1} \leq \underbrace{\frac{1}{x^2+1}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x^2+1} \right) = 0$$

$$2) \lim_{x \rightarrow +\infty} \left(\frac{x + \sin x}{x^2 + \cos x} \right) = \lim_{x \rightarrow +\infty} \frac{x^2 \left(\frac{1}{x} + \frac{\sin x}{x^2} \right)}{x^2 \left(1 + \frac{\cos x}{x^2} \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{\frac{1}{x} + \frac{\sin x}{x^2}}{1 + \frac{\cos x}{x^2}} \right) = \frac{0^+ + 0}{1 + 0} = 0^+ = 0$$

pourquoi : $\lim_{x \rightarrow +\infty} \frac{\sin x}{x^2} = \lim_{x \rightarrow +\infty} \frac{\cos x}{x^2} = 0$?

Car $-1 \leq \sin x \leq 1$ et $-1 \leq \cos x \leq 1$

$$\Rightarrow \frac{-1}{x^2} \leq \frac{\sin x}{x^2} \leq \frac{1}{x^2} \text{ et } \frac{-1}{x^2} \leq \frac{\cos x}{x^2} \leq \frac{1}{x^2}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left(\frac{\sin x}{x^2} \right) = \lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x^2} \right) = 0$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{2x + \cos x}{3x + \sin x} \right) = \lim_{x \rightarrow +\infty} \frac{x \left(2 + \frac{\cos x}{x} \right)}{x \left(3 + \frac{\sin x}{x} \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{2 + \frac{\cos x}{x}}{3 + \frac{\sin x}{x}} \right) = \frac{2 + 0}{3 + 0} = \frac{2}{3}$$

Pourquoi $\lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x} \right) = \lim_{x \rightarrow +\infty} \left(\frac{\sin x}{x} \right) = 0$

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$$\text{car : } \left| \begin{array}{ccc} \underbrace{\frac{-1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq \frac{\cos x}{x} \leq \underbrace{\frac{1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \\ \underbrace{\frac{-1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq \frac{\sin x}{x} \leq \underbrace{\frac{1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \end{array} \right.$$

$$4) \lim_{x \rightarrow +\infty} \frac{E(\sqrt{x})}{x} = 0^+$$

$$\text{Car : } \sqrt{x} - 1 \leq E(\sqrt{x}) < \sqrt{x}$$

$$\text{Ainsi : } \frac{\sqrt{x}}{x} - \frac{1}{x} \leq \frac{E(\sqrt{x})}{x} < \frac{\sqrt{x}}{x}$$

$$c - \text{à} - d \quad \underbrace{\frac{1}{\sqrt{x}} - \frac{1}{x}}_{\substack{\text{tend vers } 0^+ \\ \text{quand } x \rightarrow +\infty}} \leq \frac{E(\sqrt{x})}{x} < \underbrace{\frac{1}{\sqrt{x}}}_{\substack{\text{tend vers } 0^+ \\ \text{quand } x \rightarrow +\infty}}$$

$$5) \lim_{x \rightarrow -\infty} \left(\frac{\sin x}{x^2 + 1} \right) = 0$$

$$\text{Car On a : } -1 \leq \sin x \leq 1$$

$$\text{Alors : } \underbrace{\frac{-1}{x^2 + 1}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow -\infty}} \leq \frac{\sin x}{x^2 + 1} \leq \underbrace{\frac{1}{x^2 + 1}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow -\infty}}$$

$$6) \lim_{x \rightarrow +\infty} \left(\frac{2 - \cos x}{1 + \sqrt{x}} \right) = 0$$

$$\text{Car On a : } -1 \leq \cos x \leq 1$$

$$\Rightarrow 1 \leq 2 - \cos x \leq 3$$

$$C - \text{à} - d \quad \underbrace{\frac{1}{1 + \sqrt{x}}}_{\substack{\text{tend vers } 0^+ \\ \text{quand } x \rightarrow +\infty}} \leq \frac{2 - \cos x}{1 + \sqrt{x}} \leq \underbrace{\frac{3}{1 + \sqrt{x}}}_{\substack{\text{tend vers } 0^+ \\ \text{quand } x \rightarrow +\infty}}$$

Solution N° 35 :

$$1) \lim_{x \rightarrow 0} \left(x^2 + x + \frac{1}{x^2} \right) = 0 + 0 + \frac{1}{(0^\pm)^2} = \frac{1}{0^+} = +\infty$$

$$\begin{aligned} 2) \lim_{x \rightarrow 1^+} \frac{x^3 + 1}{(x - 1)^3} &= \lim_{\substack{t \rightarrow 0^+ \\ t = x - 1}} \frac{(t + 1)^3 + 1}{t^3} \\ &= \lim_{t \rightarrow 0^+} \left(\left(\frac{t + 1}{t} \right)^3 + \left(\frac{1}{t} \right)^3 \right) \\ &= \lim_{t \rightarrow 0^+} \left(\left(1 + \frac{1}{t} \right)^3 + \left(\frac{1}{t} \right)^3 \right) \\ &= \lim_{\substack{u \rightarrow +\infty \\ u = \frac{1}{t}}} ((1 + u)^3 + u^3) = +\infty \end{aligned}$$

$$\begin{aligned} 3) \lim_{x \rightarrow +\infty} \frac{x^3 + 1}{(x - 1)^3} &= \lim_{\substack{t \rightarrow +\infty \\ t = x - 1}} \frac{(t + 1)^3 + 1}{t^3} \\ &= \lim_{t \rightarrow +\infty} \left(\left(\frac{t + 1}{t} \right)^3 + \left(\frac{1}{t} \right)^3 \right) \\ &= \lim_{t \rightarrow +\infty} \left(\left(1 + \frac{1}{t} \right)^3 + \left(\frac{1}{t} \right)^3 \right) \\ &= \lim_{\substack{u \rightarrow 0^+ \\ u = \frac{1}{t}}} ((1 + u)^3 + u^3) = 1 \end{aligned}$$

$$\begin{aligned} 4) \lim_{x \rightarrow +\infty} (\sqrt{x} + x^2) &= \lim_{x \rightarrow +\infty} x^2 \left(1 + \frac{\sqrt{x}}{x^2} \right) \\ &= \lim_{x \rightarrow +\infty} x^2 \left(1 + \frac{1}{x\sqrt{x}} \right) = (+\infty)(1 + 0) = +\infty \end{aligned}$$

$$\begin{aligned} 5) \lim_{x \rightarrow -\infty} (x^3 - x^2 + x + 1) \\ &= \lim_{x \rightarrow -\infty} x^3 \left(1 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right) \end{aligned}$$

$$= (-\infty)(1 - 0 + 0 + 0) = -\infty$$

On a souvent l'habitude d'écrire :

$$\lim_{x \rightarrow -\infty} (x^3 - x^2 + x + 1) = \lim_{x \rightarrow -\infty} x^3 = -\infty$$

$$\begin{aligned} 6) \quad \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x}}{x^2 + 2} \right) &= \lim_{x \rightarrow +\infty} \left(\frac{1}{x^2 + 2} \right) \left(1 + \frac{1}{x} \right) \\ &= \left(\frac{1}{+\infty} \right) \left(1 + \frac{1}{+\infty} \right) = 0(1 + 0) = 0 \end{aligned}$$

Solution N° 36 :

$$1) \quad \lim_{x \rightarrow +\infty} (-3x^2 + x + 1)$$

$$= \lim_{x \rightarrow +\infty} (-3x^2) \left(1 - \frac{1}{3x} - \frac{1}{3x^2} \right)$$

$$= (-\infty)(1 - 0 - 0) = -\infty$$

On a souvent l'habitude d'écrire :

$$\lim_{x \rightarrow +\infty} (-3x^2 + x + 1) = \lim_{x \rightarrow +\infty} -3x^2 = -\infty$$

$$2) \quad \lim_{x \rightarrow -\infty} \left(\frac{3x + 5}{x^2 + 1} \right) = \lim_{x \rightarrow -\infty} \left(\frac{3x}{x^2} \right) \left(\frac{1 + \frac{5}{3x}}{1 + \frac{1}{x^2}} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{3}{x} \right) \left(\frac{1 + \frac{5}{3x}}{1 + \frac{1}{x^2}} \right) = (0^-) \left(\frac{1 + 0}{1 + 0} \right) = 0$$

On peut écrire directement :

$$\lim_{x \rightarrow -\infty} \left(\frac{3x + 5}{x^2 + 1} \right) = \lim_{x \rightarrow -\infty} \left(\frac{3x}{x^2} \right) = \lim_{x \rightarrow -\infty} \left(\frac{3}{x} \right) = 0$$

$$3) \quad \lim_{x \rightarrow +\infty} \left(\frac{x^2 - 4x}{2x^2 + 1} \right) = \lim_{x \rightarrow +\infty} \left(\frac{x^2}{2x^2} \right) = \frac{1}{2}$$

$$4) \quad \lim_{x \rightarrow +\infty} \left(\frac{x^2 - 7}{x^3 + 2x + 1} \right) = \lim_{x \rightarrow +\infty} \left(\frac{x^2}{x^3} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{x} \right) = 0$$

$$5) \quad \lim_{x \rightarrow -\infty} \left(\frac{x^3 + x^2 + x + 1}{x^3 + 3} \right) = \lim_{x \rightarrow -\infty} \left(\frac{x^3}{x^3} \right) = 1$$

$$6) \quad \lim_{x \rightarrow +\infty} \left(\frac{x^4 + x^2 + 1}{x - 1} \right) = \lim_{x \rightarrow +\infty} \left(\frac{x^4}{x} \right) = +\infty$$

Solution N° 37 :

$$1) \quad \lim_{x \rightarrow -\infty} (3x^2 - x + 4)$$

$$= \lim_{x \rightarrow -\infty} (3x^2) \left(1 - \frac{1}{3x} + \frac{4}{x^2} \right)$$

$$= (+\infty)(1 - 0 + 0) = +\infty$$

On peut écrire directement :

$$\lim_{x \rightarrow -\infty} (3x^2 - x + 4) = \lim_{x \rightarrow -\infty} (3x^2) = +\infty$$

$$1) \quad \lim_{x \rightarrow 0^-} \sqrt{3 - \frac{1}{x}} = \sqrt{3 - \frac{1}{0^-}} = \sqrt{3 + \infty} = +\infty$$

$$3) \quad \lim_{x \rightarrow 1} \left(\frac{\sqrt{2x - 1} - \sqrt{x^2 - x + 1}}{\sqrt{x} - 1} \right)$$

$$= \lim_{x \rightarrow 1} \left(\frac{\sqrt{2x - 1} - \sqrt{x^2 - x + 1}}{\sqrt{x} - 1} \right)$$

$$\times \frac{(\sqrt{2x - 1} + \sqrt{x^2 - x + 1})}{(\sqrt{2x - 1} + \sqrt{x^2 - x + 1})}$$

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$$\begin{aligned}
&= \lim_{x \rightarrow 1} \left(\frac{(\sqrt{2x-1})^2 - (\sqrt{x^2-x+1})^2}{(\sqrt{x}-1)((\sqrt{2x-1} + \sqrt{x^2-x+1}))} \right) \\
&= \lim_{x \rightarrow 1} \left(\frac{(2x-1) - (x^2-x+1)}{(\sqrt{x}-1)((\sqrt{2x-1} + \sqrt{x^2-x+1}))} \right) \\
&= \lim_{x \rightarrow 1} \left(\frac{-x^2+3x-2}{(\sqrt{x}-1)((\sqrt{2x-1} + \sqrt{x^2-x+1}))} \right) \\
&= \lim_{x \rightarrow 1} \frac{(-x^2+3x-2)(\sqrt{x}+1)}{(\sqrt{x}-1)((\sqrt{2x-1} + \sqrt{x^2-x+1}))(\sqrt{x}+1)} \\
&= \lim_{x \rightarrow 1} \frac{(-x^2+3x-2)(\sqrt{x}+1)}{((\sqrt{2x-1} + \sqrt{x^2-x+1}))(x-1)} \\
&= \lim_{x \rightarrow 1} \left(\frac{-x^2+3x-2}{x-1} \right) \left(\frac{\sqrt{x}+1}{\sqrt{2x-1} + \sqrt{x^2-x+1}} \right) \\
&= \lim_{x \rightarrow 1} (-x+2) \left(\frac{\sqrt{x}+1}{\sqrt{2x-1} + \sqrt{x^2-x+1}} \right) \\
&= (-1+2) \left(\frac{\sqrt{1}+1}{\sqrt{2-1} + \sqrt{1-1+1}} \right) = 1
\end{aligned}$$

$$\begin{aligned}
4) \quad \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2+1}{x-1}} &= \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2}{x} \left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}} \right)} \\
&= \lim_{x \rightarrow +\infty} \sqrt{x \left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}} \right)} = (+\infty) \sqrt{\frac{1+0}{1-0}} = +\infty
\end{aligned}$$

$$\begin{aligned}
5) \quad \lim_{x \rightarrow 1} \left(\frac{x^3+2x-3}{x^2+2x-3} \right) \\
&= \lim_{x \rightarrow 1} \left(x-2 + \frac{9(x-1)}{x^2+2x-3} \right)
\end{aligned}$$

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$$\begin{aligned}
&= \lim_{x \rightarrow 1} \left(x-2 + \frac{9(x-1)}{(x+3)(x-1)} \right) \\
&= \lim_{x \rightarrow 1} \left(x-2 + \frac{9}{(x+3)} \right) = \frac{5}{4} \\
6) \quad \lim_{x \rightarrow 9} \left(\frac{\sqrt{x+7} - \sqrt{x} - 1}{\sqrt{x+16} - \sqrt{x} - 2} \right) \\
&= \lim_{x \rightarrow 9} \left(\frac{\sqrt{x+7} - (\sqrt{x}+1)}{\sqrt{x+16} - (\sqrt{x}+2)} \right) \left(\frac{\sqrt{x+7} + (\sqrt{x}+1)}{\sqrt{x+7} + (\sqrt{x}+1)} \right) \\
&= \lim_{x \rightarrow 9} \left(\frac{(\sqrt{x+7})^2 - (\sqrt{x}+1)^2}{(\sqrt{x+16} - (\sqrt{x}+2))(\sqrt{x+7} + (\sqrt{x}+1))} \right) \\
&= \lim_{x \rightarrow 9} \left(\frac{(x+7) - (\sqrt{x}+1)^2}{(\sqrt{x+16} - (\sqrt{x}+2))(\sqrt{x+7} + (\sqrt{x}+1))} \right) \\
&= \lim_{x \rightarrow 9} \left(\frac{6-2\sqrt{x}}{(\sqrt{x+16} - (\sqrt{x}+2))(\sqrt{x+7} + (\sqrt{x}+1))} \right) \\
&= \lim_{x \rightarrow 9} \left(\frac{(6-2\sqrt{x})(\sqrt{x+16} + (\sqrt{x}+2))}{(\sqrt{x+16} - (\sqrt{x}+2))(\sqrt{x+7} + (\sqrt{x}+1))} \right) \\
&\quad \times \frac{1}{\sqrt{x+16} - (\sqrt{x}+2)} \\
&= \lim_{x \rightarrow 9} \frac{(6-2\sqrt{x})(\sqrt{x+16} + (\sqrt{x}+2))}{((\sqrt{x+16})^2 - (\sqrt{x}+2)^2)(\sqrt{x+7} + (\sqrt{x}+1))} \\
&= \lim_{x \rightarrow 9} \frac{(6-2\sqrt{x})(\sqrt{x+16} + (\sqrt{x}+2))}{(12-4\sqrt{x})(\sqrt{x+7} + (\sqrt{x}+1))} \\
&= \lim_{x \rightarrow 9} \frac{2(3-\sqrt{x})(\sqrt{x+16} + (\sqrt{x}+2))}{4(3-\sqrt{x})(\sqrt{x+7} + (\sqrt{x}+1))}
\end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 9} \frac{2(\sqrt{x+16} + (\sqrt{x} + 2))}{4(\sqrt{x+7} + (\sqrt{x} + 1))} \\
 &= \frac{2(\sqrt{9+16} + (\sqrt{9} + 2))}{4(\sqrt{9+7} + (\sqrt{9} + 1))} = \frac{5}{8}
 \end{aligned}$$

Solution N° 38 :

$$\begin{aligned}
 1) \quad &\lim_{x \rightarrow (-1)^+} \left(\frac{x+1}{|2x+3|-1} \right) \\
 &= \lim_{\substack{x \rightarrow 0^+ \\ t=x+1}} \left(\frac{t}{|2t+1|-1} \right) \\
 &= \lim_{t \rightarrow 0^+} \left(\frac{t}{2t+1-1} \right) = \lim_{t \rightarrow 0^+} \left(\frac{1}{2} \right) = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad &\lim_{x \rightarrow 2} \left(\frac{x^2 - 2x}{x - \sqrt{x+2}} \right) = \lim_{x \rightarrow 2} \frac{x(x-2)(x+\sqrt{x+2})}{x^2 - (x+2)} \\
 &= \lim_{x \rightarrow 2} \frac{x(x-2)(x+\sqrt{x+2})}{(x+1)(x-2)} \\
 &= \lim_{x \rightarrow 2} \frac{x(x+\sqrt{x+2})}{(x+1)} \\
 &= \frac{2(2+\sqrt{2+2})}{(2+1)} = \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad &\lim_{x \rightarrow 1^-} \left(\frac{\sqrt{x+2} - \sqrt{1+2x}}{\sqrt{1-x}} \right) \\
 &= \lim_{x \rightarrow 1^-} \frac{(\sqrt{x+2} - \sqrt{1+2x})(\sqrt{x+2} + \sqrt{1+2x})}{(\sqrt{1-x})(\sqrt{x+2} + \sqrt{1+2x})} \\
 &= \lim_{x \rightarrow 1^-} \frac{(\sqrt{x+2})^2 - (\sqrt{1+2x})^2}{(\sqrt{1-x})(\sqrt{x+2} + \sqrt{1+2x})} \\
 &= \lim_{x \rightarrow 1^-} \frac{(x+2) - (1+2x)}{(\sqrt{1-x})(\sqrt{x+2} + \sqrt{1+2x})}
 \end{aligned}$$

$$= \lim_{x \rightarrow 1^-} \frac{1-x}{(\sqrt{1-x})(\sqrt{x+2} + \sqrt{1+2x})}$$

$$= \lim_{x \rightarrow 1^-} \frac{(\sqrt{1-x})(\sqrt{1-x})}{(\sqrt{1-x})(\sqrt{x+2} + \sqrt{1+2x})}$$

$$= \lim_{x \rightarrow 1^-} \frac{\sqrt{1-x}}{(\sqrt{x+2} + \sqrt{1+2x})} = \frac{0}{\sqrt{2} + \sqrt{1}} = 0$$

$$4) \quad \lim_{x \rightarrow (-1)^-} \left(\frac{\sqrt{x^2+x} + 3x + 3}{x+1} \right)$$

$$= \lim_{\substack{t \rightarrow 0^- \\ t=x+1}} \frac{\sqrt{(t-1)^2 + (t-1)} + 3(t-1) + 3}{t}$$

$$= \lim_{t \rightarrow 0^-} \frac{\sqrt{t^2 - t} + 3t}{t} = \lim_{t \rightarrow 0^-} \frac{\sqrt{t^2 \left(1 - \frac{1}{t}\right)}}{t} + 3$$

$$= \lim_{t \rightarrow 0^-} \frac{\sqrt{t^2} \times \sqrt{\left(1 - \frac{1}{t}\right)}}{t} + 3$$

$$= \lim_{t \rightarrow 0^-} \frac{|t| \times \sqrt{\left(1 - \frac{1}{t}\right)}}{t} + 3$$

$$= \lim_{t \rightarrow 0^-} \frac{-t \times \sqrt{\left(1 - \frac{1}{t}\right)}}{t} + 3$$

$$= \lim_{t \rightarrow 0^-} -\sqrt{\left(1 - \frac{1}{t}\right)} + 3$$

$$= -\sqrt{\left(1 - \frac{1}{0^-}\right)} + 3 = -\infty$$

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$$\begin{aligned}
 5) \quad & \lim_{x \rightarrow 0^+} \left(\frac{x + \sqrt{x^2 + x}}{\sqrt{x^2 + x + 1} - 1} \right) \\
 &= \lim_{x \rightarrow 0^+} \frac{(x + \sqrt{x^2 + x})(\sqrt{x^2 + x + 1} + 1)}{(\sqrt{x^2 + x + 1} - 1)(\sqrt{x^2 + x + 1} + 1)} \\
 &= \lim_{x \rightarrow 0^+} \frac{(x + \sqrt{x^2 + x})(\sqrt{x^2 + x + 1} + 1)}{(\sqrt{x^2 + x + 1})^2 - 1^2} \\
 &= \lim_{x \rightarrow 0^+} \frac{(x + \sqrt{x^2 + x})(\sqrt{x^2 + x + 1} + 1)}{x^2 + x} \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{x + \sqrt{x^2 + x}}{x^2 + x} \right) (\sqrt{x^2 + x + 1} + 1) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1}{x + 1} + \frac{1}{\sqrt{x^2 + x}} \right) (\sqrt{x^2 + x + 1} + 1) \\
 &= \left(\frac{1}{0 + 1} + \frac{1}{0^+} \right) (\sqrt{0^2 + 0 + 1} + 1) \\
 &= (+\infty) \times 2 = +\infty
 \end{aligned}$$

$$6) \quad \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{\sqrt{x+1} - \sqrt{3x+1}} \right)$$

D'abord pour en avoir une idée sur la valeur de cette limite, On applique dans le brouillon la règle de l'Hôpital juste pour avoir la valeur numérique de la limite en question. En suite d'essayer de la rechercher par une méthode reconnue par le programme officiel pour rédiger la réponse.

La substitution directe donne la forme indéterminée zéro/zéro. Donc on peut appliquer la règle de l'hôpital :

$$\begin{aligned}
 & \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{\sqrt{x+1} - \sqrt{3x+1}} \right) \\
 &= \lim_{x \rightarrow 3} \left(\frac{\frac{1}{2}(x+1)^{-\frac{1}{2}} - 2x + 1}{\frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3} \right) \\
 &= \left(\frac{\frac{1}{2}(3+1)^{-\frac{1}{2}} - 6 + 1}{\frac{1}{2}(3+1)^{-\frac{1}{2}} - \frac{1}{2}(9)^{-\frac{1}{2}} \cdot 3} \right) \\
 &= \left(\frac{\frac{1}{2} \times \frac{1}{2} - 6 + 1}{\frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{1}{3} \times 3} \right) = 19
 \end{aligned}$$

Cherchons maintenant une méthode légale pour retrouver 19. Et pour ce faire on adoptera la technique de multiplication par la quantité conjuguée :

$$\begin{aligned}
 & \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{\sqrt{x+1} - \sqrt{3x+1}} \right) \\
 &= \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - (x^2 - x - 4)}{\sqrt{x+1} - (\sqrt{3x} - 1)} \right) \\
 &\quad \times \left(\frac{\sqrt{x+1} + (\sqrt{3x} - 1)}{\sqrt{x+1} + (\sqrt{3x} - 1)} \right) \\
 &\quad \times \left(\frac{\sqrt{x+1} + (x^2 - x - 4)}{\sqrt{x+1} + (x^2 - x - 4)} \right) \\
 &= \lim_{x \rightarrow 3} \frac{((x+1) - (x^2 - x - 4)^2)(\sqrt{x+1} + (\sqrt{3x} - 1))}{((x+1) - (\sqrt{3x} - 1)^2)(\sqrt{x+1} + x^2 - x - 4)} \\
 &= \lim_{x \rightarrow 3} \frac{1}{2} \left(\frac{-7x - 15 - x^4 + 2x^3 + 7x^2}{\sqrt{3x} - x} \right) \left(\frac{\sqrt{x+1} + (\sqrt{3x} - 1)}{\sqrt{x+1} + x^2 - x - 4} \right)
 \end{aligned}$$

On multiplie le numérateur et le dénominateur par la quantité $(\sqrt{3x} + x)$
On obtient :

$$= \lim_{x \rightarrow 3} \frac{1}{2} \left(\frac{-x^4 + 2x^3 + 7x^2 - 7x - 15}{-x^2 + 3x} \right) \\ \times \left(\frac{\sqrt{x+1} + (\sqrt{3x}-1)}{\sqrt{x+1} + x^2 - x - 4} \right) \left(\frac{\sqrt{3x} + x}{1} \right)$$

On effectue ensuite la division euclidienne du polynôme du numérateur par celle du dénominateur on trouve :

$$= \lim_{x \rightarrow 3} \frac{1}{2} \left(x^2 + x - 4 - \frac{5}{x} \right) \\ \times \left(\frac{\sqrt{x+1} + (\sqrt{3x}-1)}{\sqrt{x+1} + x^2 - x - 4} \right) \left(\frac{\sqrt{3x} + x}{1} \right)$$

On remplace x par 3 on obtient :

$$= \frac{1}{2} \left(\frac{19}{3} \right) \left(\frac{4}{4} \right) \left(\frac{6}{1} \right) = 19$$

Retenez que si vous avez une fonction fraction rationnelle $P(x)/Q(x)$ on pense directement à effectuer éventuellement la division euclidienne.

Solution N° 39 :

$$1) \lim_{x \rightarrow +\infty} (2x - \sqrt{x}) = \lim_{x \rightarrow +\infty} x \left(2 - \frac{1}{\sqrt{x}} \right)$$

$$= (+\infty)(2 - 0) = +\infty$$

$$2) \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 + 3x - 7} + 2x + 5 \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 \left(4 + \frac{3}{x} - \frac{7}{x^2} \right)} + 2x + 5 \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\sqrt{x^2} \cdot \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} + 2x + 5 \right)$$

$$= \lim_{x \rightarrow -\infty} \left(|x| \cdot \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} + 2x + 5 \right)$$

$$= \lim_{x \rightarrow -\infty} \left(-x \cdot \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} + 2x + 5 \right)$$

$$= 5 + \lim_{x \rightarrow -\infty} \left(2x - x \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)$$

$$= 5 + \lim_{x \rightarrow -\infty} \frac{x \left(2 - \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right) \left(2 + \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)}{\left(2 + \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)}$$

$$= 5 + \lim_{x \rightarrow -\infty} \frac{x \left(4 - \left(4 + \frac{3}{x} - \frac{7}{x^2} \right) \right)}{\left(2 + \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)}$$

$$= 5 + \lim_{x \rightarrow -\infty} \frac{x \left(-\frac{3}{x} + \frac{7}{x^2} \right)}{\left(2 + \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)}$$

$$= 5 + \lim_{x \rightarrow -\infty} \frac{\left(-3 + \frac{7}{x} \right)}{\left(2 + \sqrt{4 + \frac{3}{x} - \frac{7}{x^2}} \right)}$$

$$= 5 + \left(\frac{(-3 + 0)}{(2 + \sqrt{4 + 0 - 0})} \right) = 5 - \frac{3}{4} = \frac{17}{4}$$

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$$\begin{aligned}
3) \quad & \lim_{x \rightarrow +\infty} \left(\sqrt{2x^2 + 1} - \sqrt{x^2 - x - 2} \right) \\
= & \lim_{x \rightarrow +\infty} \left(\sqrt{2x^2 + 1} - \sqrt{x^2 - x - 2} \right) \\
& \times \frac{(\sqrt{2x^2 + 1} + \sqrt{x^2 - x - 2})}{(\sqrt{2x^2 + 1} + \sqrt{x^2 - x - 2})} \\
= & \lim_{x \rightarrow +\infty} \frac{(\sqrt{2x^2 + 1})^2 - (\sqrt{x^2 - x - 2})^2}{\sqrt{x^2 \left(2 + \frac{1}{x^2}\right)} + \sqrt{x^2 \left(1 - \frac{1}{x} - \frac{2}{x^2}\right)}} \\
= & \lim_{x \rightarrow +\infty} \frac{(2x^2 + 1) - (x^2 - x - 2)}{\sqrt{x^2} \times \left(\sqrt{2 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x} - \frac{2}{x^2}} \right)} \\
= & \lim_{x \rightarrow +\infty} \frac{x^2 + x + 3}{|x| \times \left(\sqrt{2 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x} - \frac{2}{x^2}} \right)} \\
= & \lim_{x \rightarrow +\infty} \frac{x \cdot \left(x + 1 + \frac{3}{x} \right)}{x \cdot \left(\sqrt{2 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x} - \frac{2}{x^2}} \right)} \\
= & \lim_{x \rightarrow +\infty} \left(\frac{x + 1 + \frac{3}{x}}{\sqrt{2 + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x} - \frac{2}{x^2}}} \right) \\
& \frac{+\infty + 1 + 0}{\sqrt{2 + 0} + \sqrt{1 - 0 - 0}} = +\infty
\end{aligned}$$

$$\begin{aligned}
4) \quad & \lim_{x \rightarrow +\infty} \left(\sqrt{5x^2 + x - 1} - 4x + 3 \right) \\
= & 3 + \lim_{x \rightarrow +\infty} \left(\sqrt{5x^2 + x - 1} - 4x \right) \\
& \times \frac{(\sqrt{5x^2 + x - 1} + 4x)}{(\sqrt{5x^2 + x - 1} + 4x)} \\
= & 3 + \lim_{x \rightarrow +\infty} \frac{(\sqrt{5x^2 + x - 1})^2 - (4x)^2}{(\sqrt{5x^2 + x - 1} + 4x)}
\end{aligned}$$

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$$\begin{aligned}
& = 3 + \lim_{x \rightarrow +\infty} \frac{5x^2 + x - 1 - 16x^2}{\left(\sqrt{x^2 \left(5 + \frac{1}{x} - \frac{1}{x^2}\right)} + 4x \right)} \\
& = 3 + \lim_{x \rightarrow +\infty} \frac{-11x^2 + x - 1}{\left(\sqrt{x^2} \cdot \sqrt{\left(5 + \frac{1}{x} - \frac{1}{x^2}\right)} + 4x \right)} \\
& = 3 + \lim_{x \rightarrow +\infty} \frac{-11x^2 + x - 1}{\left(|x| \cdot \sqrt{\left(5 + \frac{1}{x} - \frac{1}{x^2}\right)} + 4x \right)} \\
& = 3 + \lim_{x \rightarrow +\infty} \frac{-11x^2 + x - 1}{\left(x \cdot \sqrt{\left(5 + \frac{1}{x} - \frac{1}{x^2}\right)} + 4x \right)} \\
& = 3 + \lim_{x \rightarrow +\infty} \frac{x \cdot \left(-11x + 1 - \frac{1}{x} \right)}{x \cdot \left(\sqrt{\left(5 + \frac{1}{x} - \frac{1}{x^2}\right)} + 4 \right)} \\
& = 3 + \lim_{x \rightarrow +\infty} \left(\frac{-11x + 1 - \frac{1}{x}}{\sqrt{5 + \frac{1}{x} - \frac{1}{x^2}} + 4} \right) \\
& = 3 + \left(\frac{-\infty + 1 - 0}{\sqrt{5 + 0 - 0} + 4} \right) = -\infty
\end{aligned}$$

$$\begin{aligned}
5) \quad & \lim_{x \rightarrow -\infty} \left(x + \sqrt{x^2 + 1} \right) \\
& = \lim_{x \rightarrow -\infty} \left(x + \sqrt{x^2} \cdot \sqrt{1 + \frac{1}{x^2}} \right) \\
& = \lim_{x \rightarrow -\infty} \left(x + |x| \cdot \sqrt{1 + \frac{1}{x^2}} \right)
\end{aligned}$$

$$= \lim_{x \rightarrow -\infty} \left(x - x \sqrt{1 + \frac{1}{x^2}} \right)$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 - \sqrt{1 + \frac{1}{x^2}} \right) \left(1 + \sqrt{1 + \frac{1}{x^2}} \right)}{\left(1 + \sqrt{1 + \frac{1}{x^2}} \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1^2 - \sqrt{1 + \frac{1}{x^2}}^2 \right)}{\left(1 + \sqrt{1 + \frac{1}{x^2}} \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 - \left(1 + \frac{1}{x} \right) \right)}{1 + \sqrt{1 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(\frac{-1}{x^2} \right)}{1 + \sqrt{1 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{\frac{-1}{x}}{1 + \sqrt{1 + \frac{1}{x^2}}} \right)$$

$$= \left(\frac{-0^-}{1 + \sqrt{1 + 0^+}} \right) = 0^+ = 0$$

6) Soit $l = \lim_{|x| \rightarrow +\infty} (\sqrt{x^2 + x + 1} - x - 3)$

On pose $|x| = t \Leftrightarrow x = \pm t \Leftrightarrow t = \pm x$

Si $x = +t$ Alors :

$$l = \lim_{t \rightarrow +\infty} (\sqrt{t^2 + t + 1} - t - 3)$$

$$= -3 + \lim_{t \rightarrow +\infty} (\sqrt{t^2 + t + 1} - t)$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{(\sqrt{t^2 + t + 1} - t)(\sqrt{t^2 + t + 1} + t)}{(\sqrt{t^2 + t + 1} + t)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t^2 + t + 1 - t^2}{(\sqrt{t^2 + t + 1} + t)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t + 1}{\left(\sqrt{t^2 \left(1 + \frac{1}{t} + \frac{1}{t^2} \right)} + t \right)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t \left(1 + \frac{1}{t} \right)}{\left(\sqrt{t^2} \sqrt{\left(1 + \frac{1}{t} + \frac{1}{t^2} \right)} + t \right)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t \left(1 + \frac{1}{t} \right)}{\left(|t| \sqrt{\left(1 + \frac{1}{t} + \frac{1}{t^2} \right)} + t \right)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t \left(1 + \frac{1}{t} \right)}{\left(t \sqrt{\left(1 + \frac{1}{t} + \frac{1}{t^2} \right)} + t \right)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \frac{t \left(1 + \frac{1}{t} \right)}{t \left(\sqrt{\left(1 + \frac{1}{t} + \frac{1}{t^2} \right)} + 1 \right)}$$

$$= -3 + \lim_{t \rightarrow +\infty} \left(\frac{1 + \frac{1}{t}}{\sqrt{1 + \frac{1}{t} + \frac{1}{t^2}} + 1} \right)$$

$$-3 + \left(\frac{1 + 0}{\sqrt{1 + 0 + 0} + 1} \right) = \frac{-5}{2}$$

Ça c'est pour le cas où $x = +t$

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Si maintenant on ait $x = -t$ Alors :

$$l = \lim_{t \rightarrow +\infty} (\sqrt{t^2 - t + 1} + t - 3)$$

$$= \sqrt{+\infty} + \infty - 3 = +\infty$$

Donc l n'existe pas car la limite si elle existe elle est unique.

Solution N° 40 :

$$1) \lim_{x \rightarrow +\infty} (\sqrt{x^3 + 3x^2 + 4} - x^2 + 2)$$

$$\times \frac{(\sqrt{x^3 + 3x^2 + 4} + (x^2 - 2))}{(\sqrt{x^3 + 3x^2 + 4} + (x^2 - 2))}$$

$$= \lim_{x \rightarrow +\infty} \frac{((\sqrt{x^3 + 3x^2 + 4})^2) - (x^2 - 2)^2}{\sqrt{x^3 + 3x^2 + 4} + (x^2 - 2)}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^3 + 3x^2 + 4 - (x^2 - 2)^2}{\sqrt{x^3 + 3x^2 + 4} + (x^2 - 2)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^4 + x^3 + 7x^2}{\sqrt{x^4 \left(\frac{1}{x} + \frac{3}{x^2} + \frac{4}{x^4}\right)} + (x^2 - 2)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^4 + x^3 + 7x^2}{x^2 \sqrt{\frac{1}{x} + \frac{3}{x^2} + \frac{4}{x^4}} + x^2 - 2}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-x^2 + x + 7}{\sqrt{\frac{1}{x} + \frac{3}{x^2} + \frac{4}{x^4}} + 1 - \frac{2}{x^2}} \right)$$

$$= \left(\frac{-\infty}{\sqrt{0 + 0 + 0} + 1 - 0} \right) = -\infty$$

$$2) \lim_{x \rightarrow -\infty} (\sqrt{3x^2 - 6x - 1} + 2x - 5)$$

$$\lim_{x \rightarrow -\infty} (\sqrt{3x^2 - 6x - 1} + 2x - 5)$$

$$\times \frac{(\sqrt{3x^2 - 6x - 1} - (2x - 5))}{(\sqrt{3x^2 - 6x - 1} - (2x - 5))}$$

$$= \lim_{x \rightarrow -\infty} \frac{(3x^2 - 6x - 1) - (2x - 5)^2}{\sqrt{3x^2 - 6x - 1} - (2x - 5)}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^2 + 14x - 26}{\sqrt{x^2 \left(3 - \frac{6}{x} - \frac{1}{x^2}\right)} - 2x + 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^2 + 14x - 26}{\sqrt{x^2} \sqrt{3 - \frac{6}{x} - \frac{1}{x^2}} - 2x + 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^2 + 14x - 26}{|x| \sqrt{3 - \frac{6}{x} - \frac{1}{x^2}} - 2x + 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^2 + 14x - 26}{-x \sqrt{3 - \frac{6}{x} - \frac{1}{x^2}} - 2x + 5}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x \left(x - 14 + \frac{26}{x}\right)}{-x \left(\sqrt{3 - \frac{6}{x} - \frac{1}{x^2}} + 2 - \frac{5}{x}\right)}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x - 14 + \frac{26}{x}}{\sqrt{3 - \frac{6}{x} - \frac{1}{x^2}} + 2 - \frac{5}{x}} \right)$$

$$= \left(\frac{-\infty - 14 + 0}{\sqrt{3 - 0 - 0} + 2 - 0} \right) = -\infty$$

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$$3) \text{ Soit } l = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} + xm \right); m \in \mathbb{R}$$

$$\text{Si } m = 0; \quad l = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} + xm \right)$$

$$= \sqrt{+\infty} + 0 = +\infty$$

$$\text{Si } m > 0; \quad l = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} + xm \right)$$

$$= \sqrt{+\infty} + 0 = +\infty$$

$$\text{Si } m < 0; \quad l = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} + xm \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} + xm \right) \times \frac{(\sqrt{x^2 + x + 1} - xm)}{(\sqrt{x^2 + x + 1} - xm)}$$

$$= \lim_{x \rightarrow +\infty} \frac{(x^2 + x + 1) - x^2 m^2}{(\sqrt{x^2 + x + 1} - xm)}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - m^2)x^2 + x + 1}{(\sqrt{x^2 + x + 1} - xm)}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - m^2)x^2 + x + 1}{\left(\sqrt{x^2 \left(1 + \frac{1}{x} + \frac{1}{x^2} \right)} - xm \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - m^2)x^2 + x + 1}{\sqrt{x^2} \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - xm}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - m^2)x^2 + x + 1}{|x| \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - xm}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - m^2)x^2 + x + 1}{x \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - xm}$$

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$$= \lim_{x \rightarrow +\infty} \frac{x \left((1 - m^2)x + 1 + \frac{1}{x} \right)}{x \left(\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - m \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{(1 - m^2)x + 1 + \frac{1}{x}}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - m} \right)$$

$$\text{Si } (1 - m^2) < 0 \quad \text{et} \quad m < 0$$

$$C - \text{à} - d \quad m < -1 \quad \text{Alors :}$$

$$l = \left(\frac{(1 - m^2)(+\infty) + 1 + 0}{\sqrt{1 + 0 + 0} - m} \right) = \frac{-\infty}{1 - m} = -\infty$$

$$\text{Si } (1 - m^2) > 0 \quad \text{et} \quad m < 0$$

$$C - \text{à} - d \quad -1 < m < 0 \quad \text{Alors :}$$

$$l = \left(\frac{(1 - m^2)(+\infty) + 1 + 0}{\sqrt{1 + 0 + 0} - m} \right) = \frac{+\infty}{1 - m} = +\infty$$

$$\text{Si } m = -1 \quad \text{Alors :}$$

$$l = \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + x + 1} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x}}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1} \right)$$

$$= \left(\frac{1 + 0}{\sqrt{1 + 0 + 0} + 1} \right) = \frac{1}{2}$$

D'après cette analyse de limites on peut finalement résumer notre travail dans le système suivant :

$$l = \begin{cases} -\infty & \text{si } m < -1 \\ \frac{1}{2} & \text{si } m = -1 \\ +\infty & \text{si } m > -1 \end{cases}$$

$$\begin{aligned}
4) \quad & \lim_{x \rightarrow -\infty} (\sqrt{1-x^3} + x - 1) \\
&= \lim_{x \rightarrow -\infty} (\sqrt{(1-x)(1+x+x^2)} - (1-x)) \\
&= \lim_{x \rightarrow -\infty} (1-x) \left(\frac{\sqrt{(1-x)(1+x+x^2)}}{(1-x)} - 1 \right) \\
&= \lim_{x \rightarrow -\infty} (1-x) \left(\frac{\sqrt{(1-x)(1+x+x^2)}}{(1-x)^2} - 1 \right) \\
&= \lim_{x \rightarrow -\infty} (1-x) \left(\sqrt{\frac{1+x+x^2}{1-x}} - 1 \right) \\
&= (1+\infty) (\sqrt{-(-\infty)} - 1) = +\infty
\end{aligned}$$

$$\begin{aligned}
5) \quad & \text{Soit } l = \lim_{x \rightarrow -\infty} (x + \sqrt{ax^2 + bx + c}) \\
&= \lim_{x \rightarrow -\infty} \left(x + \sqrt{ax^2 + bx + c} \right) \\
&\quad \times \frac{(x - \sqrt{ax^2 + bx + c})}{(x - \sqrt{ax^2 + bx + c})} \\
&= \lim_{x \rightarrow -\infty} \left(\frac{x^2 - (ax^2 + bx + c)}{x - \sqrt{ax^2 + bx + c}} \right) \\
&= \lim_{x \rightarrow -\infty} \left(\frac{(1-a)x^2 - bx - c}{x - \sqrt{x^2 \left(a + \frac{b}{x} + \frac{c}{x^2} \right)}} \right) \\
&= \lim_{x \rightarrow -\infty} \left(\frac{(1-a)x^2 - bx - c}{x - \sqrt{x^2} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right) \\
&= \lim_{x \rightarrow -\infty} \left(\frac{(1-a)x^2 - bx - c}{x - |x| \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)
\end{aligned}$$

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$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \left(\frac{(1-a)x^2 - bx - c}{x + x \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right) \\
&= \lim_{x \rightarrow -\infty} \frac{x \left((1-a)x - b - \frac{c}{x} \right)}{x \left(1 + \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \right)} \\
&= \lim_{x \rightarrow -\infty} \left(\frac{(1-a)x - b - \frac{c}{x}}{1 + \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right) \\
&= \left(\frac{(1-a)(-\infty) - b - 0}{1 + \sqrt{a + 0 + 0}} \right) = \frac{(1-a)(-\infty)}{1 + \sqrt{a}}
\end{aligned}$$

Si $a < 0$ Alors l n'existe pas

Si $a = 0$ Alors $l = -\infty$

Si $0 < a < 1$ Alors $l = -\infty$

Si $a = 1$ Alors $l = \frac{-b}{1 + \sqrt{a}} = \frac{-b}{2}$

Si $a > 1$ Alors $l = +\infty$

Voici un résumé de cette limite :

$$l = \begin{cases} \text{n'existe pas} & \text{si } a < 0 \\ -\infty & \text{si } 0 \leq a < 1 \\ +\infty & \text{si } a > 1 \\ \frac{-b}{2} & \text{si } a = 1 \end{cases}$$

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Solution N° 41 :

$$1) \lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(3x)}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{2x} \right) \times \frac{1}{\left(\frac{\sin(3x)}{3x} \right)} \times \left(\frac{2x}{3x} \right)$$

$$= (1) \times \frac{1}{(1)} \times \left(\frac{2}{3} \right) = \frac{2}{3}$$

$$2) \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} \right) = l$$

Première méthode : il suffit de remarquer que :

$$\sqrt{3} \sin x - \cos x = 2 \left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right)$$

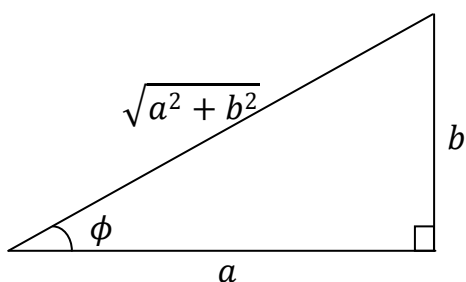
$$= 2 \left(\cos \left(\frac{\pi}{6} \right) \sin x - \sin \left(\frac{\pi}{6} \right) \cos x \right)$$

$$= 2 \sin \left(x - \frac{\pi}{6} \right)$$

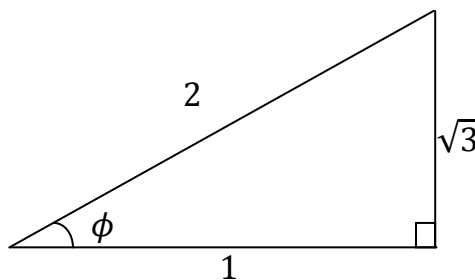
$$l = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin \left(x - \frac{\pi}{6} \right)}{x - \frac{\pi}{6}} = 2 \lim_{\substack{t \rightarrow 0 \\ t = x - \frac{\pi}{6}}} \left(\frac{\sin t}{t} \right) = 2$$

La deuxième méthode consiste à appliquer la transformation trigonométrique suivante :

$$a \cos x + b \sin x = \sqrt{a^2 + b^2} \cos(x - \phi)$$



$$\begin{aligned} \text{Donc } \sqrt{3} \sin x - \cos x &= \sqrt{3} \sin(\pi - x) + \cos(\pi - x) \\ &= \sqrt{(\sqrt{3})^2 + 1^2} \cos(\pi - x - \phi) \\ &= 2 \cos(\pi - x - \phi) \end{aligned}$$



$$\cos \phi = \frac{1}{2} = \cos \left(\frac{\pi}{3} \right)$$

$$\Rightarrow \phi \equiv \pm \frac{\pi}{3} [2\pi]$$

$$\text{On retient la valeur } \phi = \frac{\pi}{3}$$

$$D'où : \sqrt{3} \sin x - \cos x = 2 \cos \left(\pi - x - \frac{\pi}{3} \right)$$

$$\begin{aligned} &= 2 \cos \left(\frac{2\pi}{3} - x \right) = 2 \sin \left(\frac{\pi}{2} - \frac{2\pi}{3} + x \right) \\ &= 2 \sin \left(x - \frac{\pi}{6} \right) \end{aligned}$$

$$\begin{aligned} 3) \lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan x} &= \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \cdot \frac{1}{\left(\frac{\tan x}{x} \right)} \cdot \frac{3x}{x} \\ &= 1 \times \frac{1}{1} \times 3 = 3 \end{aligned}$$

$$\begin{aligned} 4) \lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(4x)} &= \lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} \cdot \frac{1}{\left(\frac{\sin 4x}{4x} \right)} \cdot \frac{5x}{4x} \\ &= 1 \times \frac{1}{1} \times \frac{5}{4} = \frac{5}{4} \end{aligned}$$

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$$5) \lim_{x \rightarrow 0} \left(\frac{\cos(\pi x) - 1}{x} \right) = \lim_{\substack{t \rightarrow 0 \\ t = \pi x}} \left(\frac{\cos t - 1}{\frac{t}{\pi}} \right)$$

$$= \pi \lim_{t \rightarrow 0} \left(\frac{\cos t - 1}{t} \right) \times \frac{1}{t}$$

$$= -\pi \lim_{t \rightarrow 0} \left(\frac{1 - \cos t}{t^2} \right) \times t$$

$$= -\pi \times \frac{1}{2} \times 0 = 0$$

$$6) \lim_{x \rightarrow 1} \frac{\tan(x-1)}{x^2-1} = \lim_{x \rightarrow 1} \frac{\tan(x-1)}{(x-1)(x+1)}$$

$$= \lim_{x \rightarrow 1} \frac{\tan(x-1)}{(x-1)} \times \lim_{x \rightarrow 1} \left(\frac{1}{x+1} \right)$$

$$= \lim_{\substack{t \rightarrow 0 \\ t = x-1}} \left(\frac{\tan t}{t} \right) \times \lim_{x \rightarrow 1} \left(\frac{1}{x+1} \right)$$

$$= 1 \times \left(\frac{1}{1+1} \right) = \frac{1}{2}$$

Solution N° 42 :

$$1) \lim_{x \rightarrow 0} \left(\frac{1 - \cos(6x)}{\sin(4x) \cdot \tan(3x)} \right)$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{1 - \cos 6x}{(6x)^2} \cdot \frac{1}{\frac{\sin 4x}{4x}} \cdot \frac{1}{\frac{\tan 3x}{3x}} \cdot \frac{(6x)^2}{(4x)(3x)} \\ &= \frac{1}{2} \times \frac{1}{1} \times \frac{1}{1} \times \frac{36}{12} = \frac{3}{2} \end{aligned}$$

$$2) \text{ Soit } l = \lim_{x \rightarrow 1} \frac{\tan(\pi x)}{x-1} = \lim_{\substack{t \rightarrow 0 \\ t = x-1}} \frac{\tan(\pi t + \pi)}{t}$$

$$\text{Dabord : } \tan(\pi t + \pi) = \frac{\tan \pi t + \tan \pi}{1 - \tan \pi t \cdot \tan \pi}$$

$$= \frac{\tan(\pi t) + 0}{1 - \tan(\pi t) \cdot 0} = \tan(\pi t)$$

Voici une autre manière pour ce faire :

$$\begin{aligned} \tan(\pi t + \pi) &= \frac{\sin(\pi t + \pi)}{\cos(\pi t + \pi)} = \frac{-\sin(\pi t)}{-\cos(\pi t)} \\ &= \tan(\pi t) \end{aligned}$$

Revenons à nouveau à la limite l :

$$l = \lim_{t \rightarrow 0} \frac{\tan(\pi t + \pi)}{t} = \lim_{t \rightarrow 0} \frac{\tan(\pi t)}{t}$$

$$= \lim_{t \rightarrow 0} \frac{\tan(\pi t)}{\pi t} \times \pi = 1 \times \pi = \pi$$

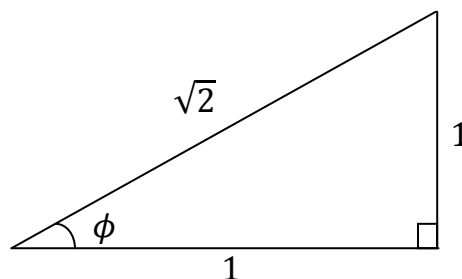
$$3) \lim_{x \rightarrow 0} \frac{\sin^3(2x)}{x^3} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^3 \times 8 = 8$$

$$4) \text{ Soit } l = \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right)$$

$$\sin x - \cos x = 1 \cdot \sin(\pi - x) + 1 \cdot \cos(\pi - x)$$

$$= \sqrt{1^2 + 1^2} \cdot \cos(\pi - x - \phi)$$

$$= \sqrt{2} \cos(\pi - x - \phi)$$



$$\cos \phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \phi \equiv \pm \frac{\pi}{4} [2\pi]$$

On retiendra la valeur $\phi = \frac{\pi}{4}$

$$D' où \sin x - \cos x = \sqrt{2} \cos\left(\pi - x - \frac{\pi}{4}\right)$$

$$= \sqrt{2} \cos\left(\frac{\pi}{2} + \frac{\pi}{4} - x\right)$$

$$= -\sqrt{2} \sin\left(\frac{\pi}{4} - x\right)$$

$$= \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$$

$$\text{Ainsi : } l = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$$

$$= \sqrt{2} \lim_{\substack{t \rightarrow 0 \\ t = x - \frac{\pi}{4}}} \frac{\sin t}{t} = \sqrt{2} \times 1 = \sqrt{2}$$

$$5) \lim_{x \rightarrow 0} \frac{\sin(2x)}{x^3} = \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \cdot \frac{2x}{x^3} = 1 \cdot \frac{2}{0^+} = +\infty$$

$$6) \lim_{x \rightarrow 0^+} \left(\frac{\cos \sqrt{x} - 1}{x} \right) = \lim_{\substack{t \rightarrow 0^+ \\ t = \sqrt{x}}} \left(\frac{\cos t - 1}{t^2} \right) = \frac{-1}{2}$$

Solution N° 43 :

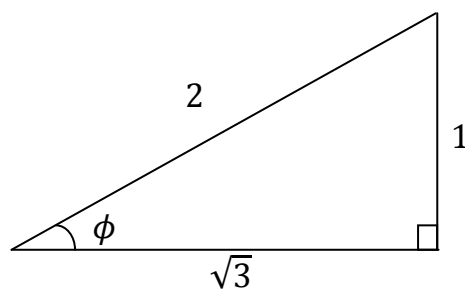
$$1) \text{ Soit } l = \lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sin x - \sqrt{3} \cos x}{x - \frac{\pi}{3}} \right)$$

$$\text{On a : } \sin x - \sqrt{3} \cos x = \sin(\pi - x) - \sqrt{3} \cos(\pi - x)$$

$$= \sqrt{(\sqrt{3})^2 + 1^2} \cos(\pi - x - \phi)$$

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$$= 2 \cos(\pi - x - \phi)$$



$$\cos \phi = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \phi \equiv \pm \frac{\pi}{6} [2\pi]$$

On retiendra la valeur $\phi = \frac{\pi}{6}$

$$D' où \sin x - \sqrt{3} \cos x = 2 \cos\left(\pi - x - \frac{\pi}{6}\right)$$

$$= 2 \cos\left(\frac{5\pi}{6} - x\right) = 2 \sin\left(\frac{\pi}{2} - \frac{5\pi}{6} + x\right) = 2 \sin\left(x - \frac{\pi}{3}\right)$$

$$D' où \quad l = \lim_{x \rightarrow \frac{\pi}{3}} \frac{2 \sin\left(x - \frac{\pi}{3}\right)}{\left(x - \frac{\pi}{3}\right)}$$

$$= 2 \lim_{\substack{t \rightarrow 0 \\ t = x - \frac{\pi}{3}}} \frac{\sin t}{t} = 2 \times 1 = 2$$

On peut éventuellement s'assurer des résultats obtenus en appliquant la règle de l'Hôpital dans le brouillon puisque c'est valable et ça donne zéro/zéro :

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{(\sin x - \sqrt{3} \cos x)}{\left(x - \frac{\pi}{3}\right)} = \lim_{x \rightarrow \frac{\pi}{3}} \frac{(\sin x - \sqrt{3} \cos x)'}{\left(x - \frac{\pi}{3}\right)'}$$

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$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x + \sqrt{3} \sin x}{1} = \frac{\left(\frac{1}{2} + \sqrt{3} \cdot \frac{\sqrt{3}}{2}\right)}{1} = 2$$

2) Soit $l = \lim_{x \rightarrow +\infty} \left(x^2 - \sin \frac{1}{x}\right)$

On a $\forall t > 0 ; \sin t < 0$

Si $x \rightarrow +\infty$ Alors $\frac{1}{x} > 0$

D'où $\sin\left(\frac{1}{x}\right) < \frac{1}{x}$

$$\Rightarrow x^2 - \sin \frac{1}{x} > \underbrace{\left(x^2 - \frac{1}{x}\right)}_{\substack{\text{tend vers } +\infty \\ \text{quand } x \rightarrow +\infty}}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left(x^2 - \sin \frac{1}{x}\right) = +\infty$$

3) On a $\forall x \in \mathbb{R} ; -1 \leq \cos x \leq 1$

$$\Rightarrow \frac{-1}{x^3} \leq \frac{\cos x}{x^3} \leq \frac{1}{x^3}$$

La quantité $\frac{1}{x^3}$ est positive car $x \rightarrow +\infty$

$$\text{D'où } \underbrace{\frac{-1}{x^3}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq \frac{\cos x}{x^3} \leq \underbrace{\frac{1}{x^3}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}}$$

Donc $\lim_{x \rightarrow +\infty} \left(\frac{\cos x}{x^3}\right) = 0$

4) $\lim_{x \rightarrow +\infty} \frac{1 + \sin x}{x^2(2 + \cos x)}$

On a : $0 \leq \sin x \leq x ; \forall x \geq 0$

Donc : $1 \leq \sin x + 1 \leq x + 1 \rightsquigarrow (1)$

On a aussi $\forall x \in \mathbb{R} ; -1 \leq \cos x \leq 1$

Donc : $1 \leq \cos x + 2 \leq 3$

Ainsi : $\frac{1}{3} \leq \frac{1}{\cos x + 2} \leq 1$

D'où $\frac{1}{3x^2} \leq \frac{1}{x^2(\cos x + 2)} \leq \frac{1}{x^2} \rightsquigarrow (2)$

$(1) \times (2) \Rightarrow \frac{1}{3x^2} \leq \frac{\sin x + 1}{x^2(2 + \cos x)} \leq \frac{x + 1}{x^2}$

D'où : $\lim_{x \rightarrow +\infty} \frac{\sin x + 1}{x^2(2 + \cos x)} = 0$

5) Soit $l = \lim_{x \rightarrow -\infty} \left(1 + \frac{x}{2 + \sqrt{x^4 + 1}}\right)$

$$= 1 + \lim_{x \rightarrow -\infty} \left(\frac{x}{2 + \sqrt{x^4 + 1}}\right)$$

$$= 1 + \lim_{x \rightarrow -\infty} \left(\frac{x}{2 + \sqrt{x^4 \left(1 + \frac{1}{x^4}\right)}}\right)$$

$$= 1 + \lim_{x \rightarrow -\infty} \left(\frac{x}{2 + x^2 \sqrt{1 + \frac{1}{x^4}}}\right)$$

$$= 1 + \lim_{x \rightarrow -\infty} \frac{x^2 \left(\frac{1}{x}\right)}{x^2 \left(\frac{2}{x^2} + \sqrt{1 + \frac{1}{x^4}}\right)}$$

$$= 1 + \lim_{x \rightarrow -\infty} \left(\frac{\frac{1}{x}}{\frac{2}{x^2} + \sqrt{1 + \frac{1}{x^4}}}\right)$$

$$= 1 + \lim_{x \rightarrow -\infty} \left(\frac{\frac{1}{x}}{\frac{2}{x^2} + \sqrt{1 + \frac{1}{x^4}}} \right)$$

$$= 1 + \frac{0}{0 + \sqrt{1 + 0}} = 1$$

6) Soit $l = \lim_{x \rightarrow +\infty} \left(\frac{2E(x) + (x - E(x))^2}{x^2} \right)$

On a : $x - 1 \leq E(x) < x$; $\forall x \in \mathbb{R}$

$$\Rightarrow 2(x - 1) \leq 2E(x) \leq 2x \rightsquigarrow (1)$$

Or, on a $-x < -E(x) \leq 1 - x$

D'où : $0 < x - E(x) \leq 1$

C - à - d $0 \leq (x - E(x))^2 < 1 \rightsquigarrow (2)$

(1) + (2) $\Rightarrow$

$$\Rightarrow 2x - 2 \leq 2E(x) + (x - E(x))^2 \leq 2x + 1$$

$$\Rightarrow \frac{2x - 2}{x^2} \leq \frac{2E(x) + (x - E(x))^2}{x^2} \leq \frac{2x + 1}{x^2}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left(\frac{2E(x) + (x - E(x))^2}{x^2} \right) = 0$$

Car $\lim_{x \rightarrow +\infty} \left(\frac{2x - 2}{x^2} \right) = \lim_{x \rightarrow +\infty} \left(\frac{2x + 1}{x^2} \right) = 0$

Solution N° 44 :

1) On effectue la division euclidienne de $(x^2 + 3x - 10)$ par $(x - 2)$ car 2 est une racine simple et on obtient ainsi :

$$\lim_{x \rightarrow 2} \left(\frac{\sqrt{2x} - 2}{x^2 + 3x - 10} \right) = \lim_{x \rightarrow 2} \left(\frac{\sqrt{2x} - 2}{(x - 2)(x + 5)} \right)$$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{2}(\sqrt{x} - \sqrt{2})}{((\sqrt{x})^2 - (\sqrt{2})^2)(x + 5)}$$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{2}(\sqrt{x} - \sqrt{2})}{(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})(x + 5)}$$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{2}}{(\sqrt{x} + \sqrt{2})(x + 5)}$$

$$= \frac{\sqrt{2}}{(\sqrt{2} + \sqrt{2})(2 + 5)} = \frac{1}{14}$$

2) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 1})$

$$= \lim_{x \rightarrow -\infty} \frac{(x + \sqrt{x^2 + 1})(x - \sqrt{x^2 + 1})}{(x - \sqrt{x^2 + 1})}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x^2 - (x^2 + 1)}{x - \sqrt{x^2 + 1}} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{-1}{x - \sqrt{x^2 + 1}} \right)$$

$$= \left(\frac{-1}{-\infty - \sqrt{+\infty}} \right) = 0^+ = 0$$

3) $\lim_{x \rightarrow 1} \left(\frac{2x^3 + 3x^2 - 4x - 1}{x^3 - 1} \right)$

On remarque que 1 est une racine simple de chacun des polynômes du numérateur et du dénominateur donc on peut effectuer la division euclidienne de ces deux polynômes par $(x - 1)$ on obtient :

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$$\begin{aligned}\lim_{x \rightarrow 1} \left(\frac{2x^3 + 3x^2 - 4x - 1}{x^3 - 1} \right) \\&= \lim_{x \rightarrow 1} \frac{(x-1)(2x^2 + 5x + 1)}{(x-1)(x^2 + x + 1)} \\&= \lim_{x \rightarrow 1} \left(\frac{2x^2 + 5x + 1}{x^2 + x + 1} \right) \\&= \left(\frac{2 + 5 + 1}{1 + 1 + 1} \right) = \frac{8}{3}\end{aligned}$$

$$\begin{aligned}4) \quad \lim_{x \rightarrow +\infty} (\sqrt{x+1} - \sqrt{x}) \\&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{(\sqrt{x+1} + \sqrt{x})} \\&= \lim_{x \rightarrow +\infty} \left(\frac{(\sqrt{x+1})^2 - (\sqrt{x})^2}{\sqrt{x+1} + \sqrt{x}} \right) \\&= \lim_{x \rightarrow +\infty} \left(\frac{x+1-x}{\sqrt{x+1} + \sqrt{x}} \right) \\&= \lim_{x \rightarrow +\infty} \left(\frac{1}{\sqrt{x+1} + \sqrt{x}} \right) = \frac{1}{+\infty} = 0^+ = 0\end{aligned}$$

$$\begin{aligned}5) \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x) \\&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{(\sqrt{x^2 + x} + x)} \\&= \lim_{x \rightarrow +\infty} \frac{(x^2 + x) - x^2}{(\sqrt{x^2 + x} + x)} \\&= \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + x} + x}\end{aligned}$$

$$\begin{aligned}&= \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 \left(1 + \frac{1}{x}\right)} + x} \\&= \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2} \cdot \sqrt{\left(1 + \frac{1}{x}\right)} + x} \\&= \lim_{x \rightarrow +\infty} \frac{x}{|x| \cdot \sqrt{\left(1 + \frac{1}{x}\right)} + x} \\&= \lim_{x \rightarrow +\infty} \frac{x}{x \cdot \sqrt{\left(1 + \frac{1}{x}\right)} + x}\end{aligned}$$

$$\begin{aligned}&= \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{\left(1 + \frac{1}{x}\right)} + 1} \\&= \frac{1}{\sqrt{(1+0)} + 1} = \frac{1}{2}\end{aligned}$$

$$6) \quad \lim_{x \rightarrow 2} \left(\frac{\sqrt{x+2} - 2}{\sqrt{2x+5} - 3} \right)$$

On multiplie par les conjugués de chacun des termes en numérateur et en dénominateur on obtient :

$$\begin{aligned}\lim_{x \rightarrow 2} \left(\frac{(\sqrt{x+2})^2 - 2^2}{(\sqrt{2x+5})^2 - 3^2} \right) \times \left(\frac{\sqrt{2x+5} + 3}{\sqrt{x+2} + 2} \right) \\&= \lim_{x \rightarrow 2} \left(\frac{x-2}{2x-4} \right) \times \left(\frac{\sqrt{2x+5} + 3}{\sqrt{x+2} + 2} \right) \\&= \lim_{x \rightarrow 2} \frac{1}{2} \times \left(\frac{\sqrt{2x+5} + 3}{\sqrt{x+2} + 2} \right) \\&= \frac{1}{2} \times \left(\frac{\sqrt{4+5} + 3}{\sqrt{2+2} + 2} \right) = \frac{3}{4}\end{aligned}$$

Solution N° 45 :

$$\begin{aligned}
1) \quad & \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - 3x) \\
&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - 3x)(\sqrt{x^2 + 1} + 3x)}{(\sqrt{x^2 + 1} + 3x)} \\
&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - 3x)(\sqrt{x^2 + 1} + 3x)}{(\sqrt{x^2 + 1} + 3x)} \\
&= \lim_{x \rightarrow +\infty} \frac{(x^2 + 1) - (3x)^2}{\sqrt{x^2 + 1} + 3x} \\
&= \lim_{x \rightarrow +\infty} \left(\frac{-8x^2 + 1}{\sqrt{x^2 + 1} + 3x} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{-8x^2 + 1}{\sqrt{x^2 \left(1 + \frac{1}{x^2}\right)} + 3x} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{-8x^2 + 1}{|x| \cdot \sqrt{\left(1 + \frac{1}{x^2}\right)} + 3x} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{-8x^2 + 1}{x \cdot \sqrt{\left(1 + \frac{1}{x^2}\right)} + 3x} \right) \\
&= \lim_{x \rightarrow +\infty} \frac{x \cdot \left(-8x + \frac{1}{x}\right)}{x \cdot \left(\sqrt{\left(1 + \frac{1}{x^2}\right)} + 3\right)} \\
&= \lim_{x \rightarrow +\infty} \left(\frac{-8x + \frac{1}{x}}{\sqrt{1 + \frac{1}{x^2}} + 3} \right) = \frac{-\infty + 0}{\sqrt{1 + 0} + 3} = -\infty
\end{aligned}$$

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$$\begin{aligned}
2) \quad & \lim_{x \rightarrow 0} \frac{\sqrt{4x^2 + x^3}}{|2x + x^3|} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2(4 + x)}}{|x(2 + x^2)|} \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{x^2} \cdot \sqrt{4 + x}}{|x| \cdot |2 + x^2|} = \lim_{x \rightarrow 0} \frac{|x| \cdot \sqrt{4 + x}}{|x| \cdot |2 + x^2|} \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{4 + x}}{|2 + x^2|} = \frac{\sqrt{4 + 0}}{|2 + 0|} = 1 \\
3) \quad & \lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{2}{x^2 - 1} \right) \\
&= \lim_{x \rightarrow 1} \left(\frac{x + 1}{x^2 - 1} - \frac{2}{x^2 - 1} \right) \\
&= \lim_{x \rightarrow 1} \left(\frac{x - 1}{x^2 - 1} \right) = \lim_{x \rightarrow 1} \frac{(x - 1)}{(x - 1)(x + 1)} \\
&= \lim_{x \rightarrow 1} \left(\frac{1}{x + 1} \right) = \frac{1}{2} \\
4) \quad & \lim_{x \rightarrow +\infty} \left(\frac{x}{\sqrt{x + 1}} - \frac{x}{\sqrt{x - 1}} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{x\sqrt{x - 1}}{\sqrt{x + 1} \cdot \sqrt{x - 1}} - \frac{x\sqrt{x + 1}}{\sqrt{x + 1} \cdot \sqrt{x - 1}} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} - \frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}} \right) \\
&= \lim_{x \rightarrow +\infty} \frac{\left(\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} - \frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}} \right) \left(\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} + \frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}} \right)}{\left(\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} + \frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}} \right)} \\
&= \lim_{x \rightarrow +\infty} \left(\frac{\left(\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} \right)^2 - \left(\frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}} \right)^2}{\frac{\sqrt{x^3 - x^2}}{\sqrt{x^2 - 1}} + \frac{\sqrt{x^3 + x^2}}{\sqrt{x^2 - 1}}} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{\left(\frac{x^3 - x^2}{x^2 - 1} \right) - \left(\frac{x^3 + x^2}{x^2 - 1} \right)}{\sqrt{\frac{x^3 - x^2}{x^2 - 1}} + \sqrt{\frac{x^3 + x^2}{x^2 - 1}}} \right)
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow +\infty} \left(\frac{\left(\frac{-2x^2}{x^2 - 1} \right)}{\sqrt{\frac{x^3 - x^2}{x^2 - 1}} + \sqrt{\frac{x^3 + x^2}{x^2 - 1}}} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{\frac{-2x^2}{x^2 \left(1 - \frac{1}{x^2} \right)}}{\sqrt{\frac{x^3 - x^2}{x^2 - 1}} + \sqrt{\frac{x^3 + x^2}{x^2 - 1}}} \right) \\
&= \lim_{x \rightarrow +\infty} \left(\frac{\frac{-2}{1 - \frac{1}{x^2}}}{\sqrt{\frac{x^3 - x^2}{x^2 - 1}} + \sqrt{\frac{x^3 + x^2}{x^2 - 1}}} \right) \\
&= \left(\frac{\frac{-2}{1 - 0}}{\sqrt{+\infty} + \sqrt{+\infty}} \right) = \frac{-2}{+\infty} = 0^- = 0
\end{aligned}$$

$$\begin{aligned}
5) \quad \lim_{x \rightarrow 0} \left(\frac{x + \sqrt{|x|}}{x - \sqrt{|x|}} \right) \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{|x|} \cdot \left(\frac{x}{\sqrt{|x|}} + 1 \right)}{\sqrt{|x|} \cdot \left(\frac{x}{\sqrt{|x|}} - 1 \right)} \\
&= \lim_{x \rightarrow 0} \frac{\left(\frac{x}{\sqrt{|x|}} + 1 \right)}{\left(\frac{x}{\sqrt{|x|}} - 1 \right)} = \frac{0 + 1}{0 - 1} = -1
\end{aligned}$$

Mais pourquoi $\lim_{x \rightarrow 0} \frac{x}{\sqrt{|x|}} = 0$?

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{x}{\sqrt{|x|}} \right) = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \left(\frac{x}{\sqrt{x}} \right) = \lim_{\substack{x \rightarrow 0 \\ x > 0}} \sqrt{x} = 0$$

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} \left(\frac{x}{\sqrt{|x|}} \right) = \lim_{\substack{x \rightarrow 0 \\ x < 0}} \left(\frac{x}{\sqrt{-x}} \right) = \lim_{\substack{x \rightarrow 0 \\ x < 0}} -\sqrt{-x} = 0$$

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$$\begin{aligned}
6) \quad \lim_{x \rightarrow +\infty} \left(x \sqrt{\frac{x}{x-1}} - x - 1 \right) \\
&= -1 + \lim_{x \rightarrow +\infty} \left(\sqrt{\frac{x^3}{x-1}} - x \right) \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{\left(\sqrt{\frac{x^3}{x-1}} - x \right) \left(\sqrt{\frac{x^3}{x-1}} + x \right)}{\left(\sqrt{\frac{x^3}{x-1}} + x \right)} \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{\left(\sqrt{\frac{x^3}{x-1}} \right)^2 - x^2}{\left(\sqrt{\frac{x^3}{x-1}} + x \right)} \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{\left(\frac{x^3 - x^3 + x^2}{x-1} \right)}{\left(\sqrt{\frac{x^3}{x-1}} + x \right)} \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{\left(\frac{x^2}{x-1} \right)}{\left(\sqrt{\frac{x^3}{x-1}} + x \right)} \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{x \left(\frac{x}{x-1} \right)}{x \left(\frac{1}{x} \sqrt{\frac{x^3}{x-1}} + 1 \right)} \\
&= -1 + \lim_{x \rightarrow +\infty} \frac{\left(\frac{x}{x-1} \right)}{\left(\sqrt{\frac{x^3}{x^3 - x^2}} + 1 \right)} \\
&= -1 + \frac{1}{\sqrt{1} + 1} = \frac{-1}{2}
\end{aligned}$$

Solution N° 46 :

$$\begin{aligned}
 1) \quad & \lim_{x \rightarrow 0} \left(\frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\tan x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{(\sqrt{1 + \sin x})^2 - (\sqrt{1 - \sin x})^2}{\tan x \cdot (\sqrt{1 + \sin x} + \sqrt{1 - \sin x})} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{(1 + \sin x) - (1 - \sin x)}{\tan x \cdot (\sqrt{1 + \sin x} + \sqrt{1 - \sin x})} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{2 \sin x}{\frac{\sin x}{\cos x} \cdot (\sqrt{1 + \sin x} + \sqrt{1 - \sin x})} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{2 \cos x}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}} \right) \\
 &= \left(\frac{2 \cos 0}{\sqrt{1 + \sin 0} + \sqrt{1 - \sin 0}} \right) = 1
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin x}{\cos x} - \sin x}{x \cdot x^2} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1}{\cos x} - 1 \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{1}{\cos x} \right) \\
 &= 1 \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}
 \end{aligned}$$

$$3) \text{ Soit } l = \lim_{x \rightarrow 0} \left(\frac{x(1 - \cos x)}{\sin(3x) - 3 \sin x} \right)$$

Je rappelle juste que :

$$\sin^3(x) = \frac{1}{4} (3 \sin x - \sin(3x))$$

$$\begin{aligned}
 \text{Donc } l &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{x^3}{\sin(3x) - 3 \sin x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{x^3}{-4 \sin^3 x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{x}{\sin x} \right)^3 \cdot \left(\frac{-1}{4} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{1}{\frac{\sin x}{x}} \right)^3 \cdot \left(\frac{-1}{4} \right) \\
 &= \frac{1}{2} \cdot \left(\frac{1}{1} \right)^3 \cdot \left(\frac{-1}{4} \right) = \frac{-1}{8}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos x}{\sqrt{x}} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos x}{\sqrt{x}} \right) \left(\frac{x\sqrt{x}}{x\sqrt{x}} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos x}{x^2} \right) (x\sqrt{x}) \\
 &= \frac{1}{2} \times 0 = 0
 \end{aligned}$$

Solution N° 47 :

$$\begin{aligned}
 1) \quad & \lim_{x \rightarrow -2} \left(\frac{-1}{3} x^2 - 5x + 7 \right) \\
 &= \frac{-1}{3} (-2)^2 - 5(-2) + 7 = \frac{47}{3}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \lim_{x \rightarrow 3} \frac{1}{(3 - x)^2} = \lim_{x \rightarrow 3^\pm} \frac{1}{(3 - x)^2} \\
 &= \lim_{\substack{t \rightarrow 0^\pm \\ t = x - 3}} \left(\frac{1}{t^2} \right) = \frac{1}{0^+} = +\infty
 \end{aligned}$$

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$$\begin{aligned} 3) \quad \lim_{x \rightarrow +\infty} (x^3 + x) &= \lim_{x \rightarrow +\infty} x^3 \left(1 + \frac{1}{x^2}\right) \\ &= (+\infty)(1 + 0) = +\infty \end{aligned}$$

On pourrait écrire directement :

$$\lim_{x \rightarrow +\infty} (x^3 + x) = \lim_{x \rightarrow +\infty} x^3 = +\infty$$

$$\begin{aligned} 4) \quad \lim_{x \rightarrow \sqrt{2}} (\sqrt{2}x^3 - 3x^2 - x) \\ = \sqrt{2}(2)^3 - 3(\sqrt{2})^2 - \sqrt{2} = -(2 + \sqrt{2}) \end{aligned}$$

$$\begin{aligned} 5) \quad \lim_{x \rightarrow 0} \left(\frac{2}{x} + \frac{1}{x^2}\right) &= \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{2}{x} + 1\right) = +\infty \\ &= \begin{cases} \frac{1}{0^+} \left(\frac{2}{0^+} + 1\right) = (+\infty)(+\infty) = +\infty & \text{si } x > 0 \\ \frac{1}{0^-} \left(\frac{2}{0^-} + 1\right) = (-\infty)(-\infty) = +\infty & \text{si } x < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} 6) \quad \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) &= \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{x}} \left(\frac{1}{\sqrt{x}} - 1\right) \\ &= \frac{1}{\sqrt{0^+}} \left(\frac{1}{\sqrt{0^+}} - 1\right) = (+\infty)(+\infty - 1) = +\infty \end{aligned}$$

Solution N° 48 :

$$\begin{aligned} 1) \quad \lim_{x \rightarrow +\infty} \left(\frac{x^5 + x^4 + 2}{x^2 - 1}\right) \\ = \lim_{x \rightarrow +\infty} \frac{x^5 \left(1 + \frac{1}{x} + \frac{2}{x^5}\right)}{x^2 \left(1 - \frac{1}{x^2}\right)} \\ = \lim_{x \rightarrow +\infty} \left(\frac{x^5}{x^2}\right) \left(\frac{1 + \frac{1}{x} + \frac{2}{x^5}}{1 - \frac{1}{x^2}}\right) \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow +\infty} (x^3) \left(\frac{1 + \frac{1}{x} + \frac{2}{x^5}}{1 - \frac{1}{x^2}}\right) \\ &= (+\infty) \left(\frac{1 + 0^+ + 0^+}{1 - 0^+}\right) = +\infty \end{aligned}$$

On peut directement répondre ainsi :

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(\frac{x^5 + x^4 + 2}{x^2 - 1}\right) &= \lim_{x \rightarrow +\infty} \left(\frac{x^5}{x^2}\right) = \lim_{x \rightarrow +\infty} x^3 \\ &= +\infty \end{aligned}$$

$$\begin{aligned} 2) \quad \lim_{x \rightarrow -\infty} \left(\frac{x - 1}{x^2 + x + 5}\right) &= \lim_{x \rightarrow -\infty} \frac{x \left(1 - \frac{1}{x}\right)}{x^2 \left(1 + \frac{1}{x} + \frac{5}{x^2}\right)} \\ &= \lim_{x \rightarrow -\infty} \left(\frac{x}{x^2}\right) \left(\frac{1 - \frac{1}{x}}{1 + \frac{1}{x} + \frac{5}{x^2}}\right) \\ &= \lim_{x \rightarrow -\infty} \left(\frac{1}{x}\right) \left(\frac{1 - \frac{1}{x}}{1 + \frac{1}{x} + \frac{5}{x^2}}\right) \\ &= \left(\frac{1}{-\infty}\right) \left(\frac{1 - 0^-}{1 + 0^- + 0^+}\right) = 0^- = 0 \end{aligned}$$

On pourrait répondre directement ainsi :

$$\lim_{x \rightarrow -\infty} \left(\frac{x - 1}{x^2 + x + 5}\right) = \lim_{x \rightarrow -\infty} \left(\frac{x}{x^2}\right) = 0$$

$$\begin{aligned} 3) \quad \lim_{x \rightarrow -\infty} \frac{(x + 1)^2 \times (2x - 7)^2}{4x^3 + x + 5} \\ = \lim_{x \rightarrow -\infty} \frac{(x^2)(2x)^2}{4x^3} = \lim_{x \rightarrow -\infty} x = -\infty \end{aligned}$$

$$4) \quad \lim_{x \rightarrow +\infty} \left(\frac{1 - 3x}{4x + 7}\right) = \lim_{x \rightarrow +\infty} \frac{-3x \left(1 - \frac{1}{3x}\right)}{4x \left(1 + \frac{7}{4x}\right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x}{4x} \right) \left(\frac{1 - \frac{1}{3x}}{1 + \frac{1}{4x}} \right)$$

$$= \left(\frac{-3}{4} \right) \left(\frac{1 - 0^+}{1 + 0^+} \right) = \frac{-3}{4}$$

On pourrait répondre directement ainsi :

$$\lim_{x \rightarrow +\infty} \left(\frac{1 - 3x}{4x + 7} \right) = \lim_{x \rightarrow +\infty} \left(\frac{-3x}{4x} \right) = \frac{-3}{4}$$

$$5) \lim_{x \rightarrow -\infty} \left(\frac{-4x^3 + 5x + 9}{7x^3 - 6} \right) = \lim_{x \rightarrow -\infty} \left(\frac{-4x^3}{7x^3} \right) = \frac{-4}{7}$$

$$6) \lim_{|x| \rightarrow +\infty} \frac{\sqrt{5} x^2 (2 - x^2)^3}{(x^4 - 1)^2}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{\sqrt{5} x^2 (-x^2)^3}{(x^4)^2} = \lim_{x \rightarrow \pm\infty} \frac{-\sqrt{5} x^8}{x^8} = -\sqrt{5}$$

Solution N° 49 :

$$1) \lim_{|x| \rightarrow +\infty} \frac{3x - x^4 + x(1 - 5x^2)}{(x^2 + 1)(2 - 3x^3)}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{-x^4}{(x^2)(-3x^3)} = \lim_{x \rightarrow \pm\infty} \left(\frac{x^4}{x^5} \right) \left(\frac{1}{3} \right)$$

$$= \lim_{x \rightarrow \pm\infty} \left(\frac{1}{x} \times \frac{1}{3} \right) = \left(\frac{1}{\pm\infty} \right) \left(\frac{1}{3} \right) = 0^{\pm} \cdot \left(\frac{1}{3} \right) = 0$$

$$2) \lim_{x \rightarrow 3} \left(\frac{x - 3}{x^2 - 2x - 3} \right) = \lim_{x \rightarrow 3} \frac{(x - 3)}{(x - 3)(x + 1)}$$

$$= \lim_{x \rightarrow 3} \left(\frac{1}{x + 1} \right) = \frac{1}{4}$$

On pourrait utiliser la division euclidienne juste pour factoriser le dénominateur car 3 est une racine simple pour ce polynôme

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$$3) \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{2x^2 + x - 3} \right)$$

On effectue la division euclidienne des polynômes du numérateur et du dénominateur par $(x - 1)$ puisque 1 est une racine simple des dites polynômes et On obtient ainsi :

$$\lim_{x \rightarrow 1} \frac{(x - 1)(x + 2)}{(x - 1)(2x + 3)} = \lim_{x \rightarrow 1} \left(\frac{x + 2}{2x + 3} \right)$$

$$= \frac{1 + 2}{2 + 3} = \frac{3}{5}$$

$$4) \lim_{x \rightarrow 2} \left(\frac{4x^3 - 5x - 22}{x^2 - x - 2} \right)$$

On effectue la division euclidienne des polynômes du numérateur et du dénominateur par $(x - 2)$ puisque 2 est une racine simple des dites polynômes et On obtient ainsi :

$$\lim_{x \rightarrow 2} \frac{(x - 2)(4x^2 + 8x + 11)}{(x - 2)(x + 1)}$$

$$= \lim_{x \rightarrow 2} \left(\frac{4x^2 + 8x + 11}{x + 1} \right) = \frac{16 + 16 + 11}{2 + 1} = \frac{43}{3}$$

$$5) \lim_{x \rightarrow 2} \left(\frac{2x^3 - 7x^2 + 4x + 4}{x^3 - x^2 - 8x + 12} \right)$$

On effectue la division euclidienne des polynômes du numérateur et du dénominateur par $(x - 2)$ puisque 2 est une racine simple des dites polynômes et On obtient ainsi :

$$\lim_{x \rightarrow 2} \frac{(x - 2)(2x^2 - 3x - 2)}{(x - 2)(x^2 + x - 6)}$$

$$= \lim_{x \rightarrow 2} \left(\frac{2x^2 - 3x - 2}{x^2 + x - 6} \right) = \lim_{x \rightarrow 2} \frac{(x - 2)(2x + 1)}{(x - 2)(x + 3)}$$

$$= \lim_{x \rightarrow 2} \left(\frac{2x+1}{x+3} \right) = \frac{4+1}{2+3} = \frac{5}{5} = 1$$

$$\begin{aligned} 6) \quad \lim_{x \rightarrow 3} \left(\frac{x^4 + 3x^3 - 7x^2 - 27x - 18}{x^4 - 3x^3 - 7x^2 + 27x - 18} \right) \\ = \lim_{x \rightarrow 3} \frac{(x-3)(x^3 + 6x^2 + 11x + 6)}{(x-3)(x^3 - 7x + 6)} \\ = \frac{27 + 54 + 33 + 6}{27 - 21 + 6} = 10 \end{aligned}$$

Solution N° 50 :

$$\begin{aligned} 1) \quad \lim_{x \rightarrow -\sqrt{3}} \left(\frac{x + \sqrt{3}}{3 - x^2} \right) &= \lim_{x \rightarrow -\sqrt{3}} \frac{(x + \sqrt{3})}{(\sqrt{3} - x)(\sqrt{3} + x)} \\ &= \lim_{x \rightarrow -\sqrt{3}} \left(\frac{1}{\sqrt{3} - x} \right) = \frac{1}{2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} 2) \quad \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right) \\ = \lim_{x \rightarrow 1} \left(\frac{x+1}{x^2-1} - \frac{2}{x^2-1} \right) \\ = \lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(x+1)} \\ = \lim_{x \rightarrow 1} \left(\frac{1}{x+1} \right) = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 3) \quad \lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x^2 - 4} \right) &= \lim_{x \rightarrow 2} \left(\frac{x^3 - 2^3}{x^2 - 2^2} \right) \\ &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)} \\ &\quad \lim_{x \rightarrow 2} \left(\frac{x^2 + 2x + 4}{x+2} \right) \\ &= \frac{4 + 4 + 4}{2 + 2} = 3 \end{aligned}$$

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$$4) \quad \lim_{x \rightarrow +\infty} \sqrt{2x+9} = \sqrt{+\infty} = +\infty$$

$$\begin{aligned} 5) \quad \lim_{x \rightarrow +\infty} (x - \sqrt{x}) &= \lim_{x \rightarrow +\infty} \sqrt{x}(\sqrt{x} - 1) \\ &= \sqrt{+\infty}(\sqrt{+\infty} - 1) = +\infty \end{aligned}$$

$$6) \quad \lim_{x \rightarrow -\infty} \sqrt{3-x} = \sqrt{3 - (-\infty)} = \sqrt{+\infty} = +\infty$$

Solution N° 51 :

$$1) \quad \lim_{x \rightarrow +\infty} (x + \sqrt{x^2 + 1}) = +\infty + \infty = +\infty$$

$$\begin{aligned} 2) \quad \lim_{x \rightarrow +\infty} (x - \sqrt{4x^2 + 1}) \\ = \lim_{x \rightarrow +\infty} \frac{(x - \sqrt{4x^2 + 1})(x + \sqrt{4x^2 + 1})}{(x + \sqrt{4x^2 + 1})} \end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x^2 - (4x^2 + 1)}{x + \sqrt{4x^2 + 1}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x^2 - 1}{x + \sqrt{4x^2 + 1}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x^2 - 1}{x + \sqrt{x^2 \left(4 + \frac{1}{x^2} \right)}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x^2 - 1}{x + \sqrt{x^2} \cdot \sqrt{4 + \frac{1}{x^2}}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x^2 - 1}{x + |x| \cdot \sqrt{4 + \frac{1}{x^2}}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x^2 - 1}{x + x\sqrt{4 + \frac{1}{x^2}}} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(-3x - \frac{1}{x} \right)}{x \left(1 + \sqrt{4 + \frac{1}{x^2}} \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-3x - \frac{1}{x}}{1 + \sqrt{4 + \frac{1}{x^2}}} \right)$$

$$= \frac{-\infty - 0}{1 + \sqrt{4 + 0}} = -\infty$$

$$3) \lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 + 3x - 1}}{x + 5} = \lim_{x \rightarrow +\infty} \sqrt{\frac{4x^2 + 3x - 1}{x^2 + 10x + 25}}$$

$$= \sqrt{\lim_{x \rightarrow +\infty} \left(\frac{4x^2}{x^2} \right)} = \sqrt{4} = 2$$

Remarque : On a le droit d'écrire :

$$x + 5 = \sqrt{(x + 5)^2}$$

Car $x + 5 > 0$ selon $x \rightarrow +\infty$

$$4) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+1} - 1}{x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}{x(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x+1})^2 - 1^2}{x(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{2}$$

$$5) \lim_{x \rightarrow +\infty} (\sqrt{x+1} - \sqrt{x})$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{(\sqrt{x+1} + \sqrt{x})}$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+1})^2 - (\sqrt{x})^2}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x+1-x}{\sqrt{x+1} + \sqrt{x}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{\sqrt{x+1} + \sqrt{x}} \right) = \frac{1}{+\infty} = 0^+ = 0$$

$$6) \lim_{x \rightarrow +\infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x + \sqrt{x}} - \sqrt{x})(\sqrt{x + \sqrt{x}} + \sqrt{x})}{(\sqrt{x + \sqrt{x}} + \sqrt{x})}$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x + \sqrt{x}})^2 - (\sqrt{x})^2}{(\sqrt{x + \sqrt{x}} + \sqrt{x})}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x + \sqrt{x} - \sqrt{x}}{\sqrt{x + \sqrt{x}} + \sqrt{x}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x}}{\sqrt{x + \sqrt{x}} + \sqrt{x}} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x}}{\sqrt{x \left(1 + \frac{1}{\sqrt{x}} \right)} + \sqrt{x}} \right)$$

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$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{\sqrt{x} \left(\sqrt{1 + \frac{1}{\sqrt{x}}} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{\sqrt{1 + \frac{1}{\sqrt{x}}} + 1} \right) = \frac{1}{\sqrt{1 + 0} + 1} = \frac{1}{2}$$

Solution N° 52 :

Pour qu'on ait $\lim_{x \rightarrow 1} g(x) = 4$

Il faut et il suffit de vérifier :

$$\lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^-} g(x) = 4$$

$$C - \text{à} - d : \begin{cases} \text{et } \lim_{x \rightarrow 1^+} (x + \alpha)(x + \beta) = 4 \\ \text{et } \lim_{x \rightarrow 1^-} (x + \alpha)(x + 2) = 4 \end{cases}$$

$$C - \text{à} - d : \begin{cases} \text{et } (1 + \alpha)(1 + \beta) = 4 \\ \text{et } (1 + \alpha)(1 + 2) = 4 \end{cases}$$

$$C - \text{à} - d : \begin{cases} \text{et } \beta = 2 \\ \text{et } \alpha = \frac{1}{3} \end{cases}$$

Solution N° 53 :

1) On a : $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$; $\forall x \neq 0$

$$\Rightarrow 1 \leq 2 + \sin\left(\frac{1}{x}\right) \leq 3$$

$$\Rightarrow \frac{1}{x^2} \leq \left(\frac{2 + \sin\left(\frac{1}{x}\right)}{x^2} \right) \leq \frac{3}{x^2}$$

$$\Rightarrow (\forall x \in \mathbb{R}^*) ; \frac{1}{x^2} \leq f(x)$$

2) comme $\lim_{x \rightarrow 0^\pm} \left(\frac{1}{x^2} \right) = \frac{1}{0^+} = +\infty$

$$\text{Et comme } \frac{1}{x^2} \leq f(x)$$

Alors d'après les propriétés des limites et ordre, on en déduit que :

$$\lim_{x \rightarrow 0} f(x) = +\infty$$

Solution N° 54 :

1) On a : $-1 \leq \cos x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow \frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x} ; \forall x > 0$$

$$\Rightarrow |g(x)| \leq \frac{1}{x} ; \forall x \in \mathbb{R}_+$$

2) comme $\underbrace{\frac{-1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}} \leq g(x) \leq \underbrace{\frac{1}{x}}_{\substack{\text{tend vers } 0 \\ \text{quand } x \rightarrow +\infty}}$

$$\Rightarrow \lim_{x \rightarrow +\infty} g(x) = 0$$

On peut répondre encore ainsi :

$$\text{Comme } 0 \leq |g(x)| \leq \frac{1}{x} \xrightarrow{x \rightarrow \infty} 0$$

$$\text{Alors : } \lim_{x \rightarrow +\infty} g(x) = 0$$

Solution N° 55 :

1) On a : $\left| \sin\left(\frac{1}{x^3}\right) \right| \leq 1$; $\forall x \neq 0$

$$\Rightarrow \left| x^2 \sin\left(\frac{1}{x^3}\right) \right| \leq x^2 ; \forall x \neq 0$$

$$\Rightarrow 0 \leq |h(x)| \leq x^2 \rightarrow 0$$

$$\Rightarrow \lim_{x \rightarrow 0} h(x) = 0$$

Solution N° 56 :

1) On a $-1 \leq \sin x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow -3 \leq -3 \sin x \leq 3$$

$$\Rightarrow x^2 - 3 \leq x^2 - 3 \sin x \leq 3 + x^2$$

$$\Rightarrow (x^2 - 3) \leq k(x) ; \quad \forall x \in \mathbb{R}$$

2) comme $(x^2 - 3) \leq k(x) ; \quad \forall x \in \mathbb{R}$

Et comme $\lim_{x \rightarrow \pm\infty} (x^2 - 3) = +\infty$

Alors $\lim_{x \rightarrow -\infty} k(x) = \lim_{x \rightarrow +\infty} k(x) = +\infty$

Solution N° 57 :

1) comme $-1 \leq \cos x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow -1 - |x + 1| \leq \cos x - |x + 1| \leq 1 - |x + 1|$$

$$\Rightarrow \forall x \leq -1 :$$

$$-1 - (x + 1) \leq \cos x - (x + 1) \leq 1 - (x + 1)$$

Remarque $x \leq -1 \Rightarrow |x + 1| = -(x + 1)$

$$\Rightarrow \frac{2 + x}{x} \leq \frac{\cos x - (x + 1)}{x} \leq \frac{x}{x} ; \quad \frac{1}{x} < 0$$

$$\Rightarrow \frac{2}{x} + 1 \leq f(x) \leq 1$$

$$\Rightarrow \frac{2}{x} \leq f(x) - 1 \leq 0$$

$$\Rightarrow \frac{2}{x} \leq f(x) - 1 \leq 0 \leq -\frac{2}{x}$$

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$$\Rightarrow |f(x) - 1| \leq \frac{-2}{x}$$

2) comme $0 \leq |f(x) - 1| \leq \frac{-2}{x} \rightarrow 0$

Alors selon les propriétés des limites et ordre on en déduit que :

$$\lim_{x \rightarrow -\infty} |f(x) - 1| = 0$$

$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = 1$$

3) Quand $x \rightarrow +\infty$ Alors $x > 0$

$$\Rightarrow f(x) = \frac{\sin x}{\sqrt{x}}$$

On a : $-1 \leq \sin x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow -\frac{1}{\sqrt{x}} \leq \frac{\sin x}{\sqrt{x}} \leq \frac{1}{\sqrt{x}} ; \quad \forall x \in \mathbb{R}_+^*$$

Comme $\lim_{x \rightarrow +\infty} \frac{-1}{\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = 0$

Alors d'après les propriétés des limites et ordre on en déduit que :

$$\lim_{x \rightarrow +\infty} \frac{\sin x}{\sqrt{x}} = 0$$

Solution N° 58 :

1) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{\sqrt{1 + \sin x} - 1}{x} \right)$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{1 + \sin x} - 1)(\sqrt{1 + \sin x} + 1)}{x(\sqrt{1 + \sin x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1 + \sin x - 1}{x(\sqrt{1 + \sin x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x(\sqrt{1 + \sin x} + 1)}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1}{\sqrt{1 + \sin x} + 1} \right)$$

$$= 1 \cdot \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2}$$

2) On a : $-1 \leq \sin x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow 0 \leq 1 + \sin x \leq 2$$

$$\Rightarrow -1 \leq \sqrt{1 + \sin x} - 1 \leq 1$$

$$\Rightarrow |\sqrt{1 + \sin x} - 1| \leq 1$$

$$\Rightarrow \frac{|\sqrt{1 + \sin x} - 1|}{|x|} \leq \frac{1}{|x|} ; \forall x \in \mathbb{R}^*$$

$$\Rightarrow |f(x)| \leq \frac{1}{|x|} ; \forall x \in \mathbb{R}^*$$

3) comme $0 \leq |f(x)| \leq \frac{1}{|x|}$; $\forall x \in \mathbb{R}^*$

Et comme $\lim_{x \rightarrow \pm\infty} \frac{1}{|x|} = \lim_{x \rightarrow \pm\infty} \frac{1}{|x|} = 0$

Alors d'après les propriétés des limites et ordre on en déduit que :

$$\lim_{x \rightarrow \pm\infty} |f(x)| = \lim_{x \rightarrow \pm\infty} f(x) = 0$$

Solution N° 59 :

1) On a : $x - 1 < E(x) \leq x$; $\forall x \in \mathbb{R}$

Et on a $-1 \leq \sin x \leq 1$; $\forall x \in \mathbb{R}$

$$\Rightarrow x - 2 < E(x) + \sin x \leq x + 1$$

$$\Rightarrow \forall x \in \mathbb{R} ; x - 2 < f(x) \leq x + 1$$

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2) comme $\underbrace{(x - 2)}_{\substack{\text{tend vers } +\infty \\ \text{quand } x \rightarrow +\infty}} < f(x)$

Alors : $\lim_{x \rightarrow +\infty} f(x) = +\infty$

Comme $f(x) \leq \underbrace{(x + 1)}_{\substack{\text{tend vers } -\infty \\ \text{quand } x \rightarrow -\infty}}$

Alors : $\lim_{x \rightarrow -\infty} f(x) = -\infty$

On a : $\forall x \in \mathbb{R} ; x - 2 < f(x) \leq x + 1$

$$\Rightarrow \forall x > 0 ; \frac{x - 2}{x} < \frac{f(x)}{x} \leq \frac{x + 1}{x}$$

$$\Rightarrow \forall x > 0 ; 1 - \frac{2}{x} < \frac{f(x)}{x} \leq 1 + \frac{1}{x}$$

comme $\lim_{x \rightarrow \pm\infty} \left(1 - \frac{2}{x}\right) = \lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right) = 1$

Alors d'après les propriétés des limites et ordre on en déduit que :

$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = 1$$

3) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} E(x) + \lim_{x \rightarrow 0^+} \sin x = 0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} E(x) + \lim_{x \rightarrow 0^-} \sin x = -1$$

Donc f n'est pas du tout continue en 0

Solution N° 60 :

1) On a : $\forall x \in \mathbb{R}^* ; \frac{1}{x} - 1 \leq E\left(\frac{1}{x}\right) < \frac{1}{x}$

$$\Rightarrow \frac{x^2}{x} - x^2 \leq x^2 E\left(\frac{1}{x}\right) < x$$

$$\Rightarrow x - x^2 - x \leq x^2 E\left(\frac{1}{x}\right) - x < x - x$$

$$\Rightarrow -x^2 \leq f(x) - x < 0$$

$$\Rightarrow -x^2 \leq 0 \leq f(x) - x < x^2$$

$$\Rightarrow -x^2 \leq f(x) - x < x^2$$

$$\Rightarrow |f(x) - x| < x^2$$

2) comme $0 \leq |f(x) - x| < x^2$

Et comme $\lim_{x \rightarrow 0} x^2 = \lim_{x \rightarrow 0} 0 = 0$

Alors : $\lim_0 |f(x) - x| = 0$

$$\Rightarrow \begin{cases} \text{oubien} & \lim_{x \rightarrow 0} (f(x) - x) = 0 \\ \text{oubien} & \lim_{x \rightarrow 0} (x - f(x)) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \text{oubien} & \lim_{x \rightarrow 0} f(x) - \lim_{x \rightarrow 0} x = 0 \\ \text{oubien} & \lim_{x \rightarrow 0} x - \lim_{x \rightarrow 0} f(x) = 0 \end{cases}$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = 0$$

3) $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x^2 E\left(\frac{1}{x}\right)$

$$= \left(\lim_{x \rightarrow -\infty} x^2\right) \cdot \left(\lim_{x \rightarrow -\infty} E\left(\frac{1}{x}\right)\right)$$

$$= \left(\lim_{x \rightarrow -\infty} x^2\right) \cdot \left(\lim_{\substack{t \rightarrow 0 \\ t = \frac{1}{x}}} E(t)\right)$$

$$= (+\infty) \cdot (-1) = -\infty$$

Calculons maintenant la 2^{ème} limite :

Quand $x \rightarrow +\infty$ on peut prendre $x > 1$

Rappel : $\forall t \in]0,1[; E(t) = 0$

Soit $x > 1$ alors $0 < \frac{1}{x} < 1$

$$\Rightarrow x > 1 \Rightarrow 0 < \frac{1}{x} < 1 \Rightarrow E\left(\frac{1}{x}\right) = 0$$

Pour $x > 1$ on ait $\frac{1}{x^3} > 0$

$$\Rightarrow \forall x > 1 ; 0 \leq E\left(\frac{1}{x}\right) < \frac{1}{x^2}$$

$$\Rightarrow \forall x > 1 ; 0 \leq x^2 E\left(\frac{1}{x}\right) < \frac{1}{x}$$

$$\Rightarrow \forall x > 1 ; \frac{-1}{x} < 0 \leq x^2 E\left(\frac{1}{x}\right) < \frac{1}{x}$$

$$\Rightarrow \forall x > 1 ; \frac{-1}{x} < f(x) < \frac{1}{x}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = 0 ; \text{ car } \begin{cases} \lim_{x \rightarrow +\infty} \frac{-1}{x} = 0 \\ \lim_{x \rightarrow +\infty} \frac{1}{x} = 0 \end{cases}$$

Solution N° 61 :

1) $\lim_{x \rightarrow 1} \left(\frac{x\sqrt{x} - 1}{x^2 - 1} \right)$

$$= \lim_{x \rightarrow 1} \frac{(x\sqrt{x} - 1)(x\sqrt{x} + 1)}{(x - 1)(x + 1)(x\sqrt{x} + 1)}$$

$$= \lim_{x \rightarrow 1} \frac{x^3 - 1}{(x - 1)(x + 1)(x\sqrt{x} + 1)}$$

$$= \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 1)(x\sqrt{x} + 1)}$$

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$$= \lim_{x \rightarrow 1} \frac{(x^2 + x + 1)}{(x + 1)(x\sqrt{x} + 1)}$$

$$= \frac{(1^2 + 1 + 1)}{(1 + 1)(1\sqrt{1} + 1)} = \frac{3}{4}$$

$$2) \lim_{x \rightarrow -4} \left(\frac{x^3 + 64}{3x^2 + 14x + 8} \right)$$

On remarque que -4 est une racine simple des polynômes du numérateur et du dénominateur, donc en effectuant la division euclidienne de ces deux polynômes par $(x + 4)$ on obtient :

$$\lim_{x \rightarrow -4} \frac{(x + 4)(x^2 - 4x + 16)}{(x + 4)(3x + 2)}$$

$$= \lim_{x \rightarrow -4} \left(\frac{x^2 - 4x + 16}{3x + 2} \right) = \frac{16 + 16 + 16}{-10} = \frac{-24}{5}$$

$$3) \lim_{x \rightarrow -1} \left(\frac{\sqrt{1 - 3x} - 2}{x^2 + 4x + 3} \right)$$

On procédera ainsi : d'abord on multiplie par le conjugué du numérateur puis on effectuera la division euclidienne du polynôme du dénominateur par $(x + 1)$ puisque -1 est une racine simple. On obtient alors :

$$\lim_{x \rightarrow -1} \frac{(\sqrt{1 - 3x} - 2)(\sqrt{1 - 3x} + 2)}{(x + 1)(x + 3)(\sqrt{1 - 3x} + 2)}$$

$$= \lim_{x \rightarrow -1} \frac{(\sqrt{1 - 3x})^2 - 2^2}{(x + 1)(x + 3)(\sqrt{1 - 3x} + 2)}$$

$$= \lim_{x \rightarrow -1} \frac{-3(x + 1)}{(x + 1)(x + 3)(\sqrt{1 - 3x} + 2)}$$

$$= \lim_{x \rightarrow -1} \frac{-3}{(x + 3)(\sqrt{1 - 3x} + 2)}$$

$$= \frac{-3}{(-1 + 3)(\sqrt{1 - 3} + 2)} = \frac{-3}{8}$$

$$4) \lim_{x \rightarrow (-1)^+} \frac{2x^2 - 2}{\sqrt{x + 1}} = \lim_{x \rightarrow (-1)^+} \frac{2(x - 1)(x + 1)}{\sqrt{x + 1}}$$

$$= \lim_{x \rightarrow (-1)^+} \frac{2 \cdot (x - 1) \cdot \sqrt{x + 1} \cdot \sqrt{x + 1}}{\sqrt{x + 1}}$$

$$= \lim_{x \rightarrow (-1)^+} 2(x - 1)\sqrt{x + 1} = 2(-2)\sqrt{0} = 0$$

$$5) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x^4 + 1} - 1}{x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x^4 + 1} - 1)(\sqrt{x^4 + 1} + 1)}{x(\sqrt{x^4 + 1} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x^4}{x(\sqrt{x^4 + 1} + 1)} = \lim_{x \rightarrow 0} \frac{x^3}{\sqrt{x^4 + 1} + 1} = 0$$

$$6) \lim_{x \rightarrow 1^-} \frac{\sqrt{1 - x^2}}{x - 1} = \lim_{\substack{t \rightarrow 0^- \\ t = x - 1}} \frac{\sqrt{-t^2 - 2t}}{t}$$

$$= \lim_{t \rightarrow 0^-} \frac{\sqrt{t^2 \left(-1 - \frac{2}{t}\right)}}{t} = \lim_{t \rightarrow 0^-} \frac{\sqrt{t^2} \cdot \sqrt{-1 - \frac{2}{t}}}{t}$$

$$= \lim_{t \rightarrow 0^-} \frac{|t| \cdot \sqrt{-1 - \frac{2}{t}}}{t} = \lim_{t \rightarrow 0^-} \frac{-t \sqrt{-1 - \frac{2}{t}}}{t}$$

$$= \lim_{t \rightarrow 0^-} -\sqrt{-1 - \frac{2}{t}} = -\sqrt{-1 - \frac{2}{0^-}} = -\infty$$

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Solution N° 62 :

$$\begin{aligned}
 1) \quad \lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{x^2-1} \right) \\
 = \lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{(x-1)(x+1)} \right) \\
 = \lim_{x \rightarrow 1^+} \frac{1}{x-1} \left(1 - \frac{1}{x+1} \right) \\
 = \lim_{\substack{t \rightarrow 0^+ \\ t=x-1}} \frac{1}{t} \left(1 - \frac{1}{t+2} \right) = \frac{1}{0^+} \left(1 - \frac{1}{2} \right) = +\infty
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \lim_{x \rightarrow 3} \left(\frac{\sqrt{2x+3} - x}{x^2 - 3x} \right) \\
 = \lim_{x \rightarrow 3} \frac{(\sqrt{2x+3} - x)(\sqrt{2x+3} + x)}{x(x-3)(\sqrt{2x+3} + x)} \\
 = \lim_{x \rightarrow 3} \frac{-x^2 + 2x + 3}{x(x-3)(\sqrt{2x+3} + x)} \\
 = \lim_{x \rightarrow 3} \frac{(x-3)(-x-1)}{x(x-3)(\sqrt{2x+3} + x)} \\
 = \lim_{x \rightarrow 3} \frac{-x-1}{x(\sqrt{2x+3} + x)} = \frac{-3-1}{3(\sqrt{9}+3)} = \frac{-2}{9}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \lim_{x \rightarrow 1^+} \left(\frac{\sqrt{x+3} - \sqrt{3x+1}}{\sqrt{x-1}} \right) \\
 = \lim_{x \rightarrow 1^+} \frac{(\sqrt{x+3} - \sqrt{3x+1})(\sqrt{x+3} + \sqrt{3x+1})}{\sqrt{x-1} \cdot (\sqrt{x+3} + \sqrt{3x+1})} \\
 = \lim_{x \rightarrow 1^+} \frac{-2(x-1)}{\sqrt{x-1} \cdot (\sqrt{x+3} + \sqrt{3x+1})}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 1^+} \frac{-2\sqrt{x-1} \sqrt{x-1}}{\sqrt{x-1} \cdot (\sqrt{x+3} + \sqrt{3x+1})} \\
 &= \lim_{x \rightarrow 1^+} \left(\frac{-2\sqrt{x-1}}{\sqrt{x+3} + \sqrt{3x+1}} \right) = \frac{-2\sqrt{0}}{\sqrt{4} + \sqrt{4}} = 0
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \lim_{x \rightarrow 9} \left(\frac{\sqrt{x} - 3}{\sqrt{x+7} - 4} \right) \\
 = \lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)(\sqrt{x+7} + 4)}{(\sqrt{x+7} - 4)(\sqrt{x+7} + 4)(\sqrt{x} + 3)} \\
 = \lim_{x \rightarrow 9} \frac{((\sqrt{x})^2 - 3^2)(\sqrt{x+7} + 4)}{((\sqrt{x+7})^2 - 4^2)(\sqrt{x} + 3)} \\
 = \lim_{x \rightarrow 9} \frac{(x-9)(\sqrt{x+7} + 4)}{(x-9)(\sqrt{x} + 3)}
 \end{aligned}$$

$$= \lim_{x \rightarrow 9} \left(\frac{\sqrt{x+7} + 4}{\sqrt{x} + 3} \right) = \frac{\sqrt{9+7} + 4}{\sqrt{9} + 3} = \frac{4}{3}$$

$$\begin{aligned}
 5) \quad \lim_{x \rightarrow 2} \left(\frac{\sqrt{2x+1} - \sqrt{3+x}}{\sqrt{x^2-1} - \sqrt{2x^2-5}} \right) \\
 = \lim_{x \rightarrow 2} \frac{(\sqrt{2x+1} - \sqrt{3+x})(\sqrt{2x+1} + \sqrt{3+x})}{(\sqrt{x^2-1} - \sqrt{2x^2-5})(\sqrt{x^2-1} + \sqrt{2x^2-5})} \\
 \quad \times \frac{(\sqrt{x^2-1} + \sqrt{2x^2-5})}{(\sqrt{2x+1} + \sqrt{3+x})}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x^2-1} + \sqrt{2x^2-5})}{-(x-2)(x+2)(\sqrt{2x+1} + \sqrt{3+x})} \\
 &= \lim_{x \rightarrow 2} \frac{-(\sqrt{x^2-1} + \sqrt{2x^2-5})}{(x+2)(\sqrt{2x+1} + \sqrt{3+x})} \\
 &= \frac{-(\sqrt{4-1} + \sqrt{8-5})}{(2+2)(\sqrt{5} + \sqrt{5})} = \frac{-1}{4} \sqrt{\frac{3}{5}}
 \end{aligned}$$

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$$\begin{aligned}
6) \quad & \lim_{x \rightarrow 0^-} \left(\frac{\sqrt{x^2 - x} - x}{\sqrt{1 + x + x^2} - 1} \right) \\
&= \lim_{x \rightarrow 0^-} \frac{(\sqrt{x^2 - x} - x)(\sqrt{x^2 - x} + x)}{(\sqrt{1 + x + x^2} - 1)(\sqrt{1 + x + x^2} + 1)} \\
&\quad \times \frac{(\sqrt{1 + x + x^2} + 1)}{(\sqrt{x^2 - x} + x)} \\
&= \lim_{x \rightarrow 0^-} \frac{-x(\sqrt{1 + x + x^2} + 1)}{(x + x^2)(\sqrt{x^2 - x} + x)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{(1 + x)(\sqrt{x^2 - x} + x)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{(1 + x) \left(\sqrt{x^2 \left(1 - \frac{1}{x} \right)} + x \right)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{(1 + x) \left(|x| \sqrt{1 - \frac{1}{x}} + x \right)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{(1 + x) \left(-x \sqrt{1 - \frac{1}{x}} + x \right)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{x(1 + x) \left(1 - \sqrt{1 - \frac{1}{x}} \right)} \\
&= \lim_{x \rightarrow 0^-} \frac{-(\sqrt{1 + x + x^2} + 1)}{(1 + x)} \left(\frac{\frac{1}{x}}{1 - \sqrt{1 - \frac{1}{x}}} \right) \\
&= \frac{-(\sqrt{1 + 0 + 0} + 1)}{(1 + 0)} (+\infty) = -\infty \\
&\text{Pourquoi } \lim_{x \rightarrow 0^-} \left(\frac{\frac{1}{x}}{1 - \sqrt{1 - \frac{1}{x}}} \right) = +\infty ?
\end{aligned}$$

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Voici pourquoi :

$$\begin{aligned}
& \lim_{x \rightarrow 0^-} \left(\frac{\frac{1}{x}}{1 - \sqrt{1 - \frac{1}{x}}} \right) = \lim_{\substack{t \rightarrow -\infty \\ t = \frac{1}{x}}} \left(\frac{t}{1 - \sqrt{1 - t}} \right) \\
&= \lim_{t \rightarrow -\infty} \frac{t(1 + \sqrt{1 - t})}{(1 - \sqrt{1 - t})(1 + \sqrt{1 - t})} \\
&= \lim_{t \rightarrow -\infty} \frac{t(1 + \sqrt{1 - t})}{t} \\
&= \lim_{t \rightarrow -\infty} (1 + \sqrt{1 - t}) = 1 + \sqrt{1 - (-\infty)} \\
&= +\infty
\end{aligned}$$

Solution N° 63 :

$$\begin{aligned}
1) \quad & \lim_{x \rightarrow 1^+} \left(\frac{\sqrt{2x - 1} - \sqrt{x - 1} - 1}{x - 1} \right) \\
&= \lim_{x \rightarrow 1^+} \frac{(\sqrt{2x - 1} - (\sqrt{x - 1} + 1))(\sqrt{2x - 1} + (\sqrt{x - 1} + 1))}{(x - 1)((\sqrt{2x - 1} + (\sqrt{x - 1} + 1)))} \\
&= \lim_{x \rightarrow 1^+} \frac{((\sqrt{2x - 1})^2 - (\sqrt{x - 1} + 1)^2)}{(x - 1)(\sqrt{2x - 1} + \sqrt{x - 1} + 1)} \\
&= \lim_{x \rightarrow 1^+} \frac{x - 1 - 2\sqrt{x - 1}}{(x - 1)(\sqrt{2x - 1} + \sqrt{x - 1} + 1)} \\
&= \lim_{x \rightarrow 1^+} \frac{(x - 1)}{(x - 1)(\sqrt{2x - 1} + \sqrt{x - 1} + 1)} \\
&\quad - 2 \lim_{x \rightarrow 1^+} \frac{\sqrt{x - 1}}{(x - 1)(\sqrt{2x - 1} + \sqrt{x - 1} + 1)}
\end{aligned}$$

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$$\begin{aligned}
&= \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{2x-1} + \sqrt{x-1} + 1} \\
&\quad - 2 \lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x-1}(\sqrt{2x-1} + \sqrt{x-1} + 1)} \\
&= \frac{1}{\sqrt{1} + \sqrt{0} + 1} - 2 \left(\frac{1}{\sqrt{1-1}} \right) \left(\frac{1}{\sqrt{1} + \sqrt{0} + 1} \right) \\
&= \frac{1}{2} - 2(+\infty) \left(\frac{1}{2} \right) = -\infty
\end{aligned}$$

$$\begin{aligned}
2) \quad &\lim_{x \rightarrow 1} \left(\frac{\sqrt{x} - 3x + 2}{x - 1} \right) \\
&= \lim_{x \rightarrow 1} \frac{(\sqrt{x} - (3x - 2))(\sqrt{x} + (3x - 2))}{(x - 1)(\sqrt{x} + 3x - 2)} \\
&= \lim_{x \rightarrow 1} \frac{(\sqrt{x})^2 - (3x - 2)^2}{(x - 1)(\sqrt{x} + 3x - 2)} \\
&= \lim_{x \rightarrow 1} \left(\frac{-9x^2 + 13x - 4}{x - 1} \right) \times \left(\frac{1}{\sqrt{x} + 3x - 2} \right)
\end{aligned}$$

On effectue la division euclidienne parce que 1 est une racine simple du polynôme $-9x^2 + 13x - 4$ par $x - 1$:

$$\begin{aligned}
&= \lim_{x \rightarrow 1} (-9x + 4) \times \left(\frac{1}{\sqrt{x} + 3x - 2} \right) \\
&= (-9 + 4) \times \left(\frac{1}{\sqrt{1} + 3 - 2} \right) = \frac{-5}{2} \\
3) \quad &\lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - x^2 + x + 4}{x - 3} \right) \\
&= \lim_{x \rightarrow 3} \frac{(\sqrt{x+1} - (x^2 - x - 4))((\sqrt{x+1} + (x^2 - x - 4)))}{(x - 3)(\sqrt{x+1} + x^2 - x - 4)}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 3} \left(\frac{(\sqrt{x+1})^2 - (x^2 - x - 4)^2}{x - 3} \right) \left(\frac{1}{\sqrt{x+1} + x^2 - x - 4} \right) \\
&= \lim_{x \rightarrow 3} \left(\frac{-x^4 + 2x^3 + 7x^2 - 7x - 15}{x - 3} \right) \left(\frac{1}{\sqrt{x+1} + x^2 - x - 4} \right)
\end{aligned}$$

On effectue la division euclidienne du polynôme du numérateur par $(x - 3)$ car 3 est une racine simple pour le polynôme du numérateur on obtient ainsi :

$$\begin{aligned}
&= \lim_{x \rightarrow 3} (-x^3 - x^2 + 4x + 5) \left(\frac{1}{\sqrt{x+1} + x^2 - x - 4} \right) \\
&= \frac{-27 - 9 + 12 + 5}{2 + 9 - 3 - 4} = \frac{-19}{4}
\end{aligned}$$

$$\begin{aligned}
4) \quad &\lim_{x \rightarrow 9} \left(\frac{\sqrt{x} - 3}{\sqrt{x+7} - 4} \right) \\
&= \lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)(\sqrt{x+7} + 4)}{(\sqrt{x+7} - 4)(\sqrt{x+7} + 4)(\sqrt{x} + 3)} \\
&= \lim_{x \rightarrow 9} \frac{((\sqrt{x})^2 - 3^2)(\sqrt{x+7} + 4)}{((\sqrt{x+7})^2 - 4^2)(\sqrt{x} + 3)} \\
&= \lim_{x \rightarrow 9} \frac{(x - 9)(\sqrt{x+7} + 4)}{(x - 9)(\sqrt{x} + 3)}
\end{aligned}$$

$$= \lim_{x \rightarrow 9} \left(\frac{\sqrt{x+7} + 4}{\sqrt{x} + 3} \right) = \frac{\sqrt{9+7} + 4}{\sqrt{9} + 3} = \frac{4}{3}$$

$$\begin{aligned}
5) \quad &\lim_x \left(\frac{\sqrt{4x+6} - 2x^2 - 3x - 3}{2x + 1} \right) \\
&\quad \times \frac{\sqrt{4x+6} + (2x^2 + 3x + 3)}{\sqrt{4x+6} + (2x^2 + 3x + 3)}
\end{aligned}$$

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$$\begin{aligned}
 &= \lim_{x \rightarrow \frac{-1}{2}} \frac{(\sqrt{4x+6})^2 - (2x^2 + 3x + 3)^2}{(2x+1)(\sqrt{4x+6} + (2x^2 + 3x + 3))} \\
 &= \lim_{x \rightarrow \frac{-1}{2}} - \left(\frac{4x^4 + 12x^3 + 21x^2 + 14x + 3}{2x+1} \right) \\
 &\quad \times \left(\frac{1}{\sqrt{4x+6} + 2x^2 + 3x + 3} \right)
 \end{aligned}$$

On effectue la division euclidienne du polynôme du numérateur par celui du dénominateur on obtient ainsi :

$$\begin{aligned}
 &\lim_{x \rightarrow \frac{-1}{2}} \frac{-(2x^3 + 5x^2 + 8x + 3)}{\sqrt{4x+6} + 2x^2 + 3x + 3} \\
 &= \frac{-\left(\frac{-2}{8} + \frac{5}{4} - \frac{8}{2} + 3\right)}{\sqrt{4} + \frac{2}{4} - \frac{3}{2} + 3} = 0
 \end{aligned}$$

$$\begin{aligned}
 6) \quad &\lim_{x \rightarrow 2} \left(\frac{\sqrt{x-1} + \sqrt{x+2} - 3}{x-2} \right) \\
 &\quad \times \frac{\sqrt{x-1} - (\sqrt{x+2} - 3)}{\sqrt{x-1} - (\sqrt{x+2} - 3)}
 \end{aligned}$$

$$= \lim_{x \rightarrow 2} \frac{(\sqrt{x-1})^2 - (\sqrt{x+2} - 3)^2}{(x-2)(\sqrt{x-1} - \sqrt{x+2} + 3)}$$

$$= \lim_{x \rightarrow 2} \frac{6\sqrt{x+2} - 12}{(x-2)(\sqrt{x-1} - \sqrt{x+2} + 3)}$$

$$= \lim_{x \rightarrow 2} \left(\frac{\sqrt{x+2} - 2}{x-2} \right) \left(\frac{6}{\sqrt{x-1} - \sqrt{x+2} + 3} \right)$$

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$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \left(\frac{6}{\sqrt{x-1} - \sqrt{x+2} + 3} \right) \\
 &\quad \times \frac{(\sqrt{x+2} - 2)(\sqrt{x+2} + 2)}{(x-2)(\sqrt{x+2} + 2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \left(\frac{6}{\sqrt{x-1} - \sqrt{x+2} + 3} \right) \\
 &\quad \times \frac{(\sqrt{x+2})^2 - 2^2}{(x-2)(\sqrt{x+2} + 2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \left(\frac{6}{\sqrt{x-1} - \sqrt{x+2} + 3} \right) \\
 &\quad \times \frac{(x-2)}{(x-2)(\sqrt{x+2} + 2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \left(\frac{6}{\sqrt{x-1} - \sqrt{x+2} + 3} \right) \left(\frac{1}{\sqrt{x+2} + 2} \right) \\
 &= \left(\frac{6}{\sqrt{1} - \sqrt{4} + 3} \right) \left(\frac{1}{\sqrt{4} + 2} \right) = \frac{3}{4}
 \end{aligned}$$

Solution N° 64 :

$$\begin{aligned}
 1) \quad &\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{\cos x} - 1 + \sin x}{\cos x - \cos(3x)} \right) \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sqrt{\cos x} - 1 + \sin x)(\sqrt{\cos x} - 1 - \sin x)}{(\cos x - \cos(3x))(\sqrt{\cos x} - 1 - \sin x)} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sqrt{\cos x} - 1)^2 - \sin^2 x}{(\cos x - \cos(3x))(\sqrt{\cos x} - 1 - \sin x)} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x + \cos^2 x - 2\sqrt{\cos x}}{(4 \cos x - 4 \cos^3 x)(\sqrt{\cos x} - 1 - \sin x)} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{\cos x} (\sqrt{\cos x} + \cos x \sqrt{\cos x} - 2)}{4 \cos x (1 - \cos^2 x)(\sqrt{\cos x} - 1 - \sin x)}
 \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{\cos x} (\sqrt{\cos x} + \cos x \sqrt{\cos x} - 2)}{4\sqrt{\cos x} \sqrt{\cos x} \cdot \sin^2 x (\sqrt{\cos x} - 1 - \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sqrt{\cos x} + \cos x \sqrt{\cos x} - 2)}{4 \sin^2 x (\sqrt{\cos x} - 1 - \sin x)} \cdot \frac{1}{\sqrt{\cos x}} \\
&= \frac{(0 + 0 - 2)}{4 \cdot 1^2 \cdot (0 - 1 - 1)} \cdot \frac{1}{0^+} = +\infty
\end{aligned}$$

$$\begin{aligned}
2) \quad &\lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\cos x - \sin x}{1 - \sqrt{2} \cos x} \right) \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{(1 - \sqrt{2} \cos x)(\cos x + \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{(1 - \sqrt{2} \cos x)(\cos x + \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\cos^2 x - 1}{(1 - \sqrt{2} \cos x)(\cos x + \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-(1 - \sqrt{2} \cos x)(1 + \sqrt{2} \cos x)}{(1 - \sqrt{2} \cos x)(\cos x + \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-(1 + \sqrt{2} \cos x)}{(\cos x + \sin x)} = \frac{-(1 + \sqrt{2} \cos \frac{\pi}{4})}{(\cos \frac{\pi}{4} + \sin \frac{\pi}{4})} \\
&= \frac{-\left(1 + \sqrt{2} \times \frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)} = -\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
3) \quad &\lim_{x \rightarrow 0} \left(\frac{1 - \cos(3x)}{\sin^2(5x)} \right) \\
&= \lim_{x \rightarrow 0} \frac{(1 - \cos(3x))}{(3x)^2} \cdot \frac{(5x)^2}{\sin^2(5x)} \cdot \frac{(3x)^2}{(5x)^2}
\end{aligned}$$

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$$= \frac{1}{2} \cdot \left(\frac{1}{1}\right)^2 \cdot \frac{9}{25} = \frac{9}{50}$$

$$\begin{aligned}
4) \quad &\lim_{|x| \rightarrow +\infty} (x^2 + 2x \cdot \sin x) \\
&= \lim_{x \rightarrow \pm\infty} (x^2 + 2x \cdot \sin x)
\end{aligned}$$

$$\text{On a : } -1 \leq \sin x \leq 1 ; \quad \forall x \in \mathbb{R}$$

$$\Rightarrow -2x \leq 2x \cdot \sin x \leq 2x ; \quad \forall x \in \mathbb{R}$$

$$\Rightarrow \underbrace{(x^2 - 2x)}_{\substack{\text{tend vers } +\infty \\ \text{quand } x \rightarrow \pm\infty}} \leq x^2 + 2x \cdot \sin x ; \quad \forall x \in \mathbb{R}$$

$$\Rightarrow \lim_{x \rightarrow \pm\infty} (x^2 + 2x \sin x) = +\infty$$

$$\begin{aligned}
5) \quad &\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 + \cos(2x)}{1 - \sin x} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{2 \cos^2 x}{1 - \sin x} \right) \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(2 \cos^2 x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(2 \cos^2 x)(1 + \sin x)}{1 - \sin^2 x} \\
&= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(2 \cos^2 x)(1 + \sin x)}{\cos^2 x}
\end{aligned}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} 2(1 + \sin x) = 2 \left(1 + \sin \frac{\pi}{2}\right) = 4$$

$$6) \quad \text{On a } -1 < \sin\left(\frac{2}{x}\right) < 1 ; \quad \forall x \neq 0$$

$$\Rightarrow -1 < 2 \cos\left(\frac{1}{x}\right) \sin\left(\frac{1}{x}\right) < 1 ; \quad \forall x \neq 0$$

$$\Rightarrow \frac{-1}{2} < \cos\left(\frac{1}{x}\right) \sin\left(\frac{1}{x}\right) < \frac{1}{2}$$

$$\Rightarrow \frac{-x}{2} < x \cdot \cos\left(\frac{1}{x}\right) \cdot \sin\left(\frac{1}{x}\right) < \frac{x}{2}$$

$$\Rightarrow \lim_{x \rightarrow 0} \left(x \cdot \cos\left(\frac{1}{x}\right) \cdot \sin\left(\frac{1}{x}\right) \right) = 0$$

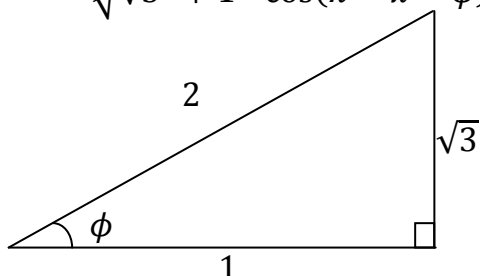
$$\text{Car : } \lim_{x \rightarrow 0} \left(\frac{x}{2}\right) = \lim_{x \rightarrow 0} \left(\frac{-x}{2}\right) = 0$$

Solution N° 65 :

$$\begin{aligned} 1) \quad & \lim_{x \rightarrow 0} \left(\frac{\sqrt{\cos x} - \cos x}{\sin(2x) \cdot \tan(3x)} \right) \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{\cos x} - \cos x)(\sqrt{\cos x} + \cos x)}{\sin(2x) \cdot \tan(3x) \cdot (\sqrt{\cos x} + \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{\cos x - \cos^2 x}{\sin(2x) \cdot \tan(3x) \cdot (\sqrt{\cos x} + \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{(1 - \cos x) \cos x}{\sin(2x) \cdot \tan(3x) \cdot (\sqrt{\cos x} + \cos x)} \\ &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin(2x) \cdot \tan(3x)} \right) \left(\frac{\cos x}{\sqrt{\cos x} + \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin(2x) \cdot \tan(3x)} \right) \left(\frac{1}{2} \right) \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{2x}{\sin 2x} \right) \left(\frac{3x}{\tan 3x} \right) \left(\frac{x^2}{6x^2} \right) \\ &= \frac{1}{2} \times \left(\frac{1}{1} \right) \times \left(\frac{1}{1} \right) \times \left(\frac{1}{1} \right) \times \left(\frac{1}{6} \right) = \frac{1}{24} \end{aligned}$$

$$2) \quad \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\sqrt{3} \sin x - \cos x}{6x - \pi} \right)$$

$$\begin{aligned} \sqrt{3} \sin x - \cos x &= \sqrt{3} \sin(\pi - x) + 1 \cos(\pi - x) \\ &= \sqrt{\sqrt{3}^2 + 1^2} \cos(\pi - x - \phi) \end{aligned}$$



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$$\cos \phi = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \phi \equiv \pm \frac{\pi}{3} [2\pi]$$

$$\text{On prendra } \phi = \frac{\pi}{3} \text{ Donc :}$$

$$\sqrt{3} \sin x - \cos x = 2 \cos\left(\pi - x - \frac{\pi}{3}\right)$$

$$= 2 \cos\left(\frac{2\pi}{3} - x\right)$$

$$= 2 \sin\left(x - \frac{\pi}{6}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\sqrt{3} \sin x - \cos x}{6x - \pi} \right) = \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin\left(x - \frac{\pi}{6}\right)}{6x - \pi}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin\left(x - \frac{\pi}{6}\right)}{6\left(x - \frac{\pi}{6}\right)} = \frac{1}{3} \lim_{t \rightarrow 0} \left(\frac{\sin t}{t} \right) = \frac{1}{3} \times 1$$

$$3) \quad \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan x - 1}{2 \sin x - \sqrt{2}} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan^2 x - 1}{(2 \sin x)^2 - \sqrt{2}^2} \right) \cdot \left(\frac{2 \sin x + \sqrt{2}}{\tan x + 1} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan^2 x - 1}{4 \sin^2 x - 2} \right) \cdot \left(\frac{2 \sin \frac{\pi}{4} + \sqrt{2}}{\tan \frac{\pi}{4} + 1} \right)$$

$$= \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{4 \sin^2 x - 2} \right)$$

$$= \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\sin^2 x - \cos^2 x}{\cos^2 x (4 \sin^2 x - 2)} \right)$$

$$= \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{1 - 2 \cos^2 x}{\cos^2 x (4 - 4 \cos^2 x - 2)} \right)$$

$$= \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{1 - 2 \cos^2 x}{2 \cos^2 x (1 - 2 \cos^2 x)} \right)$$

$$= \sqrt{2} \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{1}{2 \cos^2 x} \right) = \sqrt{2} \cdot \frac{1}{2 \left(\frac{2}{4} \right)} = \sqrt{2}$$

$$4) \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2-1} = \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x-1)(x+1)}$$

$$= \lim_{\substack{t \rightarrow 0 \\ t=x-1}} \left(\frac{\sin t}{t} \right) \times \left(\frac{1}{t+2} \right) = 1 \times \frac{1}{2} = \frac{1}{2}$$

$$5) \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{\frac{\sin x}{1+\cos x}} - 1}{x - \frac{\pi}{2}} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\left(\sqrt{\frac{\sin x}{1+\cos x}} - 1 \right) \left(\sqrt{\frac{\sin x}{1+\cos x}} + 1 \right)}{\left(x - \frac{\pi}{2} \right) \left(\sqrt{\frac{\sin x}{1+\cos x}} + 1 \right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\frac{\sin x}{1+\cos x} - 1}{x - \frac{\pi}{2}} \right) \left(\frac{1}{\sqrt{\frac{\sin x}{1+\cos x}} + 1} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1 - \cos x}{\left(x - \frac{\pi}{2} \right) (1 + \cos x)} \left(\frac{1}{\sqrt{\frac{\sin x}{1+\cos x}} + 1} \right)$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin x - 1 - \cos x}{x - \frac{\pi}{2}} \right) \times \frac{1}{(1 + \cos x) \left(\sqrt{\frac{\sin x}{1+\cos x}} + 1 \right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin x - 1 - \cos x}{x - \frac{\pi}{2}} \right) \times \frac{1}{(1)(\sqrt{1} + 1)}$$

$$= \frac{1}{2} \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin x - 1 - \cos x}{x - \frac{\pi}{2}} \right)$$

$$= \frac{1}{2} \lim_{\substack{t \rightarrow 0 \\ t=x-\frac{\pi}{2}}} \left(\frac{\sin \left(t + \frac{\pi}{2} \right) - 1 - \cos \left(t + \frac{\pi}{2} \right)}{t} \right)$$

$$= \frac{1}{2} \lim_{t \rightarrow 0} \left(\frac{\cos t - 1 + \sin t}{t} \right)$$

$$= \frac{1}{2} \lim_{t \rightarrow 0} \left(-t \left(\frac{1 - \cos t}{t^2} \right) + \frac{\sin t}{t} \right)$$

$$= \frac{1}{2} \left(-0 \left(\frac{1}{2} \right) + 1 \right) = \frac{1}{2}$$

Solution N° 66 :

$$1) \lim_{x \rightarrow 0^-} \frac{\sqrt{\cos(ax)} - \cos(bx)}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{-\sqrt{\cos(ax)} - \cos(bx)}{-x}$$

$$= \lim_{x \rightarrow 0^-} \frac{-\sqrt{\cos(ax)} - \cos(bx)}{|x|}$$

$$= \lim_{x \rightarrow 0^-} \frac{-\sqrt{\cos(ax)} - \cos(bx)}{\sqrt{x^2}}$$

$$= \lim_{x \rightarrow 0^-} -\sqrt{\frac{\cos(ax) - \cos(bx)}{x^2}}$$

$$= \lim_{x \rightarrow 0^-} -\sqrt{\frac{\cos(ax) - \cos(bx) - 1 + 1}{x^2}}$$

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$$= \lim_{x \rightarrow 0^-} -\sqrt{b^2 \left(\frac{1 - \cos(bx)}{(bx)^2} \right) - a^2 \left(\frac{1 - \cos(ax)}{(ax)^2} \right)}$$

$$= -\sqrt{b^2 \left(\frac{1}{2} \right) - a^2 \left(\frac{1}{2} \right)} = -\sqrt{\frac{b^2 - a^2}{2}}$$

$$2) \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x + 1} - \alpha x - \beta)$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 - 3x + 1})^2 - (\alpha x + \beta)^2}{\sqrt{x^2 - 3x + 1} + \alpha x + \beta}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - \alpha^2)x - (3 + 2\alpha\beta)x + (1 - \beta^2)}{\sqrt{x^2 - 3x + 1} + \alpha x + \beta}$$

Pour que cette limite soit égale à zéro il suffit qu'on ait la limite du numérateur soit un nombre réel et que la limite du dénominateur soit $+\infty$. Ce que traduisent les conditions suivantes :

$$\begin{cases} \text{Et } (1 - \alpha^2) = 0 \\ \text{Et } (3 + 2\alpha\beta) = 0 \\ \text{Et } \alpha \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{Et } \alpha = \pm 1 \\ \text{Et } \beta = \mp \frac{3}{2} \\ \text{Et } \alpha \geq 0 \end{cases}$$

Donc on retient le cas où :

$$\alpha = 1 \quad \text{et} \quad \beta = \frac{-3}{2}$$

Solution N° 67 :

$$1) \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2 \left(\frac{\sin 2x}{2x} \right) - 2 = 0$$

$$= 2(1) - 2 = 0$$

$$\text{Et } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x(x+1)}{x^2 - 4} = \frac{0}{-4} = 0$$

$$\text{Donc : } \lim_{x \rightarrow 0} f(x)$$

$$2) \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x(x+1)}{x^2 - 4}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x^2 + x}{x^2 - 4} \right) = \lim_{x \rightarrow -\infty} \left(\frac{x^2}{x^2} \right) = 1$$

$$3) \text{ Soit } x > 0 \text{ On a : } -1 \leq \sin 2x \leq 2$$

$$\Rightarrow \frac{-1}{x} \leq \frac{\sin 2x}{x} \leq \frac{1}{x}$$

$$\Rightarrow \frac{-1}{x} - 2 \leq \frac{\sin 2x}{x} - 2 \leq \frac{1}{x} - 2$$

$$4) \text{ On a : } \underbrace{\frac{-1}{x} - 2}_{-2} \leq \frac{\sin 2x}{x} - 2 \leq \underbrace{\frac{1}{x} - 2}_{-2}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = -2$$

$$5) \lim_{x \rightarrow (-2)^+} f(x) = \lim_{x \rightarrow (-2)^+} \frac{x(x+1)}{x^2 - 4}$$

$$= \lim_{x \rightarrow (-2)^+} \frac{x(x+1)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow (-2)^+} \left(\frac{x(x+1)}{x-2} \right) \times \lim_{x \rightarrow (-2)^+} \left(\frac{1}{x+2} \right)$$

$$= \left(\frac{-2(-2+1)}{-2-2} \right) \times \lim_{t \rightarrow 0^+} \left(\frac{1}{t} \right)$$

$$= \frac{-1}{2} \times \frac{1}{0^+} = \frac{-1}{2} \times (+\infty) = -\infty$$

$$\lim_{x \rightarrow (-2)^-} f(x) = \text{même procédé}$$

$$= \frac{-1}{2} \times \frac{1}{0^-} = \frac{-1}{2} \times (-\infty) = +\infty$$

Solution N° 68 :

$$\begin{aligned}
1) \quad & \lim_{x \rightarrow -\infty} x \left(x + \sqrt{x^2 + 1} \right) \\
&= \lim_{x \rightarrow -\infty} x \left(x + \sqrt{x^2 \left(1 + \frac{1}{x^2} \right)} \right) \\
&= \lim_{x \rightarrow -\infty} x \left(x + \sqrt{x^2} \cdot \sqrt{1 + \frac{1}{x^2}} \right) \\
&= \lim_{x \rightarrow -\infty} x \left(x + |x| \cdot \sqrt{1 + \frac{1}{x^2}} \right) \\
&= \lim_{x \rightarrow -\infty} x \left(x - x \sqrt{1 + \frac{1}{x^2}} \right) \\
&= \lim_{x \rightarrow -\infty} x^2 \left(1 - \sqrt{1 + \frac{1}{x^2}} \right) \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \sqrt{1 + \frac{1}{x^2}} \right) \left(1 + \sqrt{1 + \frac{1}{x^2}} \right)}{\left(1 + \sqrt{1 + \frac{1}{x^2}} \right)} \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1^2 - \left(\sqrt{1 + \frac{1}{x^2}} \right)^2 \right)}{\left(1 + \sqrt{1 + \frac{1}{x^2}} \right)} \\
&= \lim_{x \rightarrow -\infty} \left(\frac{-1}{1 + \sqrt{1 + \frac{1}{x^2}}} \right) = \frac{-1}{1 + \sqrt{1 + 0}} = \frac{-1}{2}
\end{aligned}$$

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$$\begin{aligned}
2) \quad & \lim_{x \rightarrow +\infty} (\sqrt{x} + 4x^2 - x + 5) \\
&= \lim_{x \rightarrow +\infty} x^2 \left(\frac{1}{x\sqrt{x}} + 4 - \frac{1}{x} + \frac{5}{x^2} \right) \\
&= (+\infty)(0 + 4 - 0 + 0) = +\infty \\
3) \quad & \lim_{x \rightarrow +\infty} \left(\frac{x + 2\sqrt{x}}{x - 3} \right) = \lim_{x \rightarrow +\infty} \frac{x \left(1 + \frac{2}{\sqrt{x}} \right)}{x \left(1 - \frac{3}{x} \right)} \\
&= \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{2}{\sqrt{x}}}{1 - \frac{3}{x}} \right) = \left(\frac{1 + 0}{1 - 0} \right) = 1 \\
4) \quad & \lim_{x \rightarrow -\infty} \sqrt{\frac{x + 5}{2x - 4}} = \lim_{x \rightarrow -\infty} \sqrt{\frac{x \left(1 + \frac{5}{x} \right)}{x \left(2 - \frac{4}{x} \right)}} \\
&= \lim_{x \rightarrow -\infty} \sqrt{\left(\frac{1 + \frac{5}{x}}{2 - \frac{4}{x}} \right)} = \sqrt{\frac{1 + 0}{2 - 0}} = \frac{\sqrt{2}}{2} \\
5) \quad & \lim_{x \rightarrow -\infty} \sqrt{x^2 - 3x + 5} = \lim_{x \rightarrow -\infty} \sqrt{x^2 \left(1 - \frac{3}{x} + \frac{5}{x^2} \right)} \\
&= \lim_{x \rightarrow -\infty} \sqrt{x^2} \cdot \sqrt{1 - \frac{3}{x} + \frac{5}{x^2}} \\
&= \lim_{x \rightarrow -\infty} |x| \cdot \sqrt{1 - \frac{3}{x} + \frac{5}{x^2}} \\
&= \lim_{x \rightarrow -\infty} -x \sqrt{1 - \frac{3}{x} + \frac{5}{x^2}} = +\infty \cdot \sqrt{1} = +\infty \\
6) \quad & \lim_{x \rightarrow +\infty} \frac{\sqrt{2x + 1}}{\sqrt{x + 1}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{2x + 1}{x + 1}}
\end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{x \left(2 + \frac{1}{x}\right)}}{\sqrt{x \left(1 + \frac{1}{x}\right)}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{2 + \frac{1}{x}}{1 + \frac{1}{x}}}$$

$$= \sqrt{\frac{2+0}{1+0}} = \sqrt{2}$$

Solution N° 69 :

$$1) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x} - 1}{\sqrt{x} + 3} \right) = \lim_{x \rightarrow +\infty} \frac{\sqrt{x} \left(1 - \frac{1}{\sqrt{x}}\right)}{\sqrt{x} \left(1 + \frac{3}{\sqrt{x}}\right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1 - \frac{1}{\sqrt{x}}}{1 + \frac{3}{\sqrt{x}}} \right) = \frac{1-0}{1+0} = 1$$

$$2) \lim_{x \rightarrow 2} \left(\frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} \right) = \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x \left((\sqrt{x})^2 - (\sqrt{2})^2 \right)}$$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})}$$

$$= \lim_{x \rightarrow 2} \frac{1}{x(\sqrt{x} + \sqrt{2})} = \frac{1}{2(\sqrt{2} + \sqrt{2})} = \frac{\sqrt{2}}{8}$$

$$3) \lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 - 3x} + 2x - 5 \right)$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{4x^2 - 3x} + 2x - 5)(\sqrt{4x^2 - 3x} - (2x - 5))}{(\sqrt{4x^2 - 3x} - 2x + 5)}$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{4x^2 - 3x})^2 - (2x - 5)^2}{(\sqrt{4x^2 - 3x} - 2x + 5)}$$

$$= \lim_{x \rightarrow -\infty} \frac{17x + 25}{\left(\sqrt{x^2 \left(4 - \frac{3}{x}\right)} - 2x + 5 \right)}$$

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$$= \lim_{x \rightarrow -\infty} \frac{x \left(17 + \frac{25}{x}\right)}{\left(\sqrt{x^2} \cdot \sqrt{4 - \frac{3}{x}} - 2x + 5 \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(17 + \frac{25}{x}\right)}{\left(|x| \cdot \sqrt{4 - \frac{3}{x}} - 2x + 5 \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(17 + \frac{25}{x}\right)}{\left(-x \sqrt{4 - \frac{3}{x}} - 2x + 5 \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(17 + \frac{25}{x}\right)}{x \left(-\sqrt{4 - \frac{3}{x}} - 2 + \frac{5}{x} \right)}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{17 + \frac{25}{x}}{-\sqrt{4 - \frac{3}{x}} - 2 + \frac{5}{x}} \right)$$

$$= \frac{17+0}{-\sqrt{4}-2+0} = \frac{-17}{4}$$

$$4) \lim_{x \rightarrow +\infty} \frac{\sqrt{x+2}}{x^2} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x+2}{x^4}}$$

$$= \lim_{x \rightarrow +\infty} \sqrt{\frac{x \left(1 + \frac{2}{x}\right)}{x^4}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{1 + \frac{2}{x}}{x^3}}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{x\sqrt{x}} \sqrt{1 + \frac{2}{x}} = 0\sqrt{1+0} = 0$$

$$5) \lim_{x \rightarrow -\infty} \sqrt{\frac{x^3 + 1}{2x + 3}} = \lim_{x \rightarrow -\infty} \sqrt{\frac{x^3 \left(1 + \frac{1}{x^3}\right)}{x \left(2 + \frac{3}{x}\right)}}$$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \sqrt{x^2 \left(\frac{1 + \frac{1}{x^3}}{2 + \frac{3}{x}} \right)} = \lim_{x \rightarrow -\infty} \sqrt{x^2} \sqrt{\frac{1 + \frac{1}{x^3}}{2 + \frac{3}{x}}} \\
&= \lim_{x \rightarrow -\infty} |x| \cdot \sqrt{\frac{1 + \frac{1}{x^3}}{2 + \frac{3}{x}}} = \lim_{x \rightarrow -\infty} -x \sqrt{\frac{1 + \frac{1}{x^3}}{2 + \frac{3}{x}}} \\
&= -(-\infty) \sqrt{\frac{1+0}{2+0}} = +\infty
\end{aligned}$$

$$6) \lim_{x \rightarrow -\infty} (x + 7 + \sqrt{4 - 2x})$$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \frac{(x + 7 + \sqrt{4 - 2x})(x + 7 - \sqrt{4 - 2x})}{(x + 7 - \sqrt{4 - 2x})} \\
&= \lim_{x \rightarrow -\infty} \frac{(x + 7)^2 - (\sqrt{4 - 2x})^2}{(x + 7 - \sqrt{4 - 2x})} \\
&= \lim_{x \rightarrow -\infty} \left(\frac{x^2 + 16x + 45}{x + 7 - \sqrt{4 - 2x}} \right) \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{16}{x} + \frac{45}{x^2} \right)}{x + 7 - \sqrt{x^2 \left(\frac{4}{x^2} - \frac{2}{x} \right)}} \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{16}{x} + \frac{45}{x^2} \right)}{x + 7 - \sqrt{x^2} \cdot \sqrt{\frac{4}{x^2} - \frac{2}{x}}} \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{16}{x} + \frac{45}{x^2} \right)}{x + 7 - |x| \cdot \sqrt{\frac{4}{x^2} - \frac{2}{x}}} \\
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{16}{x} + \frac{45}{x^2} \right)}{x + 7 + x \sqrt{\frac{4}{x^2} - \frac{2}{x}}}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 + \frac{16}{x} + \frac{45}{x^2} \right)}{x \left(1 + \frac{7}{x} + \sqrt{\frac{4}{x^2} - \frac{2}{x}} \right)} \\
&= \lim_{x \rightarrow -\infty} x \left(\frac{1 + \frac{16}{x} + \frac{45}{x^2}}{1 + \frac{7}{x} + \sqrt{\frac{4}{x^2} - \frac{2}{x}}} \right) \\
&= (-\infty) \left(\frac{1 + 0 + 0}{1 + 0 + \sqrt{0 - 0}} \right) = -\infty
\end{aligned}$$

Solution N° 70 :

$$1) \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{x^4}{x^2 - x}} + 2x \right)$$

$$\begin{aligned}
&= \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{x^2 \cdot x^2}{x^2 \left(1 - \frac{1}{x} \right)}} + 2x \right) \\
&= \lim_{x \rightarrow -\infty} \left(\sqrt{\frac{x^2}{1 - \frac{1}{x}}} + 2x \right) \\
&= \lim_{x \rightarrow -\infty} \left(\sqrt{x^2} \cdot \sqrt{\frac{1}{1 - \frac{1}{x}}} + 2x \right) \\
&= \lim_{x \rightarrow -\infty} \left(|x| \cdot \sqrt{\frac{1}{1 - \frac{1}{x}}} + 2x \right) \\
&= \lim_{x \rightarrow -\infty} \left(-x \cdot \sqrt{\frac{1}{1 - \frac{1}{x}}} + 2x \right)
\end{aligned}$$

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$$= \lim_{x \rightarrow -\infty} x \left(-\sqrt{\frac{1}{1 - \frac{1}{x}}} + 2 \right)$$

$$= (-\infty) \left(-\sqrt{\frac{1}{1 - 0}} + 2 \right) = (-\infty) \times 1 = -\infty$$

$$2) \lim_{x \rightarrow -\infty} \left(\frac{x-2}{\sqrt{x^2+5x}} \right) = \lim_{x \rightarrow -\infty} \left(\frac{x-2}{\sqrt{x^2 \left(1 + \frac{5}{x}\right)}} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x-2}{\sqrt{x^2} \cdot \sqrt{1 + \frac{5}{x}}} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x-2}{|x| \cdot \sqrt{1 + \frac{5}{x}}} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{x-2}{-x \sqrt{1 + \frac{5}{x}}} \right)$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 - \frac{2}{x}\right)}{-x \sqrt{1 + \frac{5}{x}}} = \lim_{x \rightarrow -\infty} \frac{\left(1 - \frac{2}{x}\right)}{-\sqrt{1 + \frac{5}{x}}}$$

$$= \frac{(1-0)}{-\sqrt{1+0}} = -1$$

$$3) \lim_{x \rightarrow 1} \frac{-2}{|\sqrt{x}-1|} = \lim_{\substack{t \rightarrow 0 \\ t=\sqrt{x}-1}} \frac{-2}{|t|}$$

$$= \begin{cases} \text{oubien} & \lim_{\substack{t \rightarrow 0 \\ t > 0}} \left(\frac{-2}{t} \right) = -\infty \\ \text{oubien} & \lim_{\substack{t \rightarrow 0 \\ t < 0}} \left(\frac{-2}{-t} \right) = -\infty \end{cases}$$

$$\text{Donc} \quad \lim_{x \rightarrow 1} \frac{-2}{|\sqrt{x}-1|} = -\infty$$

$$4) \lim_{x \rightarrow +\infty} \left(\sqrt{\sqrt{x^4 - x^3}} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{\sqrt{x^4 - x^3}} - x)(\sqrt{\sqrt{x^4 - x^3}} + x)}{(\sqrt{\sqrt{x^4 - x^3}} + x)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x^4 - x^3} - x^2}{\sqrt{\sqrt{x^4 - x^3}} + x} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^4 - x^3} - x^2)(\sqrt{x^4 - x^3} + x^2)}{(\sqrt{\sqrt{x^4 - x^3}} + x)((\sqrt{x^4 - x^3} + x^2))}$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^4 - x^3})^2 - x^4}{(\sqrt{\sqrt{x^4 - x^3}} + x)((\sqrt{x^4 - x^3} + x^2))}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^3}{\left(\sqrt{\sqrt{x^4 \left(1 - \frac{1}{x}\right)}} + x \right) \left(\left(\sqrt{x^4 \left(1 - \frac{1}{x}\right)} + x^2 \right) \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^3}{x \left(\sqrt{\sqrt{\left(1 - \frac{1}{x}\right)}} + 1 \right) x^2 \left(\sqrt{\left(1 - \frac{1}{x}\right)} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-1}{\left(\sqrt{\sqrt{\left(1 - \frac{1}{x}\right)}} + 1 \right) \left(\sqrt{\left(1 - \frac{1}{x}\right)} + 1 \right)}$$

$$= \frac{-1}{\left(\sqrt{\sqrt{(1-0)}+1}\right)\left(\sqrt{(1-0)}+1\right)} = \frac{-1}{4}$$

$$\begin{aligned} 5) \quad \lim_{x \rightarrow 0^+} \left(\frac{x - \sqrt{x}}{x + \sqrt{x}} \right) &= \lim_{x \rightarrow 0^+} \frac{\sqrt{x}(\sqrt{x} - 1)}{\sqrt{x}(\sqrt{x} + 1)} \\ &= \lim_{x \rightarrow 0^+} \frac{(\sqrt{x} - 1)}{(\sqrt{x} + 1)} = \frac{(\sqrt{0} - 1)}{(\sqrt{0} + 1)} = -1 \end{aligned}$$

$$\begin{aligned} 6) \quad \lim_{x \rightarrow 1^+} \left(\frac{x^2 - \sqrt{x}}{\sqrt{x} - 1} \right) &= \lim_{x \rightarrow 1^+} \frac{\sqrt{x}(x\sqrt{x} - 1)(\sqrt{x} + 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)} \\ &= \lim_{x \rightarrow 1^+} \frac{\sqrt{x}(x^2 + x\sqrt{x} - \sqrt{x} - 1)}{(\sqrt{x})^2 - 1^2} \\ &= \lim_{x \rightarrow 1^+} \frac{\sqrt{x}(x^2 - 1 + \sqrt{x}(x - 1))}{x - 1} \\ &= \lim_{x \rightarrow 1^+} \frac{\sqrt{x}((x-1)(x+1) + \sqrt{x}(x-1))}{x - 1} \\ &= \lim_{x \rightarrow 1^+} \frac{\sqrt{x}(x-1)(x+1 + \sqrt{x})}{x - 1} \\ &= \lim_{x \rightarrow 1^+} \sqrt{x}(x+1 + \sqrt{x}) \\ &= \sqrt{1}(\sqrt{1} + 1 + 1) = 3 \end{aligned}$$

Solution N° 71 :

$$\begin{aligned} 1) \quad \lim_{x \rightarrow +\infty} \left(\sqrt{3x^2 + x + 4} - 2x + 1 \right) &\times \frac{(\sqrt{3x^2 + x + 4} + 2x - 1)}{(\sqrt{3x^2 + x + 4} + 2x - 1)} \end{aligned}$$

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$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{3x^2 + x + 4})^2 - (2x - 1)^2}{(\sqrt{3x^2 + x + 4} + 2x - 1)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^2 + 5x + 3}{(\sqrt{3x^2 + x + 4} + 2x - 1)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^2 + 5x + 3}{\left(\sqrt{x^2 \left(3 + \frac{1}{x} + \frac{4}{x^2}\right)} + 2x - 1\right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^2 + 5x + 3}{\sqrt{x^2} \sqrt{\left(3 + \frac{1}{x} + \frac{4}{x^2}\right)} + 2x - 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^2 + 5x + 3}{|x| \sqrt{\left(3 + \frac{1}{x} + \frac{4}{x^2}\right)} + 2x - 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{-x^2 + 5x + 3}{x \sqrt{\left(3 + \frac{1}{x} + \frac{4}{x^2}\right)} + 2x - 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(-x + 5 + \frac{3}{x}\right)}{x \left(\sqrt{\left(3 + \frac{1}{x} + \frac{4}{x^2}\right)} + 2 - \frac{1}{x}\right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-x + 5 + \frac{3}{x}}{\sqrt{3 + \frac{1}{x} + \frac{4}{x^2}} + 2 - \frac{1}{x}} \right)$$

$$= \left(\frac{-\infty + 5 + 0}{\sqrt{3 + 0 + 0} + 2 - 0} \right) = -\infty$$

$$\begin{aligned} 2) \quad \lim_{x \rightarrow +\infty} \left(\sqrt{9x^2 + 4x} - 3x + 8 \right) &\times \frac{\sqrt{9x^2 + 4x} + 3x - 8}{\sqrt{9x^2 + 4x} + 3x - 8} \end{aligned}$$

$$\lim_{x \rightarrow +\infty} \frac{(\sqrt{9x^2 + 4x})^2 - (3x - 8)^2}{\sqrt{x^2 \left(9 + \frac{4}{x}\right)} + 3x - 8}$$

$$= \lim_{x \rightarrow +\infty} \frac{52x - 64}{|x| \sqrt{9 + \frac{4}{x}} + 3x - 8}$$

$$= \lim_{x \rightarrow +\infty} \frac{52x - 64}{x \sqrt{9 + \frac{4}{x}} + 3x - 8}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(52 - \frac{64}{x} \right)}{x \left(\sqrt{9 + \frac{4}{x}} + 3 - \frac{8}{x} \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{52 - \frac{64}{x}}{\sqrt{9 + \frac{4}{x}} + 3 - \frac{8}{x}} \right)$$

$$= \left(\frac{52 - 0}{\sqrt{9 + 0} + 3 - 0} \right) = \frac{26}{3}$$

$$3) \lim_{x \rightarrow -\infty} \left(\frac{x - 2}{\sqrt{x^2 + 5x}} \right) = \lim_{x \rightarrow -\infty} \left(\frac{x - 2}{\sqrt{x^2 \left(1 + \frac{5}{x} \right)}} \right)$$

$$= \lim_{x \rightarrow -\infty} \frac{x - 2}{\sqrt{x^2} \cdot \sqrt{1 + \frac{5}{x}}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x - 2}{|x| \cdot \sqrt{1 + \frac{5}{x}}}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 - \frac{2}{x} \right)}{-x \sqrt{1 + \frac{5}{x}}} = \lim_{x \rightarrow -\infty} \frac{1 - \frac{2}{x}}{-\sqrt{1 + \frac{5}{x}}}$$

$$= \frac{1 - 0}{-\sqrt{1 + 0}} = -1$$

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$$4) \lim_{x \rightarrow -\infty} \left(\frac{2 - 7x}{3x + 5} \right) \sqrt{1 - 2x}$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(\frac{2}{x} - 7 \right)}{x \left(3 + \frac{5}{x} \right)} \sqrt{1 - 2x}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{\frac{2}{x} - 7}{3 + \frac{5}{x}} \right) \sqrt{1 - 2x}$$

$$= \left(\frac{0 - 7}{3 + 0} \right) \lim_{\substack{t \rightarrow +\infty \\ t = 1 - 2x}} \sqrt{1 - 2x}$$

$$= \frac{-7}{3} \cdot (+\infty) = -\infty$$

$$5) \lim_{x \rightarrow 4} \left(\frac{\sqrt{x + 5} - \sqrt{x} - 1}{\sqrt{x + 12} - \sqrt{x} - 2} \right)$$

$$= \lim_{x \rightarrow 4} \frac{(\sqrt{x + 5})^2 - (\sqrt{x} + 1)^2}{(\sqrt{x + 12})^2 - (\sqrt{x} + 2)^2} \cdot \left(\frac{\sqrt{x + 12} + \sqrt{x} + 2}{\sqrt{x + 5} + \sqrt{x} + 1} \right)$$

$$= \lim_{x \rightarrow 4} \left(\frac{4 - 2\sqrt{x}}{8 - 4\sqrt{x}} \right) \cdot \left(\frac{\sqrt{x + 12} + \sqrt{x} - 2}{\sqrt{x + 5} + \sqrt{x} - 1} \right)$$

$$= \lim_{x \rightarrow 4} \frac{1}{2} \left(\frac{4 - 2\sqrt{x}}{4 - 2\sqrt{x}} \right) \cdot \left(\frac{\sqrt{x + 12} + \sqrt{x} - 2}{\sqrt{x + 5} + \sqrt{x} - 1} \right)$$

$$= \frac{1}{2} \lim_{x \rightarrow 4} \left(\frac{\sqrt{x + 12} + \sqrt{x} - 2}{\sqrt{x + 5} + \sqrt{x} - 1} \right)$$

$$= \frac{1}{2} \left(\frac{\sqrt{4 + 12} + \sqrt{4} - 2}{\sqrt{4 + 5} + \sqrt{4} - 1} \right) = \frac{2}{3}$$

Solution N° 72 :

$$\begin{aligned}
 1) \quad \lim_{x \rightarrow \frac{1}{3}} \left| \frac{x^2 - 6x}{3x - 1} \right| &= \lim_{x \rightarrow \frac{1}{3}} \left| \frac{x(x - 6)}{x(x - \frac{1}{3})} \right| = \lim_{x \rightarrow \frac{1}{3}} \left| \frac{x - 6}{x - \frac{1}{3}} \right| \\
 &= \lim_{x \rightarrow \frac{1}{3}} |x - 6| \times \lim_{x \rightarrow \frac{1}{3}} \frac{1}{|x - \frac{1}{3}|} \\
 &= \frac{17}{3} \lim_{\substack{t \rightarrow 0^+ \\ t = x - \frac{1}{3}}} \frac{1}{|t|} = \frac{17}{3} \left(\frac{1}{|0^+|} \right) = +\infty
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \lim_{x \rightarrow (\frac{-3}{2})^+} \frac{3|x - 5| + 2}{4x^2 - 9} &= \lim_{x \rightarrow (\frac{-3}{2})^+} \left(\frac{3|x - 5| + 2}{2x - 3} \right) \left(\frac{1}{2x + 3} \right) \\
 &= \frac{-67}{24} \lim_{\substack{t \rightarrow 0^+ \\ t = x + \frac{3}{2}}} \frac{1}{t} = \frac{-67}{24} (+\infty) = -\infty
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \lim_{x \rightarrow -4} \frac{x + 5}{|x^2 + 4x|} &= \lim_{x \rightarrow -4} \left(\frac{x + 5}{|x|} \right) \cdot \left(\frac{1}{|x + 4|} \right) \\
 &= \frac{1}{4} \lim_{\substack{t \rightarrow 0^+ \\ t = x + 4}} \frac{1}{|t|} = \frac{1}{4} \times \frac{1}{0^+} = +\infty
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \lim_{x \rightarrow 0} \left(\frac{x^2 + |x|}{x^2 - |x|} \right) &= \lim_{x \rightarrow 0} \frac{|x| \left(\frac{x^2}{|x|} + 1 \right)}{|x| \left(\frac{x^2}{|x|} - 1 \right)} \\
 &= \lim_{x \rightarrow 0} \frac{(\pm x + 1)}{(\pm x - 1)} = \frac{(\pm 0 + 1)}{(\pm 0 - 1)} = -1
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \lim_{x \rightarrow +\infty} \frac{2x - 3}{|-5x + 7|} &= \lim_{x \rightarrow +\infty} \frac{x \left(2 - \frac{3}{x} \right)}{|x| \cdot \left| -5 + \frac{7}{x} \right|} \\
 &= \lim_{x \rightarrow +\infty} \frac{x \left(2 - \frac{3}{x} \right)}{x \left| -5 + \frac{7}{x} \right|} \\
 &= \lim_{x \rightarrow +\infty} \frac{\left(2 - \frac{3}{x} \right)}{\left| -5 + \frac{7}{x} \right|} = \frac{(2 - 0)}{|-5 + 0|} = \frac{2}{5}
 \end{aligned}$$

$$6) \quad \lim_{x \rightarrow 4} \left(\frac{|x^2 - 2x| - 8}{x^2 - 5x + 4} \right)$$

Quand $x \rightarrow 4$ alors on peut légalement considérer $x > 2$ puisque x prend des valeurs au voisinage de 4.

$$\begin{aligned}
 D' où \quad x(x - 2) &> 0 \\
 c - à - d \quad x^2 - 2x &> 0 \\
 Ainsi \quad |x^2 - 2x| &= x^2 - 2x
 \end{aligned}$$

Ainsi notre limite devient alors :

$$\begin{aligned}
 \lim_{x \rightarrow 4} \left(\frac{x^2 - 2x - 8}{x^2 - 5x + 4} \right) &= \lim_{x \rightarrow 4} \frac{(x - 4)(x + 2)}{(x - 4)(x - 1)} \\
 &= \lim_{x \rightarrow 4} \left(\frac{x + 2}{x - 1} \right) = \left(\frac{4 + 2}{4 - 1} \right) = 2
 \end{aligned}$$

Solution N° 73 :

$$\begin{aligned}
 1) \quad \lim_{x \rightarrow 4^-} \left(\frac{x}{x - 4} \right) &= \lim_{\substack{t \rightarrow 0^- \\ t = x - 4}} \left(\frac{t + 4}{t} \right) = \lim_{t \rightarrow 0^-} \left(1 + \frac{4}{t} \right) \\
 &= 1 + \frac{4}{0^-} = -\infty
 \end{aligned}$$

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$$\begin{aligned}
 2) \quad \lim_{x \rightarrow 3^-} \left(\frac{2x^2 - x + 1}{(3-x)(-1-x)} \right) \\
 &= \lim_{x \rightarrow 3^-} \left(\frac{2x^2 - x + 1}{-1-x} \right) \left(\frac{1}{3-x} \right) \\
 &= \left(\frac{18 - 3 + 1}{-1 - 3} \right) \lim_{\substack{t \rightarrow 0^+ \\ t=3-x}} \left(\frac{1}{t} \right) \\
 &= -4 \left(\frac{1}{0^+} \right) = -4(+\infty) = -\infty
 \end{aligned}$$

$$3) \quad \lim_{x \rightarrow 0^+} (x\sqrt{x} + 2) = 0\sqrt{0} + 2 = 2$$

$$\begin{aligned}
 4) \quad \lim_{x \rightarrow 2^+} \left(\frac{2-3x}{2-x} \right) &= \lim_{x \rightarrow 2^+} (2-3x) \times \lim_{x \rightarrow 2^+} \left(\frac{1}{2-x} \right) \\
 &= -4 \times \lim_{\substack{t \rightarrow 0^- \\ t=2-x}} \left(\frac{1}{t} \right) = -4(-\infty) = +\infty
 \end{aligned}$$

$$5) \quad \lim_{x \rightarrow 3^+} \frac{-7}{\sqrt{x}-3} = \lim_{\substack{t \rightarrow 0^+ \\ t=x-3}} \frac{-7}{\sqrt{t}} = \frac{-7}{0^+} = -\infty$$

$$\begin{aligned}
 6) \quad \lim_{x \rightarrow 2^+} \left(\frac{x^3 - 8}{x^2 - 4x + 4} \right) \\
 &= \lim_{x \rightarrow 2^+} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x-2)} \\
 &= \lim_{x \rightarrow 2^+} \left(\frac{x^2 + 2x + 4}{x-2} \right) \\
 &= \lim_{x \rightarrow 2^+} (x^2 + 2x + 4) \times \lim_{x \rightarrow 2^+} \left(\frac{1}{x-2} \right) \\
 &= 12 \lim_{\substack{t \rightarrow 0^+ \\ t=x-2}} \left(\frac{1}{t} \right) = 12 \left(\frac{1}{0^+} \right) = +\infty
 \end{aligned}$$

Solution N° 74 :

$$\begin{aligned}
 1) \quad \lim_{x \rightarrow 1^-} \frac{(x-1)^2}{(x^2-1)^5} &= \lim_{x \rightarrow 1^-} \frac{(x-1)^2}{(x-1)^5(x+1)^5} \\
 &= \lim_{x \rightarrow 1^-} \frac{1}{(x+1)^5} \times \lim_{x \rightarrow 1^-} \frac{1}{(x-1)^3} \\
 &= \left(\frac{1}{2} \right)^5 \lim_{\substack{t \rightarrow 0^- \\ t=x-1}} \left(\frac{1}{t^3} \right) = \frac{1}{32} \left(\frac{1}{0^-} \right) = -\infty
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \lim_{x \rightarrow 0^+} \left(\frac{x^3 - 1}{x^2 - x} \right) &= \lim_{x \rightarrow 0^+} \frac{(x-1)(x^2 + x + 1)}{x(x-1)} \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{x^2 + x + 1}{x} \right) \\
 &= \lim_{x \rightarrow 0^+} (x^2 + x + 1) \times \lim_{x \rightarrow 0^+} \frac{1}{x} = 1 \left(\frac{1}{0^+} \right) = +\infty
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \lim_{x \rightarrow 3^+} \left(\frac{\sqrt{x^2 - 9} + \sqrt{x} - \sqrt{3}}{\sqrt{x} - 3} \right) \\
 &= \lim_{x \rightarrow 3^+} \frac{\sqrt{x^2 - 9}}{\sqrt{x} - 3} + \lim_{x \rightarrow 3^+} \frac{\sqrt{x} - \sqrt{3}}{\sqrt{x} - 3} \\
 &= \lim_{x \rightarrow 3^+} \frac{\sqrt{x-3} \cdot \sqrt{x+3}}{\sqrt{x} - 3} \\
 &\quad + \lim_{x \rightarrow 3^+} \frac{(\sqrt{x} - \sqrt{3})(\sqrt{x} + \sqrt{3})}{\sqrt{x} - 3 (\sqrt{x} + \sqrt{3})} \\
 &= \lim_{x \rightarrow 3^+} \sqrt{x+3} + \lim_{x \rightarrow 3^+} \frac{(x-3)}{\sqrt{x-3} (\sqrt{x} + \sqrt{3})} \\
 &= \lim_{x \rightarrow 3^+} \sqrt{x+3} + \lim_{x \rightarrow 3^+} \frac{\sqrt{x-3} \cdot \sqrt{x-3}}{\sqrt{x-3} (\sqrt{x} + \sqrt{3})} \\
 &= \lim_{x \rightarrow 3^+} \sqrt{x+3} + \lim_{x \rightarrow 3^+} \frac{\sqrt{x-3}}{\sqrt{x} + \sqrt{3}} \\
 &= \sqrt{6} + \frac{\sqrt{3-3}}{\sqrt{3} + \sqrt{3}} = \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \lim_{x \rightarrow (-2)^+} \left(\frac{4x^2 - x + 5}{x^2 - 4} \right) \\
 &= \lim_{x \rightarrow (-2)^+} \left(\frac{4x^2 - x + 5}{x - 2} \right) \times \lim_{x \rightarrow (-2)^+} \left(\frac{1}{x + 2} \right) \\
 &= \left(\frac{16 + 2 + 5}{-2 - 2} \right) \lim_{\substack{t \rightarrow 0^+ \\ t = x + 2}} \left(\frac{1}{t} \right) = \frac{-23}{4} \left(\frac{1}{0^+} \right) = -\infty
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} - \frac{1}{x^3 - 1} \right) \\
 &= \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} - \frac{1}{(x - 1)(x^2 + x + 1)} \right) \\
 &= \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} \right) \left(1 - \frac{1}{x^2 + x + 1} \right) \\
 &= \left(\frac{1}{0^+} \right) \left(1 - \frac{1}{1 + 1 + 1} \right) = +\infty
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & \lim_{x \rightarrow 0^+} \left(\frac{2 - \sqrt{x^2 + 4}}{\sqrt{x} - \sqrt{2x^2}} \right) \\
 &= \lim_{x \rightarrow 0^+} \frac{(2)^2 - (\sqrt{x^2 + 4})^2}{(\sqrt{x})^2 - (\sqrt{2x^2})^2} \times \frac{\sqrt{x} + \sqrt{2x^2}}{2 + \sqrt{x^2 + 4}} \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{-x^2}{x - 2x^2} \right) \left(\frac{\sqrt{x} + \sqrt{2x^2}}{2 + \sqrt{x^2 + 4}} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{x}{x} \right) \left(\frac{-x}{1 - 2x} \right) \left(\frac{\sqrt{x}}{1} \right) \left(\frac{1 + \sqrt{2x}}{2 + \sqrt{x^2 + 4}} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{-x}{1 - 2x} \right) \left(\frac{\sqrt{x}}{1} \right) \left(\frac{1 + \sqrt{2x}}{2 + \sqrt{x^2 + 4}} \right) \\
 &= \left(\frac{-0}{1 - 0} \right) \left(\frac{\sqrt{0}}{1} \right) \left(\frac{1 + \sqrt{0}}{2 + \sqrt{4}} \right) = 0
 \end{aligned}$$

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Solution N° 75 :

$$\begin{aligned}
 1) \quad & \lim_{x \rightarrow 0} \left(\frac{\sin(2x) - 2 \sin x}{x^3} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{2 \sin x \cdot \cos x - 2 \sin x}{x^3} \right) \\
 &= \lim_{x \rightarrow 0} 2 \left(\frac{\sin x}{x} \right) \left(\frac{\cos x - 1}{x^2} \right) \\
 &= 2(1) \left(\frac{-1}{2} \right) = -1
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos \sqrt{x}}{\sin x} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos \sqrt{x}}{x} \right) \left(\frac{x}{\sin x} \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1 - \cos \sqrt{x}}{(\sqrt{x})^2} \right) \cdot \lim_{x \rightarrow 0^+} \left(\frac{1}{\frac{\sin x}{x}} \right) \\
 &= \lim_{\substack{t \rightarrow 0^+ \\ t = \sqrt{x}}} \left(\frac{1 - \cos t}{t^2} \right) \cdot \lim_{x \rightarrow 0^+} \left(\frac{1}{\frac{\sin x}{x}} \right) \\
 &= \left(\frac{1}{2} \right) \times \left(\frac{1}{1} \right) = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \lim_{x \rightarrow 0} \left(\frac{1 - \cos(4x)}{\tan 2x \cdot \sin x} \right) \\
 &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 4x}{(4x)^2} \right) \left(\frac{2x}{\tan 2x} \right) \left(\frac{x}{\sin x} \right) \left(\frac{8}{1} \right) \\
 &= \left(\frac{1}{2} \right) \left(\frac{1}{1} \right) \left(\frac{1}{1} \right) \left(\frac{8}{1} \right) = 4
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \lim_{x \rightarrow +\infty} x^2 \left(1 - \cos \left(\frac{1}{x} \right) \right) \\
 &= \lim_{x \rightarrow +\infty} \left(\frac{1 - \cos \left(\frac{1}{x} \right)}{\left(\frac{1}{x} \right)^2} \right)
 \end{aligned}$$

$$= \lim_{\substack{t \rightarrow 0^+ \\ t = \frac{1}{x}}} \left(\frac{1 - \cos t}{t^2} \right) = \frac{1}{2}$$

$$\begin{aligned} 5) \quad \lim_{x \rightarrow 0} \left(\frac{2 - \sqrt{x+4}}{\tan(5x)} \right) \\ &= \lim_{x \rightarrow 0} \frac{(2 - \sqrt{x+4})(2 + \sqrt{x+4})}{\tan(5x) \cdot (2 + \sqrt{x+4})} \\ &= \lim_{x \rightarrow 0} \frac{2^2 - (\sqrt{x+4})^2}{(\tan(5x))(2 + \sqrt{x+4})} \\ &= \lim_{x \rightarrow 0} \frac{-x}{(\tan(5x))(2 + \sqrt{x+4})} \\ &= \lim_{x \rightarrow 0} \left(\frac{5x}{\tan(5x)} \right) \left(\frac{-1}{2 + \sqrt{x+4}} \right) \frac{1}{5} \\ &= \left(\frac{1}{1} \right) \left(\frac{-1}{2 + \sqrt{4}} \right) \frac{1}{5} = \frac{-1}{20} \end{aligned}$$

$$\begin{aligned} 6) \quad \lim_{x \rightarrow 0} \left(\frac{\sin x - \tan x}{x^3} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x - \frac{\sin x}{\cos x}}{x^3} \right) \\ &= \lim_{x \rightarrow 0} \frac{(\sin x) \left(1 - \frac{1}{\cos x} \right)}{x^3} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{-1}{\cos^2 x} \right) \\ &= (1) \left(\frac{1}{2} \right) \left(\frac{-1}{1^2} \right) = \frac{-1}{2} \end{aligned}$$

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Solution N° 76 :

$$\begin{aligned} 1) \quad \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \left(\frac{1 + \sin x}{\cos x} \right) \\ &= \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \frac{(1 + \sin x)(1 - \sin x)}{(\cos x)(1 - \sin x)} \\ &= \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \frac{1 - \sin^2 x}{(\cos x)(1 - \sin x)} \\ &= \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \frac{\cos^2 x}{(\cos x)(1 - \sin x)} \\ &= \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \left(\frac{\cos x}{1 - \sin x} \right) \\ &= \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^+} \left(\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)} \right) \\ &= \lim_{\substack{t \rightarrow 0^- \\ t = \frac{\pi}{2} - x}} \left(\frac{\sin t}{1 - \cos t} \right) \\ &= \lim_{t \rightarrow 0^-} \left(\frac{\sin t}{t} \right) \left(\frac{t^2}{1 - \cos t} \right) \left(\frac{1}{t} \right) \\ &= (1) \left(\frac{2}{1} \right) \left(\frac{1}{0^-} \right) = -\infty \end{aligned}$$

$$\begin{aligned} 2) \quad \lim_{x \rightarrow \left(-\frac{\pi}{2}\right)^+} \tan x &= \lim_{x \rightarrow \left(-\frac{\pi}{2}\right)^+} -\tan(-x) \\ &= - \lim_{\substack{t \rightarrow \left(\frac{\pi}{2}\right)^- \\ t = -x}} \tan t = -(+\infty) = -\infty \end{aligned}$$

$$3) \quad \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cos x}{2x - \pi} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(x - \frac{\pi}{2}\right)}$$

$$= \frac{-1}{2} \lim_{\substack{t \rightarrow 0 \\ t = \frac{\pi}{2} - x}} \left(\frac{\sin t}{t} \right) = \left(\frac{-1}{2} \right) (1) = \frac{-1}{2}$$

$$4) \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \tan x = +\infty \quad ; \quad \text{selon le cours}$$

$$\begin{aligned} 5) \lim_{\substack{t \rightarrow 0 \\ t=x-1}} \frac{\sin(\pi t + \pi)}{t} &= \lim_{t \rightarrow 0} \frac{-\sin(\pi t)}{t} \\ &= \lim_{t \rightarrow 0} \left(\frac{\sin \pi t}{\pi t} \right) \left(\frac{-\pi}{1} \right) = (1) \left(\frac{-\pi}{1} \right) = -\pi \end{aligned}$$

Solution N° 77 :

$$\begin{aligned} 1) \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \left(\frac{x^3 - 1}{x - 1} \right) \\ &= \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{(x - 1)} \\ &= \lim_{x \rightarrow 1} (x^2 + x + 1) = (1 + 1 + 1) = 3 \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = 1$ noté $\tilde{f}$ et définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq 1 \\ \tilde{f}(1) = 3 \end{cases}$$

$$\begin{aligned} 2) \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - 1}{x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{1 + \sin x})^2 - 1^2}{x(\sqrt{1 + \sin x} + 1)} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{x(\sqrt{1 + \sin x} + 1)} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{1}{\sqrt{1 + \sin x} + 1} \right) \\ &= (1) \left(\frac{1}{\sqrt{1 + \sin 0} + 1} \right) = \frac{1}{2} \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = 0$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq 0 \\ \tilde{f}(0) = \frac{1}{2} \end{cases}$$

$$\begin{aligned} 3) \lim_{x \rightarrow -1} f(x) &= \lim_{x \rightarrow -1} \left(\frac{x^3 - 2x^2 + 3x + 6}{x + 1} \right) \\ &= \lim_{x \rightarrow -1} \frac{(x + 1)(x^2 - 3x + 6)}{(x + 1)} \quad ; \quad \left| \begin{array}{l} \text{Division} \\ \text{euclidienne} \end{array} \right. \\ &= \lim_{x \rightarrow -1} (x^2 - 3x + 6) = 1 + 3 + 6 = 10 \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = -1$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq -1 \\ \tilde{f}(-1) = 10 \end{cases}$$

$$\begin{aligned} 4) \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{x \sin x}{\cos x - 1} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{x^2}{\cos x - 1} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{-1}{\frac{\cos x - 1}{x^2}} \right) \\ &= (1) \left(\frac{-1}{\frac{1}{2}} \right) = -2 \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = 0$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq 0 \\ \tilde{f}(0) = -2 \end{cases}$$

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Solution N° 78 :

$$1) \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{|x^2 + 4x| - 3}{x + 3}$$

Comme $x \rightarrow -3$ Alors on peut prendre $x < 0$ et $x > -4$ car x prend ses valeurs dans un voisinage de -3 .

$$\begin{aligned} \Rightarrow x < 0 \quad \text{et} \quad x + 4 > 0 \\ \Rightarrow x(x + 4) < 0 \\ \Rightarrow x^2 + 4x < 0 \\ \Rightarrow |x^2 + 4x| = -x^2 - 4x \end{aligned}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow -3} f(x) &= \lim_{x \rightarrow -3} \frac{-x^2 - 4x - 3}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{(x + 3)(-x - 1)}{(x + 3)} = \lim_{x \rightarrow -3} (-x - 1) \\ &= 3 - 1 = 2 \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = 3$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq -3 \\ \tilde{f}(-3) = 2 \end{cases}$$

$$\begin{aligned} 2) \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} \left(\frac{x^3 - a^3}{x - a} \right) ; \quad a \in \mathbb{R} \\ &= \lim_{x \rightarrow a} \frac{(x - a)(x^2 + ax + a^2)}{(x - a)} \\ &= \lim_{x \rightarrow a} (x^2 + ax + a^2) = 3a^2 \in \mathbb{R} \end{aligned}$$

Donc f admet un prolongement par continuité au point $x_0 = a$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq a \\ \tilde{f}(a) = 3a^2 \end{cases}$$

$$\begin{aligned} 3) \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} \frac{(x - 2)^2}{4x - x^3} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x - 2)}{x(4 - x^2)} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x - 2)}{x(2 - x)(2 + x)} \end{aligned}$$

$$= \lim_{x \rightarrow 2} \frac{(x - 2)}{x(2 + x)} = \frac{2 - 2}{2(2 + 2)} = 0 \in \mathbb{R}$$

Donc f admet un prolongement par continuité au point $x_0 = 2$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq 2 \\ \tilde{f}(2) = 0 \end{cases}$$

$$4) \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (x - 1) \sin\left(\frac{1}{x - 1}\right)$$

$$\text{On a : } \left| \sin\left(\frac{1}{x - 1}\right) \right| < 1 ; \quad \forall x \neq 1$$

$$\Rightarrow \underbrace{0}_{\downarrow 0} < \left| (x - 1) \sin\left(\frac{1}{x - 1}\right) \right| < \underbrace{|x - 1|}_{\downarrow 0}$$

$$\Rightarrow \lim_{x \rightarrow 1} (x - 1) \sin\left(\frac{1}{x - 1}\right) = 0 \in \mathbb{R}$$

Donc f admet un prolongement par continuité au point $x_0 = 1$ noté $\tilde{f}$ définie ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad \forall x \neq 1 \\ \tilde{f}(1) = 0 \end{cases}$$

Solution N° 79 :

$$1) \text{ On pose : } f(x) = u \circ v(x) ; \quad \forall x \neq \pm 1$$

$$\text{Avec } u(x) = \sin x \quad \text{et} \quad v(x) = \frac{2x + 1}{x^2 - 1}$$

La fonction u est continue sur $\mathbb{R}$ selon le cours. Et la fonction v est continue sur $\mathbb{R} \setminus \{1; -1\}$.

Si $x \in \mathbb{R} \setminus \{-1, 1\}$ Alors $v(x) = \frac{2x+1}{x^2-1} \in \mathbb{R}$

D'où $v(\mathbb{R} \setminus \{-1, 1\}) \subseteq \mathbb{R}$

Donc $f = u \circ v$ est continue sur $\mathbb{R} \setminus \{-1, 1\}$

2) on pose $f(x) = u \circ v(x)$; $\forall x \in \mathbb{R}$

Avec : $u(x) = \cos x$ et $v(x) = \sqrt{x^2 + 1}$

La fonction u est continue sur $\mathbb{R}$ d'après le cours et la fonction v est continue sur $\mathbb{R}$ aussi.

Si $x \in \mathbb{R}$ Alors $v(x) = \sqrt{x^2 + 1} \in \mathbb{R}$

Donc $v(\mathbb{R}) \subseteq \mathbb{R}$

Donc $f = u \circ v$ est continue sur $\mathbb{R}$.

$$3) f(x) = \sqrt{\frac{x-3}{x+2}}$$

$$D_f = \left\{ x \in \mathbb{R} ; x+2 \neq 0 \text{ et } \frac{x-3}{x+2} \geq 0 \right\}$$

$$= \left\{ x \in \mathbb{R} ; \begin{array}{l} x-3 \leq 0 \text{ et } x+2 < 0 \\ \text{ou bien} \\ x-3 \geq 0 \text{ et } x+2 > 0 \end{array} \right\}$$

$$= \left\{ x \in \mathbb{R} ; \begin{array}{l} x \leq 3 \text{ et } x < -2 \\ \text{ou bien} \\ x \geq 3 \text{ et } x > -2 \end{array} \right\}$$

$$= \left\{ x \in \mathbb{R} ; \begin{array}{l} \text{ou bien } x < -2 \\ \text{ou bien } x \geq 3 \end{array} \right\}$$

$$=]-\infty, -2[\cup [3, +\infty[$$

On pose $f(x) = u \circ v(x)$; $\forall x \in D_f$

Avec : $u(x) = \sqrt{x}$ et $v(x) = \frac{x-3}{x+2}$

La fonction u est continue sur $\mathbb{R}^+$ d'après le cours et la fonction v est continue sur $\mathbb{R} \setminus \{-2\}$ Donc v est continue sur $] -\infty, -2[\cup [3, +\infty[\subset \mathbb{R} \setminus \{-2\}$

Si $x \in] -\infty, -2[\cup [3, +\infty[$

Alors $x < -2$ ou $x \geq 3$

$$\Rightarrow x+2 < 0 \text{ ou } x-3 \geq 0$$

$$\Rightarrow \begin{cases} \text{si } x+2 < 0 & \text{alors } x-3 \leq 0 \\ \text{si } x-3 \geq 0 & \text{alors } x+2 > 0 \end{cases}$$

$$\Rightarrow \begin{cases} \text{ou bien } \frac{x-3}{x+2} \geq 0 \\ \text{ou bien } \frac{x-3}{x+2} \geq 0 \end{cases} ; \text{ même résultat}$$

$$\Rightarrow \frac{x-3}{x+2} \geq 0$$

$$\Rightarrow v(x) \in \mathbb{R}^+$$

$$\Rightarrow v(]-\infty, -2[\cup [3, +\infty[) \subseteq \mathbb{R}^+$$

D'où finalement $f = u \circ v$ est continue sur $] -\infty, -2[\cup [3, +\infty[$.

$$4) f(x) = \sqrt{1 - \sin x}$$

$$D_f = \{ x \in \mathbb{R} ; 1 - \sin x \geq 0 \}$$

$$= \{ x \in \mathbb{R} ; \sin x \leq 1 \text{ toujours vraie} \}$$

$$= \mathbb{R} \text{ tout entier}$$

On pose $f(x) = u \circ v(x)$; $\forall x \in \mathbb{R}$

Avec $u(x) = \sqrt{x}$ et $v(x) = 1 - \sin x$

On a la fonction v est continue sur $\mathbb{R}$ et la fonction u est continue sur $\mathbb{R}^+$

Si $x \in \mathbb{R}$ Alors $v(x) = \sqrt{x} \in \mathbb{R}^+$

Donc $v(\mathbb{R}) \subseteq \mathbb{R}^+$.

D'où finalement on déduit que la composition $u \circ v = f$ est bien continue sur $\mathbb{R}$ tout entier.

Solution N° 80 :

1) on pose $f(x) = u \circ v(x)$; $\forall x \in \mathbb{R}$

Avec : $u(x) = \cos x$; $v(x) = 2x^2 - 3x + 4$

On a v est continue sur $\mathbb{R}$ car polynôme
Et on a u est continue sur $\mathbb{R}$.

$$\begin{aligned} x \in \mathbb{R} &\Rightarrow (2x^2 - 3x + 4) \in \mathbb{R} \\ &\Rightarrow v(x) \in \mathbb{R} \\ &\Rightarrow v(\mathbb{R}) \subseteq \mathbb{R} \end{aligned}$$

D'où $u \circ v$ est continue sur $\mathbb{R}$.

2) $f(x) = \tan\left(\frac{\pi}{x}\right)$

$$D_f = \left\{ x \in \mathbb{R} ; \frac{\pi}{x} \neq \frac{\pi}{2}[\pi] \right\}$$

$$= \left\{ x \in \mathbb{R} ; \frac{\pi}{x} \neq \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$$

$$= \left\{ x \in \mathbb{R} ; \frac{1}{x} \neq \frac{1}{2} + k ; k \in \mathbb{Z} \right\}$$

$$= \left\{ x \in \mathbb{R} ; x \neq \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}$$

$$= \mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}$$

On pose : $f(x) = u \circ v(x)$; $\forall x \in D_f$

Avec : $u(x) = \tan x$; $v(x) = \frac{\pi}{x}$

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On a v est continue sur $\mathbb{R}^*$.

Donc v est continue sur $\mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}$

Car $\mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\} \subseteq \mathbb{R}^*$

Car $\frac{2}{2k+1} \neq 0 ; \forall k \in \mathbb{Z}$

$$\Rightarrow x \neq \frac{2}{2k+1} ; k \in \mathbb{Z}$$

$$\Rightarrow \frac{\pi}{x} \neq \frac{\pi}{2} + k\pi ; k \in \mathbb{Z}$$

$$\Rightarrow v(x) \in \mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}$$

$$\begin{aligned} \Rightarrow v\left(\mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}\right) \\ \subseteq \mathbb{R} \setminus \left\{ \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\} \right\} \end{aligned}$$

Donc d'après le théorème de la continuité de la fonction composée on en déduit que la composition $u \circ v$ est continue sur :

$$\mathbb{R} \setminus \left\{ \frac{2}{2k+1} ; k \in \mathbb{Z} \right\}$$

$$3) f(x) = \sqrt{\frac{x}{1 + \sin^2 x}}$$

$$\begin{aligned} D_f &= \left\{ x \in \mathbb{R} ; \frac{x}{1 + \sin^2 x} \geq 0 \text{ et } \underbrace{1 + \sin^2 x \neq 0}_{\text{toujours vérifiée}} \right\} \\ &= \{ x \in \mathbb{R} ; x \geq 0 \} = \mathbb{R}^+ \end{aligned}$$

On pose $f(x) = u \circ v(x)$; $\forall x \geq 0$

Avec : $u(x) = \sqrt{x}$; $v(x) = \frac{x}{1 + \sin^2 x}$

On a v est continue sur $\mathbb{R}$ Donc v est continue sur $\mathbb{R}^+$ car $\mathbb{R}^+ \subset \mathbb{R}$

On a u est continue sur $\mathbb{R}^+$; (cours)

$$\begin{aligned} x \in \mathbb{R}^+ &\Rightarrow v(x) = \sqrt{x} \in \mathbb{R}^+ \\ &\Rightarrow v(\mathbb{R}^+) \subseteq \mathbb{R}^+ \end{aligned}$$

D'où finally $u \circ v$ est continue sur $\mathbb{R}^+$

$$4) f(x) = \cos(\tan^2 x)$$

$$\begin{aligned} D_f &= \left\{ x \in \mathbb{R} ; x \neq \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\} \\ &= \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\} \end{aligned}$$

On pose : $f(x) = u \circ v(x) ; \forall x \in D_f$

Avec : $u(x) = \cos x ; v(x) = \tan^2 x$

On a v est continue sur $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$

Et u est continue sur $\mathbb{R}$

$$x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\} \Rightarrow v(x) = \tan^2 x \in \mathbb{R}$$

$$\text{Donc } v\left(\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}\right) \subseteq \mathbb{R}$$

Donc la composition $u \circ v$ est continue

Sur l'ensemble $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$

Solution N° 81 :

$$1) \lim_{x \rightarrow +\infty} \left(x - 2\sqrt{x} + \frac{1}{x} \right)^3 = +\infty$$

$$2) \lim_{x \rightarrow 0} \tan\left(\frac{\pi \sin x}{3x}\right) = \sqrt{3}$$

$$3) \lim_{x \rightarrow -\infty} \cos\left(\frac{\pi x + 1}{x + 2}\right) = -1$$

$$4) \lim_{x \rightarrow +\infty} \cos\left(\sin\left(\frac{1}{x}\right)\right) = 1$$

$$5) \lim_{x \rightarrow +\infty} \cos\left(\pi \sqrt{\frac{x-1}{x+1}}\right) = -1$$

$$6) \lim_{x \rightarrow 0} \sin\left(\pi \left(\frac{1 - \cos x}{x^2}\right)\right) = 1$$

Solution N° 82 :

$$1) f(x) = x^2 + 2$$

La fonction f est continue sur $\mathbb{R}$ car c'est un polynôme. Donc f est continue sur $[-1, 3] \subset \mathbb{R}$. Pour déterminer $f([-1, 3])$ on doit d'abord déterminer le sens de variations de f sur $[-1, 3]$.

$$f'(x) = 2x ; \forall x \in \mathbb{R}$$

$$\begin{cases} x = 0 & \Leftrightarrow f'(x) = 0 \\ x > 0 & \Leftrightarrow f'(x) > 0 \\ x < 0 & \Leftrightarrow f'(x) < 0 \end{cases}$$

Donc la fonction f est décroissante sur l'intervalle $[-1, 0]$ et croissante sur $[0, 3]$.

$$\begin{aligned} \text{D'où } f([-1, 3]) &= f([-1, 0] \cup [0, 3]) \\ &= f([-1, 0]) \cup f([0, 3]) \\ &= [f(0), f(-1)] \cup [f(0), f(3)] \\ &= [2, 3] \cup [2, 11] \\ &= [2, 11] \end{aligned}$$

$$2) f(x) = \frac{x-4}{x-2}$$

f est continue sur $\mathbb{R} \setminus \{2\}$ car quotient bien défini de deux polynômes (fraction rationnelle). Donc f est continue sur $[5, 8] \subset \mathbb{R} \setminus \{2\}$ la fonction f est décroissante sur $\mathbb{R} \setminus \{2\}$ car :

$$f'(x) = \frac{2}{(x-2)^2} > 0 ; \forall x \neq 2$$

On peut montrer que f est décroissante

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En déterminant le signe du taux de variation $\left(\frac{f(a)-f(b)}{a-b}\right)$

D'où finalement :

$$f([5,8]) = [f(5); f(8)] = \left[\frac{1}{3}; \frac{2}{3}\right]$$

$$3) f(x) = 2x\sqrt{x+1}$$

La fonction f est continue sur $[-1, +\infty[= D_f$ car c'est un produit d'un polynôme et d'une composition bien définie et continue. Donc f est continue sur $[3,5] \subset [-1, +\infty[$. On a aussi f est une fonction croissante sur $[3,5]$.

$$\text{car si } x > y \Rightarrow 2x > 2y \text{ et } x+1 > y+1$$

$$\Rightarrow 2x > 2y \text{ et } \sqrt{x+1} > \sqrt{y+1}$$

$$\Rightarrow 2x\sqrt{x+1} > 2y\sqrt{y+1}$$

$$\Rightarrow f(x) > f(y)$$

$$\Rightarrow \left(\frac{f(x)-f(y)}{x-y}\right) > 0$$

$$\Rightarrow f \text{ est strictement } \nearrow$$

$$\Rightarrow f([3,5]) = [f(3), f(5)] = [12, 10\sqrt{6}]$$

$$4) f(x) = \tan x$$

On a la fonction f est continue sur chaque intervalle de l'ensemble :

$$\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$$

On a aussi la fonction f est croissante sur chaque intervalle de l'ensemble :

$$\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$$

$$\text{Car } f'(x) = 1 + \tan^2 x > 0$$

$$D' \text{ où } f\left(\left]-\frac{\pi}{2}; \frac{\pi}{2}\right[\right) = \left[\lim_{x \rightarrow -\frac{\pi}{2}^+} f(x); \lim_{x \rightarrow \frac{\pi}{2}^-} f(x)\right[\\ =]-\infty, +\infty[= \mathbb{R}$$

$$5) f(x) = \begin{cases} x+3 & ; x \leq 2 \\ x^2+1 & ; x > 2 \end{cases}$$

La fonction f est continue sur $\mathbb{R}$ tout entier car elle est définie par morceaux de polynômes. Donc f est continue sur l'intervalle $[-3,5] \subset \mathbb{R}$.

La fonction f est strictement croissante sur l'ensemble $\mathbb{R}$ car :

$$\begin{cases} \text{Si } x \leq 2 & \text{Alors } f'(x) = 1 > 0 \\ \text{Si } x > 2 & \text{Alors } f'(x) = 2x > 0 \end{cases}$$

$$\text{Donc : } f([-3,5]) = [f(-3), f(5)] = [0, 26]$$

Solution N° 83 :

$$\text{Soit : } \varphi(x) = x^4 + x^2 + 4x - 1$$

On a φ est continue sur $\mathbb{R}$ car c'est un polynôme. Donc φ est continue sur l'intervalle $[0,1]$

$$\text{On a encore } \varphi(0) = -1 \text{ et } \varphi(1) = 5$$

$$\text{Donc : } \varphi(0) \cdot \varphi(1) < 0$$

D'où selon le théorème des valeurs intermédiaires on déduit que :

$$\exists \alpha \in [0,1] ; \varphi(\alpha) = 0$$

C'est à dire que l'équation :

$$x^4 + x^2 + 4x - 1 = 0$$

Admet au moins une solution $\alpha \in [0,1]$

Soit la fonction φ définie ainsi :

$$\varphi(x) = \tan x + x^2 - 2 ; \quad \forall x \neq \frac{\pi}{2} [\pi]$$

On a φ est continue sur l'ensemble :

$$\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$$

Donc φ est continue sur $\left] \frac{\pi}{4} ; \frac{\pi}{3} \right[$ car :

$$\left] \frac{\pi}{4} ; \frac{\pi}{3} \right[\subset \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi ; k \in \mathbb{Z} \right\}$$

$$\begin{aligned} \text{Or, } \varphi\left(\frac{\pi}{4}\right) &= \tan\left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right)^2 - 2 \\ &= \left(\frac{\pi^2}{16} - 1\right) < 0 \quad \text{car } \pi < 4 \end{aligned}$$

$$\begin{aligned} \text{Et, } \varphi\left(\frac{\pi}{3}\right) &= \tan\left(\frac{\pi}{3}\right) + \left(\frac{\pi}{3}\right)^2 - 2 \\ &= \left(\sqrt{3} - 2 + \frac{\pi^2}{9}\right) < 0 \end{aligned}$$

$$\text{Donc } \varphi\left(\frac{\pi}{4}\right) \times \varphi\left(\frac{\pi}{3}\right) < 0$$

Donc d'après le théorème des valeurs intermédiaires on en déduit que :

$$\exists \alpha \in \left] \frac{\pi}{4} ; \frac{\pi}{3} \right[; \quad \varphi(\alpha) = 0$$

C'est à dire que l'équation :

$$\tan x + x^2 - 2 = 0$$

Admet au moins une solution $\alpha \in \left] \frac{\pi}{4} ; \frac{\pi}{3} \right[$

Solution N° 84 :

1) D'abord f est continue sur $[1, +\infty[$ car c'est un polynôme de $\mathbb{R}[X]$.

On a aussi f est strictmnt $\nearrow$ sur $[1, +\infty[$
Car $f'(x) = 2x - 2 \geq 0 ; \quad \forall x \geq 1$

Donc f réalise une bijection de $[1, +\infty[$ sur un intervalle $J = f([1, +\infty[)$

$$\begin{aligned} J = f([1, +\infty[) &= \left[f(1) ; \lim_{x \rightarrow +\infty} f(x) \right[\\ &= [4, +\infty[\end{aligned}$$

Donc $f : [1, +\infty[\mapsto [4, +\infty[$ est bijective

$y \in [4, +\infty[$ alors $\exists ! x \in [1, +\infty[: f(x) = y$

$$\begin{aligned} &\Leftrightarrow x^2 - 2x + 5 = y \\ &\Leftrightarrow x^2 - 2x + 5 - y = 0 \end{aligned}$$

$$\Leftrightarrow x = \frac{2 \pm 2\sqrt{y-4}}{2} ; \quad \Delta = 4(y-4) \geq 0$$

$$\Leftrightarrow x = 1 \pm \sqrt{y-4}$$

$$\Leftrightarrow x = 1 + \sqrt{y-4} \in [1, +\infty[$$

D'où finalement :

$$\begin{aligned} f^{-1} : [4, +\infty[&\mapsto [1, +\infty[\\ y &\mapsto 1 + \sqrt{y-4} \end{aligned}$$

2) D'abord f est continue sur $] -\infty, 2[$ car c'est un polynôme de $\mathbb{R}[X]$

On a aussi f est strictement croissante sur l'intervalle $] -\infty, 2[$ car :

$$f'(x) = -2x + 4 ; \quad \forall x < 2$$

Donc f réalise une bijection de $] -\infty, 2[$ Sur $J = f(] -\infty, 2[) =] -\infty, 0[$

Donc $f :]-\infty, 2[\mapsto]-\infty, 0[$ est bijective

$y \in]-\infty, 0[$ alors $\exists ! x \in]-\infty, 2[: f(x) = y$

$$\Leftrightarrow 4x - x^2 = y$$

$$\Leftrightarrow -x^2 + 4x - y = 0 \quad ; \quad \Delta = 16 - 4y \geq 0$$

$$\Leftrightarrow x = \frac{-4 \pm 2\sqrt{4-y}}{-2} \quad ; \quad \Delta = 16 - 4y \geq 0$$

$$\Leftrightarrow x = 2 \pm \sqrt{4-y}$$

$$\Leftrightarrow x = 2 - \sqrt{4-y} \in]-\infty, 2[$$

D'où finalement :

$$f^{-1} :]-\infty, 0[\mapsto]-\infty, 2[\\ y \mapsto 2 - \sqrt{4-y}$$

3) D'abord f est continue sur l'ensemble :

$$D_f =]-\infty, 0] \cup [1, +\infty[$$

Car c'est une somme d'une composition bien définie et continue $\sqrt{x^2 - x}$ et un polynôme. Donc f est continue sur $]-\infty, 0] \subset]-\infty, 0] \cup [1, +\infty[$.

On a aussi f est une fonction strictement décroissante sur $]-\infty, 0]$ car Si $x, y \in]-\infty, 0]$; $x > y$ Alors :

$$\Rightarrow x > y \quad \text{et} \quad x - 1 > y - 1$$

$$\Rightarrow x(x-1) < y(y-1)$$

$$\Rightarrow \sqrt{x(x-1)} < \sqrt{y(y-1)} \quad \text{et} \quad -x < -y$$

$$\Rightarrow \sqrt{x^2 - x} - x < \sqrt{y^2 - y} - y$$

$$\Rightarrow f(x) < f(y)$$

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$$\Rightarrow \left(\frac{f(x) - f(y)}{x - y} \right) < 0$$

$$\Rightarrow f \text{ est } \searrow \text{ sur }]-\infty, 0]$$

On peut montrer que f est décroissante en calculant $f'(x)$. Et comme f est continue et étant strictement décroissante sur $]-\infty, 0]$ alors f réalise une bijection de $]-\infty, 0]$ sur un intervalle $J = f(]-\infty, 0])$

$$J = f(]-\infty, 0]) = \left[f(0) ; \lim_{x \rightarrow -\infty} f(x) \right[\\ = [0, +\infty[$$

Voici pourquoi $\lim_{x \rightarrow -\infty} f(x) = +\infty$:

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} (\sqrt{x^2 - x} - x)$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2 - x} - x)(\sqrt{x^2 - x} + x)}{(\sqrt{x^2 - x} + x)}$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2 - x})^2 - x^2}{\sqrt{x^2 - x} + x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{x^2 \left(1 - \frac{1}{x}\right)} + x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{x^2} \cdot \sqrt{1 - \frac{1}{x}} + x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x}{|x| \cdot \sqrt{1 - \frac{1}{x}} + x}$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{-x}{-x \sqrt{1 - \frac{1}{x}} + x} \right)$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{1}{\sqrt{1 - \frac{1}{x}} - 1} \right)$$

$$= \lim_{\substack{t \rightarrow 0^+ \\ t = \sqrt{1 - \frac{1}{x}} - 1}} \left(\frac{1}{t} \right) = \frac{1}{0^+} = +\infty$$

Donc $f :]-\infty, 0] \mapsto [0, +\infty[$ est bijective

$y \in [0, +\infty[$ alors $\exists ! x \in]-\infty, 0] : f(x) = y$

$$\Leftrightarrow \sqrt{x^2 - x} - x = y$$

$$\Leftrightarrow \frac{(\sqrt{x^2 - x} - x)(\sqrt{x^2 - x} + x)}{(\sqrt{x^2 - x} + x)} = y$$

$$\Leftrightarrow \frac{(\sqrt{x^2 - x})^2 - x^2}{\sqrt{x^2 - x} + x} = y$$

$$\Leftrightarrow \frac{-x}{\sqrt{x^2 - x} + x} = y$$

$$\Leftrightarrow \frac{-x}{|x| \cdot \sqrt{1 - \frac{1}{x}} + x} = y$$

$$\Leftrightarrow \frac{-x}{-x\sqrt{1 - \frac{1}{x}} + x} = y \quad \text{car } x \leq 0$$

$$\Leftrightarrow \frac{1}{\sqrt{1 - \frac{1}{x}} - 1} = y$$

$$\Leftrightarrow \sqrt{1 - \frac{1}{x}} - 1 = \frac{1}{y}$$

$$\Leftrightarrow \sqrt{1 - \frac{1}{x}} = \frac{1}{y} + 1$$

$$\Leftrightarrow 1 - \frac{1}{x} = \left(\frac{1}{y} + 1 \right)^2$$

$$\Leftrightarrow 1 - \left(\frac{1}{y} + 1 \right)^2 = \frac{1}{x}$$

$$\Leftrightarrow x = \frac{1}{1 - \left(\frac{1}{y} + 1 \right)^2} \leq 0 \quad \text{car } y \geq 0$$

$$\Leftrightarrow x = \left(\frac{1}{1 - \left(\frac{1}{y} + 1 \right)^2} \right) \in]-\infty, 0]$$

D'où finalement :

$$f^{-1} : [0, +\infty[\mapsto]-\infty, 0]$$

$$y \mapsto \left(\frac{1}{1 - \left(\frac{1}{y} + 1 \right)^2} \right)$$

4) D'abord f est continue sur $\mathbb{R} = D_f$ car c'est un quotient bien défini de deux fonctions continues et bien définies. Donc f est continue sur $[0, \sqrt{2}] \subset \mathbb{R}$.

Montrons maintenant que f est croissante sur $[0, \sqrt{2}]$ par la méthode du taux de variations. Soient $x, y \in [0, \sqrt{2}]$.

$$\left(\frac{f(x) - f(y)}{x - y} \right) = \left(\frac{\frac{x}{x^2 + 2} - \frac{y}{y^2 + 2}}{x - y} \right)$$

$$= \frac{xy^2 + 2x - yx^2 - 2y}{(x - y)(x^2 + 2)(y^2 + 2)}$$

$$= \frac{xy^2 - yx^2 + 2x - 2y}{(x - y)(x^2 + 2)(y^2 + 2)}$$

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$$= \frac{xy(y-x) - 2(y-x)}{(x-y)(x^2+2)(y^2+2)}$$

$$= \frac{(y-x)(xy-2)}{(x-y)(x^2+2)(y^2+2)}$$

$$= \frac{2-xy}{(x^2+2)(y^2+2)} > 0 \quad ; \quad \text{car} \quad \begin{cases} x < \sqrt{2} \\ y < \sqrt{2} \end{cases}$$

Donc f est strictement croissante sur l'intervalle $[0, \sqrt{2}]$. On peut montrer cette croissance en calculant la dérivée première $f'(x)$ et c'est trop facile.

Donc f réalise une bijection (car continue et strictement monotone) de $[0, \sqrt{2}]$ sur un intervalle $J = f([0, \sqrt{2}])$

$$J = [f(0); f(\sqrt{2})] = \left[0, \frac{\sqrt{2}}{4}\right]$$

Donc $f : [0, \sqrt{2}] \mapsto \left[0, \frac{\sqrt{2}}{4}\right]$ est bijective

$y \in \left[0, \frac{\sqrt{2}}{4}\right]$ alors $\exists ! x \in [0, \sqrt{2}] : f(x) = y$

$$\Leftrightarrow \frac{x}{x^2+2} = y$$

$$\Leftrightarrow y(x^2+2) = x$$

$$\Leftrightarrow yx^2 - x + 2y = 0$$

$$\Leftrightarrow x = \frac{1 \pm \sqrt{1-8y^2}}{2y} \quad ; \quad \Delta = 1-8y^2 \geq 0$$

$$\Leftrightarrow x = \frac{1 - \sqrt{1-8y^2}}{2y} \leq \sqrt{2}$$

$$\Leftrightarrow x = \left(\frac{1 - \sqrt{1-8y^2}}{2y} \right) \in [0, \sqrt{2}]$$

D'où finalement :

$$f^{-1} : \left[0, \frac{\sqrt{2}}{4}\right] \mapsto [0, \sqrt{2}]$$

$$y \mapsto \left(\frac{1 - \sqrt{1-8y^2}}{2y} \right)$$

Solution N° 85 :

1) D'abord on remarque que x et $3x$ ont le même signe c'est à dire que les deux seraient à la fois positifs ou à la fois négatifs.

On suppose que $x < 0$ et $3x < 0$:

$$\Rightarrow \text{Arctan}(x) < 0 \text{ et } \text{Arctan}(3x) < 0$$

$$\Rightarrow \text{Arctan}(x) + \text{Arctan}(3x) < 0$$

$$\Rightarrow \frac{\pi}{3} < 0 \quad (\text{absurde})$$

$$\Rightarrow x > 0 \quad ; \quad \text{retenue}$$

$$(E) \Leftrightarrow \text{Arctan}(x) + \text{Arctan}(3x) = \frac{\pi}{3}$$

$$\Leftrightarrow \tan(\text{Arctan}(x) + \text{Arctan}(3x)) = \tan\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{\tan(\text{Arctan}(x)) + \tan(\text{Arctan}(3x))}{1 - \tan(\text{Arctan}(x))\tan(\text{Arctan}(3x))} = \sqrt{3}$$

$$\Leftrightarrow \frac{x + 3x}{1 - 3x^2} = \sqrt{3}$$

$$\Leftrightarrow 3\sqrt{3}x^2 + 4x - \sqrt{3} = 0$$

$$\Leftrightarrow x = \frac{-4 \pm \sqrt{52}}{6\sqrt{3}} \quad ; \quad \Delta = 52 > 0$$

$$\Leftrightarrow x = \frac{-4 + \sqrt{52}}{6\sqrt{3}} > 0$$

Ce nombre est la seule solution de (E)

2) Soit x une solution de l'inéquation (E)

$$\Leftrightarrow \operatorname{Arctan}(2x) + \operatorname{Arctan}(x-1) \leq 0$$

$$\Leftrightarrow \operatorname{Arctan}(x-1) \leq -\operatorname{Arctan}(2x)$$

$$\Leftrightarrow \operatorname{Arctan}(x-1) \leq \operatorname{Arctan}(-2x)$$

$$\tan(\operatorname{Arctan}(x-1)) \leq \tan(\operatorname{Arctan}(-2x))$$

Car tangente et arc tangente sont des bijections croissantes.

$$\Leftrightarrow (x-1) \leq -2x$$

$$\Leftrightarrow x \leq \frac{1}{3} \quad \Leftrightarrow x \in \left] -\infty, \frac{1}{3} \right]$$

$$\Leftrightarrow \text{Solution}(E) = \left] -\infty, \frac{1}{3} \right]$$

3) Soit $\alpha = \operatorname{Arctan}\left(\frac{1}{2}\right) + \operatorname{Arctan}\left(\frac{1}{3}\right)$

On commence par : $\begin{cases} -1 < \frac{1}{2} < 1 \\ -1 < \frac{1}{3} < 1 \end{cases}$

$$\Rightarrow \begin{cases} \operatorname{Arctan}(-1) < \operatorname{Arctan}\left(\frac{1}{2}\right) < \operatorname{Arctan}(1) \\ \operatorname{Arctan}(-1) < \operatorname{Arctan}\left(\frac{1}{3}\right) < \operatorname{Arctan}(1) \end{cases}$$

$$\Rightarrow \begin{cases} -\frac{\pi}{4} < \operatorname{Arctan}\left(\frac{1}{2}\right) < \frac{\pi}{4} \\ -\frac{\pi}{4} < \operatorname{Arctan}\left(\frac{1}{3}\right) < \frac{\pi}{4} \end{cases}$$

$$\Rightarrow -\frac{\pi}{2} < \operatorname{Arctan}\left(\frac{1}{2}\right) + \operatorname{Arctan}\left(\frac{1}{3}\right) < \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

Or ; On a $\alpha = \operatorname{Arctan}\left(\frac{1}{2}\right) + \operatorname{Arctan}\left(\frac{1}{3}\right)$

$$\Leftrightarrow \tan(\alpha) = \tan\left(\operatorname{Arctan}\left(\frac{1}{2}\right) + \operatorname{Arctan}\left(\frac{1}{3}\right)\right)$$

$$\tan \alpha = \frac{\tan\left(\operatorname{Arctan}\left(\frac{1}{2}\right)\right) + \tan\left(\operatorname{Arctan}\left(\frac{1}{3}\right)\right)}{1 - \tan\left(\operatorname{Arctan}\left(\frac{1}{2}\right)\right)\tan\left(\operatorname{Arctan}\left(\frac{1}{3}\right)\right)}$$

$$\Leftrightarrow \tan(\alpha) = \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right)$$

$$\Leftrightarrow \tan(\alpha) = 1 \quad \Leftrightarrow \tan \alpha = \tan\left(\frac{\pi}{4}\right)$$

$$\Leftrightarrow \alpha \equiv \frac{\pi}{4} [\pi] \quad \text{et} \quad \alpha \neq \frac{\pi}{2} [\pi]$$

$$\Leftrightarrow \alpha \equiv \frac{\pi}{4} + k\pi \quad ; \quad k \in \mathbb{Z}$$

$$\Leftrightarrow -\frac{\pi}{2} < \frac{\pi}{4} + k\pi < \frac{\pi}{2} \quad ; \quad k \in \mathbb{Z}$$

$$\Leftrightarrow -\frac{1}{2} < \frac{1}{4} + k < \frac{1}{2} \quad ; \quad k \in \mathbb{Z}$$

$$\Leftrightarrow -\frac{3}{4} < k < \frac{1}{4} \quad ; \quad k \in \mathbb{Z}$$

$$\Leftrightarrow -0,75 < k < 0,25 \quad ; \quad k \in \mathbb{Z}$$

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$$\Leftrightarrow k = 0 ; \text{ car } k \in \mathbb{Z}$$

$$\Leftrightarrow \alpha = \frac{\pi}{4} + 0\pi = \frac{\pi}{4}$$

$$\Leftrightarrow \operatorname{Arctan}\left(\frac{1}{2}\right) + \operatorname{Arctan}\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

L'équation (E) devient ainsi :

$$\Leftrightarrow \operatorname{Arctan}(x) = \frac{\pi}{4}$$

$$\Leftrightarrow \tan(\operatorname{Arctan}(x)) = \tan\left(\frac{\pi}{4}\right)$$

$$\Leftrightarrow x = 1$$

$$4) (E) : \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) > \frac{\pi}{3}$$

Soit φ la fonction définie sur $\mathbb{R}$ par :

$$\varphi(x) = \operatorname{Arctan}(x) + \operatorname{Arctan}(2x)$$

D'abord on devrait résoudre l'équation :

$$\varphi(x) = \frac{\pi}{3}$$

$$\Leftrightarrow \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) = \frac{\pi}{3}$$

$$\Leftrightarrow \tan(\operatorname{Arctan}(x) + \operatorname{Arctan}(2x)) = \tan\frac{\pi}{3}$$

$$\Leftrightarrow \sqrt{3} = \frac{\tan(\operatorname{Arctan}(x)) + \tan(\operatorname{Arctan}(2x))}{1 - \tan(\operatorname{Arctan}(x))\tan(\operatorname{Arctan}(2x))}$$

$$\Leftrightarrow \frac{x + 2x}{1 - 2x^2} = \sqrt{3}$$

$$\Leftrightarrow \frac{3x - \sqrt{3} + 2\sqrt{3}x^2}{1 - 2x^2} = 0$$

$$\Leftrightarrow \frac{2x^2 + \sqrt{3}x - 1}{1 - 2x^2} = 0$$

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$$\Leftrightarrow x = \frac{-\sqrt{3} \pm \sqrt{11}}{4} \text{ et } x \neq \pm \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow x = \frac{-\sqrt{3} + \sqrt{11}}{4} > 0$$

$$\text{sinon on aurait } \frac{\pi}{3} < 0$$

La fonction φ est continue sur $\mathbb{R}^+$ car somme de deux fonctions bien définies et continues sur $\mathbb{R}^+$.

On a aussi la fonction φ est strictement croissante sur $\mathbb{R}$ car :

$$\varphi'(x) = \frac{1}{x^2 + 1} + \frac{2}{x^2 + 1} > 0$$

Donc φ réalise une bijection de $[0, +\infty[$ vers $\varphi([0, +\infty[) = [0, \pi[$.

Ainsi l'inéquation (E) devient :

$$\Leftrightarrow \varphi(x) > \varphi\left(\frac{-\sqrt{3} + \sqrt{11}}{4}\right)$$

$$\Leftrightarrow \varphi^{-1}(\varphi(x)) > \varphi^{-1}\left(\varphi\left(\frac{-\sqrt{3} + \sqrt{11}}{4}\right)\right)$$

$$\Leftrightarrow x > \frac{-\sqrt{3} + \sqrt{11}}{4}$$

$$\Leftrightarrow x \in \left] \frac{-\sqrt{3} + \sqrt{11}}{4} ; +\infty \right[$$

$$\Leftrightarrow \text{Solutions}(E) = \left] \frac{-\sqrt{3} + \sqrt{11}}{4} ; +\infty \right[$$

Solution N° 86 :

$$1) \lim_{x \rightarrow 0^+} (x + 1) \operatorname{Arctan}\left(\frac{1}{x}\right)$$

$$= \lim_{\substack{t \rightarrow +\infty \\ t = \frac{1}{x}}} \left(\frac{1}{t} + 1\right) \operatorname{Arctan}(t)$$

$$= \lim_{t \rightarrow +\infty} \left(\frac{1}{t} + 1\right) \times \lim_{t \rightarrow +\infty} \operatorname{Arctan}(t)$$

$$= (0^+ + 1) \times \left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$2) \lim_{x \rightarrow 0^-} (x + 1) \operatorname{Arctan}\left(\frac{1}{x}\right)$$

$$= \lim_{\substack{t \rightarrow -\infty \\ t = \frac{1}{x}}} \left(\frac{1}{t} + 1\right) \operatorname{Arctan}(t)$$

$$= \lim_{t \rightarrow -\infty} \left(\frac{1}{t} + 1\right) \times \lim_{t \rightarrow -\infty} \operatorname{Arctan}(t)$$

$$= (0^+ + 1) \times \left(\frac{-\pi}{2}\right) = \frac{-\pi}{2}$$

$$3) \text{ Calculons } \lim_{x \rightarrow -\infty} x \left(\frac{\pi}{2} + \operatorname{Arctan}(x)\right)$$

Je rappelle juste qu'on a toujours :

$$\left| \begin{array}{l} \forall x \in \mathbb{R}^{++} ; \operatorname{Arctan}(x) + \operatorname{Arctan}\left(\frac{1}{x}\right) = \frac{\pi}{2} \\ \forall x \in \mathbb{R}^{*-} ; \operatorname{Arctan}(x) + \operatorname{Arctan}\left(\frac{1}{x}\right) = \frac{-\pi}{2} \\ \lim_{x \rightarrow 0^+} \frac{\operatorname{Arctan}(x)}{x} = 1 \end{array} \right.$$

$$\text{Donc : } \lim_{x \rightarrow -\infty} x \left(\frac{\pi}{2} + \operatorname{Arctan}(x)\right)$$

$$= \lim_{x \rightarrow -\infty} -x \operatorname{Arctan}\left(\frac{1}{x}\right)$$

$$= - \lim_{\substack{t \rightarrow 0^- \\ t = \frac{1}{x}}} \frac{\operatorname{Arctan}(t)}{t} = -1$$

$$4) \lim_{x \rightarrow 2} \frac{\operatorname{Arctan}(x - 2)}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} \frac{\operatorname{Arctan}(x - 2)}{(x - 2)} \times \frac{1}{(x + 2)}$$

$$= \lim_{\substack{t \rightarrow 0 \\ t = x - 2}} \frac{\operatorname{Arctan}(t)}{t} \times \frac{1}{t + 4}$$

$$= 1 \times \frac{1}{0 + 4} = \frac{1}{4}$$

Solution N° 87 :

$$1) f(x) = \sqrt[3]{\operatorname{Arctan} x}$$

$$D_f = \{x \in \mathbb{R} ; \operatorname{Arctan}(x) \in \mathbb{R}\} = \mathbb{R}$$

car $\sqrt[3]{x}$ est définie de $\mathbb{R}$ vers $\mathbb{R}$

$$\text{On pose } f(x) = u \circ v(x) ; \forall x \in \mathbb{R}$$

$$\text{Avec : } u(x) = \sqrt[3]{x} ; v(x) = \operatorname{Arctan}(x)$$

On a v est une fonction continue sur $\mathbb{R}$ selon le cours et on a u est une fonction continue sur $\mathbb{R}$ aussi car 3 est un nombre impair.

$$\text{Si } x \in \mathbb{R} \text{ Alors } v(x) = \operatorname{Arctan}(x) \in \mathbb{R}$$

$$\text{Donc } v(\mathbb{R}) \subseteq \mathbb{R}$$

D'où l'on déduit que la composition $u \circ v$ est continue sur $\mathbb{R}$ tout entier.

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$$g(x) = \sqrt[4]{\frac{x}{x-1}}$$

$$\begin{aligned} D_g &= \left\{ x \in \mathbb{R} \ ; \ \frac{x}{x-1} \geq 0 \ \text{et} \ x \neq 1 \right\} \\ &= \{ x \in \mathbb{R} \ ; \ x \in]-\infty, 0] \cup [1, +\infty[\ \text{et} \ x \neq 1 \} \\ &= \{ x \in \mathbb{R} \ ; \ x \notin]0, 1] \} = \mathbb{R} \setminus \{]0, 1] \} \end{aligned}$$

On pose $g(x) = u \circ v(x)$; $\forall x \in D_g$

Avec $u(x) = \sqrt[4]{x}$ et $v(x) = \frac{x}{x-1}$

On a v est continue sur $\mathbb{R} \setminus \{1\}$
Donc v est continue sur $\mathbb{R} \setminus]0, 1]$
car $\mathbb{R} \setminus]0, 1] \subset \mathbb{R} \setminus \{1\}$

On a aussi la fonction u est continue sur $\mathbb{R}^+$ car le nombre 4 est pair.

Si $x \in \mathbb{R} \setminus]0, 1]$ alors $v(x) = \frac{x}{x-1} \in \mathbb{R}^+$

Car si $x \in]-\infty, 0]$ alors $\frac{x}{x-1} \geq 0$

Et si $x \in]1, +\infty[$ alors $\frac{x}{x-1} \geq 0$

Donc $v(\mathbb{R} \setminus]0, 1]) \subseteq \mathbb{R}^+$

D'où la composition $u \circ v$ est continue sur l'ensemble $\mathbb{R} \setminus]0, 1]$

$$2) \quad \lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x} - 1}{x - 1} \right) = \lim_{\substack{t = \sqrt[3]{x} \\ t \rightarrow 1}} \left(\frac{t - 1}{t^3 - 1} \right)$$

$$= \lim_{t \rightarrow 1} \frac{(t - 1)}{(t - 1)(t^2 + t + 1)}$$

$$= \lim_{t \rightarrow 1} \left(\frac{1}{t^2 + t + 1} \right) = \frac{1}{3}$$

$$\begin{aligned} &\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x} - x \right) \\ &= \lim_{x \rightarrow +\infty} \left((x^3 + x)^{\frac{1}{3}} - x \right) \\ &= \lim_{x \rightarrow +\infty} \left((x^3 + x)^{\frac{1}{3}} - x \right) \\ &\quad \times \frac{(x^3 + x)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2}{(x^3 + x)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2} \end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{(x^3 + x) - x^3}{(x^3 + x)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x}{(x^3 + x)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x}{x^2 \left(\left(1 + \frac{1}{x^2}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x^2}\right)^{\frac{1}{3}} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{x \left(\left(1 + \frac{1}{x^2}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x^2}\right)^{\frac{1}{3}} + 1 \right)}$$

$$= \left(\frac{1}{+\infty} \right) \frac{1}{(1 + 0^+)^{\frac{2}{3}} + (1 + 0)^{\frac{1}{3}} + 1} = 0^+ = 0$$

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt[3]{x+1} - 1}{\sqrt[4]{x+1} - 1} \right) = \lim_{x \rightarrow 0} \left(\frac{(x+1)^{\frac{1}{3}} - 1}{(x+1)^{\frac{1}{4}} - 1} \right)$$

$$= \lim_{\substack{t \rightarrow 1 \\ t = x+1}} \left(\frac{t^{\frac{1}{3}} - 1}{t^{\frac{1}{4}} - 1} \right)$$

On peut calculer cette limite à l'aide de la règle de l'Hôpital dans le brouillon juste pour connaître la limite. On a le droit d'appliquer cette règle dans le bro-

Brouillon puisque ça donne la forme indéterminée zéro/zéro. On obtient :

$$\begin{aligned}\lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)}{\left(t^{\frac{1}{4}} - 1\right)} &= \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)'}{\left(t^{\frac{1}{4}} - 1\right)'} \\ &= \lim_{t \rightarrow 1} \frac{\left(\frac{1}{3} \cdot t^{-\frac{2}{3}}\right)}{\left(\frac{1}{4} \cdot t^{-\frac{3}{4}}\right)} = \lim_{t \rightarrow 1} \frac{4}{3} \cdot t^{\left(-\frac{2}{3} + \frac{3}{4}\right)} \\ &= \lim_{t \rightarrow 1} \left(\frac{4}{3} \cdot t^{\frac{1}{12}}\right) = \left(\frac{4}{3} \times 1^{\frac{1}{12}}\right) = \frac{4}{3}\end{aligned}$$

Revenons maintenant à notre limite pour la recalculer par une méthode conforme au programme officiel .

$$\begin{aligned}\lim_{\substack{t \rightarrow 1 \\ t=x+1}} \left(\frac{t^{\frac{1}{3}} - 1}{t^{\frac{1}{4}} - 1}\right) &= \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)\left(t^{\frac{1}{4}} + 1\right)}{\left(t^{\frac{1}{4}} - 1\right)\left(t^{\frac{1}{4}} + 1\right)} \\ &= \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)\left(t^{\frac{1}{4}} + 1\right)}{\left(t^{\frac{1}{4}}\right)^2 - 1^2} \\ &= \lim_{t \rightarrow 1} \left(\frac{t^{\frac{1}{3}} - 1}{t^{\frac{1}{2}} - 1}\right)\left(1^{\frac{1}{4}} + 1\right) \\ &= 2 \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)\left(t^{\frac{1}{2}} + 1\right)}{\left(t^{\frac{1}{2}} - 1\right)\left(t^{\frac{1}{2}} + 1\right)} \\ &= 2 \lim_{t \rightarrow 1} \left(\frac{t^{\frac{1}{3}} - 1}{t - 1}\right)\left(1^{\frac{1}{2}} + 1\right)\end{aligned}$$

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$$\begin{aligned}&= 4 \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}} - 1\right)\left(t^{\frac{2}{3}} + t^{\frac{1}{3}} + 1\right)}{(t - 1)\left(t^{\frac{2}{3}} + t^{\frac{1}{3}} + 1\right)} \\ &= 4 \lim_{t \rightarrow 1} \frac{\left(t^{\frac{1}{3}}\right)^3 - 1^3}{(t - 1)\left(t^{\frac{2}{3}} + t^{\frac{1}{3}} + 1\right)} \\ &= 4 \lim_{t \rightarrow 1} \frac{(t - 1)}{(t - 1)\left(t^{\frac{2}{3}} + t^{\frac{1}{3}} + 1\right)} \\ &= 4 \lim_{t \rightarrow 1} \left(\frac{1}{t^{\frac{2}{3}} + t^{\frac{1}{3}} + 1}\right) \\ &= 4 \left(\frac{1}{1 + 1 + 1}\right) = \frac{4}{3}\end{aligned}$$

$$\begin{aligned}\blacksquare \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 1} - \sqrt[3]{x^2 + 1}\right) &= \lim_{x \rightarrow +\infty} \left((x^2 + 1)^{\frac{1}{2}} - (x^2 + 1)^{\frac{1}{3}}\right) \\ &= \lim_{x \rightarrow +\infty} (x^2 + 1)^{\frac{1}{2}} \left(1 - \frac{(x^2 + 1)^{\frac{1}{3}}}{(x^2 + 1)^{\frac{1}{2}}}\right) \\ &= \lim_{x \rightarrow +\infty} (x^2 + 1)^{\frac{1}{2}} \left(1 - (x^2 + 1)^{\frac{1}{3} - \frac{1}{2}}\right) \\ &= \lim_{x \rightarrow +\infty} (x^2 + 1)^{\frac{1}{2}} \left(1 - \left(\frac{1}{x^2 + 1}\right)^{\frac{1}{6}}\right) \\ &= (+\infty) \left(1 - \frac{1}{+\infty}\right) = +\infty\end{aligned}$$

Soit à résoudre l'équation suivante :

$$\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2} = \sqrt[3]{4x}$$

D'abord il faut qu'on ait :

$$x + 1 \geq 0 \quad \text{et} \quad x - 1 \geq 0 \quad \text{et} \quad 4x \in \mathbb{R}$$

$c - a - d \quad x \geq -1 \quad \text{et} \quad x \geq 1 \quad \text{et} \quad x \in \mathbb{R}$
Donc il faut qu'on ait : $x \geq 1$

L'équation devient alors :

$$(x+1)^{\frac{2}{3}} - (x-1)^{\frac{2}{3}} = (4x)^{\frac{1}{3}}$$

$$\Leftrightarrow \left((x+1)^{\frac{2}{3}} - (x-1)^{\frac{2}{3}} \right)^3 = \left((4x)^{\frac{1}{3}} \right)^3$$

$$\Leftrightarrow (x+1)^2 - 3(x+1)^{\frac{4}{3}} \cdot (x-1)^{\frac{2}{3}} + 3(x+1)^{\frac{2}{3}} \cdot (x-1)^{\frac{4}{3}} - (x-1)^2 = 4x$$

$$\Leftrightarrow 4x + 3(x+1)^{\frac{2}{3}} \cdot (x-1)^{\frac{4}{3}} - 3(x+1)^{\frac{4}{3}} \cdot (x-1)^{\frac{2}{3}} = 4x$$

$$\Leftrightarrow 3(x+1)^{\frac{2}{3}} \cdot (x-1)^{\frac{2}{3}} \cdot \left((x-1)^{\frac{2}{3}} - (x+1)^{\frac{2}{3}} \right) = 0$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x+1=0 \\ \text{ou bien } x-1=0 \\ \text{ou bien } (x-1)^{\frac{2}{3}} = (x+1)^{\frac{2}{3}} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x=-1 \\ \text{ou bien } x=1 \\ \text{ou bien } (x-1)^2 = (x+1)^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x=-1 & \text{à rejeter} \\ \text{ou bien } x=1 & \text{à retenir} \\ \text{ou bien } x=0 & \text{à rejeter} \end{cases}$$

$$\Leftrightarrow x=1 \quad \text{car} \quad x \geq 1$$

L'équation admet donc une seule solution

Solution N° 88 :

$$\blacksquare \quad A = \frac{\sqrt[4]{32} \times \sqrt[6]{27} \times \sqrt[4]{108}}{\sqrt[4]{6}}$$

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$$= \frac{\sqrt[4]{2^5} \times \sqrt[6]{3^3} \times \sqrt[4]{(2^2 \times 3^3)}}{\sqrt[4]{3 \times 2}}$$

$$= \frac{2^{\frac{5}{4}} \times 3^{\frac{3}{6}} \times (2^2 \times 3^3)^{\frac{1}{4}}}{(3 \times 2)^{\frac{1}{4}}}$$

$$= \left(\frac{2^{\frac{5}{4}} \times 2^{\frac{2}{4}}}{2^{\frac{1}{4}}} \right) \times \left(\frac{3^{\frac{3}{6}} \times 3^{\frac{3}{4}}}{3^{\frac{1}{4}}} \right)$$

$$= \left(\frac{2^{\frac{7}{4}}}{2^{\frac{1}{4}}} \right) \times \left(\frac{3^{\frac{5}{4}}}{3^{\frac{1}{4}}} \right) = 2^{\left(\frac{7}{4}-\frac{1}{4}\right)} \times 3^{\left(\frac{5}{4}-\frac{1}{4}\right)}$$

$$= 2^{\frac{3}{2}} \times 3^1 = 3\sqrt{8}$$

$$\blacksquare \quad B = \frac{(125)^{\frac{2}{9}} \times (625)^{\frac{1}{4}} \times (25)^{\frac{5}{2}}}{(5)^{\frac{17}{3}}}$$

$$= \frac{(5^3)^{\frac{2}{9}} \times (5^4)^{\frac{1}{4}} \times (5^2)^{\frac{5}{2}}}{(5)^{\frac{17}{3}}}$$

$$= \frac{5^{\frac{6}{9}} \times 5^1 \times 5}{5^{\frac{17}{3}}} = \frac{5^{\frac{20}{3}}}{5^{\frac{17}{3}}} = 5^{\left(\frac{20}{3}-\frac{17}{3}\right)} = 5^1 = 5$$

$$\blacksquare \quad C = \frac{\left(7^{\frac{2}{3}}\right)^{\frac{1}{2}} \times \left(3^{-\frac{5}{3}}\right)^{\frac{1}{4}} \times (21)^{\frac{3}{4}}}{\left(7^{-\frac{11}{2}}\right)^{\frac{1}{6}} \times (343)^{\frac{2}{3}} \times (63^{-2})^{-\frac{1}{6}}}$$

$$= \frac{7^{\frac{1}{3}} \times 3^{-\frac{5}{12}} \times 3^{\frac{3}{4}} \times 7^{\frac{3}{4}}}{7^{\frac{-11}{12}} \times (7^3)^{\frac{2}{3}} \times (7 \times 3^2)^{\frac{2}{6}}}$$

$$= \left(\frac{7^{\frac{1}{3}} \times 7^{\frac{3}{4}}}{7^{\frac{-11}{12}} \times 7^2 \times 7^{\frac{2}{6}}} \right) \times \left(\frac{3^{-\frac{5}{12}} \times 3^{\frac{3}{4}}}{3^{\frac{4}{6}}} \right)$$

$$= \left(\frac{7^{\frac{13}{12}}}{7^{\frac{17}{12}}} \right) \times \left(\frac{3^{\frac{1}{3}}}{3^{\frac{4}{6}}} \right) = 7^{\left(\frac{13}{12} - \frac{17}{12}\right)} \times 3^{\left(\frac{1}{3} - \frac{4}{6}\right)}$$

$$= 7^{-\frac{1}{3}} \times 3^{-\frac{1}{3}} = (7 \times 3)^{-\frac{1}{3}} = 21^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{21}}$$

Solution N° 89 :

1) d'abord on remarque que :

$$\begin{cases} \sqrt[3]{3+x} \text{ est définie si } (3+x) \in \mathbb{R} \\ \sqrt[3]{3-x} \text{ est définie si } (3-x) \in \mathbb{R} \\ \sqrt[6]{4x^2} \text{ est définie si } (2x) \in \mathbb{R} \end{cases}$$

Soit $x \in \mathbb{R}$ une solution de l'équation (E)

$$\Leftrightarrow (3+x)^{\frac{1}{3}} - (3-x)^{\frac{1}{3}} = (2x)^{\frac{1}{3}}$$

$$\Leftrightarrow \left((3+x)^{\frac{1}{3}} - (3-x)^{\frac{1}{3}} \right)^3 = \left((2x)^{\frac{1}{3}} \right)^3$$

$$\Leftrightarrow (3+x) - 3(3+x)^{\frac{2}{3}}(3-x)^{\frac{1}{3}} + 3(3+x)^{\frac{1}{3}}(3-x)^{\frac{2}{3}} - (3-x) = 2x$$

$$\Leftrightarrow 3(3+x)^{\frac{1}{3}} \cdot (3-x)^{\frac{1}{3}} \cdot \left((3-x)^{\frac{1}{3}} - (3+x)^{\frac{1}{3}} \right) = 0$$

$$\Leftrightarrow \begin{cases} \text{ou bien } (3+x)^{\frac{1}{3}} = 0 \\ \text{ou bien } (3-x)^{\frac{1}{3}} = 0 \\ \text{ou bien } (3-x)^{\frac{1}{3}} - (3+x)^{\frac{1}{3}} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } (3+x) = 0 \\ \text{ou bien } (3-x) = 0 \\ \text{ou bien } (3-x)^{\frac{1}{3}} = (3+x)^{\frac{1}{3}} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x = -3 \\ \text{ou bien } x = 3 \\ \text{ou bien } x = 0 \end{cases}$$

2) soit à résoudre l'équation suivante :

$$(E) : 2x\sqrt{x} - 3x\sqrt[4]{\frac{1}{x}} = 20$$

Cette équation est définie si on ait :

$$x \geq 0 \quad \text{et} \quad x \neq 0$$

$$(E) \Leftrightarrow 2x^{\frac{3}{2}} - 3x \cdot x^{-\frac{1}{4}} = 20$$

$$\Leftrightarrow 2x^{\frac{3}{2}} - 3x^{\frac{3}{4}} = 20$$

$$\text{On pose } t = x^{\frac{3}{4}} > 0 ; \quad \forall x > 0$$

$$(E) \Leftrightarrow 2t^2 - 3t - 20 = 0$$

$$\Leftrightarrow t = \frac{3 \pm \sqrt{169}}{4} \quad \text{avec } \Delta = 169$$

$$\Leftrightarrow x^{\frac{3}{4}} = t = 4 > 0$$

$$\Leftrightarrow \left(x^{\frac{3}{4}} \right)^{\frac{4}{3}} = 4^{\frac{4}{3}}$$

$$\Leftrightarrow x = 4^{\frac{4}{3}} = \sqrt[3]{4^4} > 0$$

3) Soit à résoudre l'équation suivante :

$$(E) : \sqrt{x+1} - \sqrt[3]{x} = 1$$

Cette équation est définie si on ait :

$$(x+1) \geq 0 \quad \text{et} \quad x \in \mathbb{R}$$

$$\text{On pose } t = x^{\frac{1}{3}} \in \mathbb{R} \Leftrightarrow x = t^3 \in \mathbb{R}$$

$$(E) \Leftrightarrow (x+1)^{\frac{1}{2}} - x^{\frac{1}{3}} = 1$$

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$$\Leftrightarrow (t^3 + 1)^{\frac{1}{2}} - t = 1$$

$$\Leftrightarrow (t^3 + 1)^{\frac{1}{2}} = 1 + t$$

$$\Rightarrow \left((t^3 + 1)^{\frac{1}{2}} \right)^2 = (1 + t)^2$$

$$\Rightarrow (t^3 + 1) = 1 + t^2 + 2t$$

$$\Rightarrow t^3 - t^2 - 2t = 0$$

$$\Rightarrow t(t^2 - t - 2) = 0$$

$$\Rightarrow t(t - 2)(t + 1) = 0$$

$$\Rightarrow \begin{cases} \text{ou bien } t = 0 \\ \text{ou bien } t = 2 \\ \text{ou bien } t = -1 \end{cases}$$

$$\Rightarrow \begin{cases} \text{ou bien } x^{\frac{1}{3}} = 0 \\ \text{ou bien } x^{\frac{1}{3}} = 2 \\ \text{ou bien } x^{\frac{1}{3}} = -1 \end{cases}$$

$$\Rightarrow \begin{cases} \text{ou bien } x = 0 \geq -1 \\ \text{ou bien } x = 8 \geq -1 \\ \text{ou bien } x = -1 \geq -1 \end{cases}$$

Inversement, si on remplace x par chacune de ces trois valeurs dans (E)
On obtient que l'égalité est vérifiée :

$$\Rightarrow \begin{cases} \sqrt{0+1} - \sqrt[3]{0} = 1 \\ \sqrt{8+1} - \sqrt[3]{8} = 1 \\ \sqrt{-1+1} - \sqrt[3]{-1} = 1 \end{cases}$$

D'où l'ensemble des solutions de (E)
est définie explicitement par :
 $S = \{-1 ; 0 ; 8\}$

4) soit à résoudre l'équation (E) suivante

$$(E) : \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) = \frac{\pi}{4}$$

D'abord il faut qu'on ait $x > 0$.

Sinon on aurait :

$$\operatorname{Arctan}(x) < 0 \quad \text{et} \quad \operatorname{Arctan}(2x) < 0$$

$$\Rightarrow \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) < 0$$

$$\Rightarrow \frac{\pi}{4} < 0 \quad \text{absurde et contradiction}$$

Soit $x > 0$ une solution de l'équation (E)

$$\Leftrightarrow \operatorname{Arctan}(x) + \operatorname{Arctan}(2x) = \frac{\pi}{4}$$

$$\Leftrightarrow \tan(\operatorname{Arctan}(x) + \operatorname{Arctan}(2x)) = \tan \frac{\pi}{4}$$

$$\Leftrightarrow \frac{\tan(\operatorname{Arctan}(x)) + \tan(\operatorname{Arctan}(2x))}{1 - \tan(\operatorname{Arctan}(x)) \cdot \tan(\operatorname{Arctan}(2x))} = 1$$

$$\Leftrightarrow \left(\frac{x + 2x}{1 - x \cdot 2x} \right) = 1$$

$$\Leftrightarrow 2x^2 + 3x - 1 = 0$$

$$\Leftrightarrow x = \frac{-3 \pm \sqrt{17}}{4} \quad ; \quad \Delta = 17$$

$$\Leftrightarrow x = \frac{-3 + \sqrt{17}}{4} > 0$$

Donc finalement on en déduit que cette équation admet une seule solution.

Solution N° 90 :

$$\begin{aligned} 1) \lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x^2} - 1}{\sqrt[4]{x} - 1} \right) &= \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x^{\frac{1}{4}} - 1} \right) \left(\frac{x^{\frac{1}{4}} + 1}{x^{\frac{1}{4}} + 1} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x^{\frac{1}{2}} - 1} \right) \left(x^{\frac{1}{4}} + 1 \right) \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x^{1/2} - 1} \right) \left(1^{\frac{1}{4}} + 1 \right) \\
&= 2 \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x^{\frac{1}{2}} - 1} \right) \left(\frac{x^{\frac{1}{2}} + 1}{x^{\frac{1}{2}} + 1} \right) \\
&= 2 \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x - 1} \right) \left(x^{\frac{1}{2}} + 1 \right) \\
&= 2 \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x - 1} \right) \left(1^{\frac{1}{2}} + 1 \right) \\
&= 4 \lim_{x \rightarrow 1} \left(\frac{x^{\frac{2}{3}} - 1}{x - 1} \right) \left(\frac{x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1}{x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1} \right) \\
&= 4 \lim_{x \rightarrow 1} \frac{\left(x^{\frac{2}{3}} \right)^3 - 1^3}{(x - 1) \left(x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1 \right)} \\
&= 4 \lim_{x \rightarrow 1} \frac{(x^2 - 1)}{(x - 1) \left(x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1 \right)} \\
&= 4 \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{(x - 1) \left(x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1 \right)} \\
&= 4 \lim_{x \rightarrow 1} \left(\frac{x + 1}{x^{\frac{4}{3}} + x^{\frac{2}{3}} + 1} \right) \\
&= 4 \left(\frac{1 + 1}{1 + 1 + 1} \right) = \frac{8}{3}
\end{aligned}$$

$$\begin{aligned}
2) \quad &\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2} - x \right) \\
&= \lim_{x \rightarrow +\infty} \left((x^3 + x^2)^{\frac{1}{3}} - x \right) \\
&= \lim_{x \rightarrow +\infty} \left((x^3 + x^2)^{\frac{1}{3}} - x \right) \\
&\quad \times \frac{\left((x^3 + x^2)^{\frac{2}{3}} + x(x^3 + x^2)^{\frac{1}{3}} + x^2 \right)}{\left((x^3 + x^2)^{\frac{2}{3}} + x(x^3 + x^2)^{\frac{1}{3}} + x^2 \right)} \\
&= \lim_{x \rightarrow +\infty} \frac{\left((x^3 + x^2)^{\frac{1}{3}} \right)^3 - x^3}{\left((x^3 + x^2)^{\frac{2}{3}} + x(x^3 + x^2)^{\frac{1}{3}} + x^2 \right)} \\
&= \lim_{x \rightarrow +\infty} \frac{x^2}{\left(x^3 \left(1 + \frac{1}{x} \right)^{\frac{2}{3}} + x \left(x^3 \left(1 + \frac{1}{x} \right) \right)^{\frac{1}{3}} + x^2 \right)} \\
&= \lim_{x \rightarrow +\infty} \frac{x^2}{(x^3)^{\frac{2}{3}} \left(1 + \frac{1}{x} \right)^{\frac{2}{3}} + x(x^3)^{\frac{1}{3}} \left(1 + \frac{1}{x} \right)^{\frac{1}{3}} + x^2} \\
&= \lim_{x \rightarrow +\infty} \frac{x^2}{x^2 \left(1 + \frac{1}{x} \right)^{\frac{2}{3}} + x^2 \left(1 + \frac{1}{x} \right)^{\frac{1}{3}} + x^2} \\
&= \lim_{x \rightarrow +\infty} \frac{x^2}{x^2 \left(\left(1 + \frac{1}{x} \right)^{\frac{2}{3}} + \left(1 + \frac{1}{x} \right)^{\frac{1}{3}} + 1 \right)} \\
&= \lim_{x \rightarrow +\infty} \frac{1}{\left(1 + \frac{1}{x} \right)^{\frac{2}{3}} + \left(1 + \frac{1}{x} \right)^{\frac{1}{3}} + 1} \\
&= \frac{1}{(1 + 0)^{\frac{2}{3}} + (1 + 0)^{\frac{1}{3}} + 1} = \frac{1}{3}
\end{aligned}$$

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$$\begin{aligned}
3) \quad \lim_{x \rightarrow +\infty} \left(\frac{\sqrt[4]{x} - \sqrt[3]{x+1}}{\sqrt{x} - \sqrt[6]{x+1}} \right) \\
= \lim_{x \rightarrow +\infty} \left(\frac{x^{\frac{1}{4}} - (x+1)^{\frac{1}{3}}}{x^{\frac{1}{2}} - (x+1)^{\frac{1}{6}}} \right) \\
= \lim_{x \rightarrow +\infty} \left(\frac{x^{\frac{1}{4}} - \left(x \left(1 + \frac{1}{x} \right) \right)^{\frac{1}{3}}}{x^{\frac{1}{2}} - \left(x \left(1 + \frac{1}{x} \right) \right)^{\frac{1}{6}}} \right) \\
= \lim_{x \rightarrow +\infty} \left(\frac{x^{\frac{1}{4}} - x^{\frac{1}{3}} \left(1 + \frac{1}{x} \right)^{\frac{1}{3}}}{x^{\frac{1}{2}} - x^{\frac{1}{6}} \left(1 + \frac{1}{x} \right)^{\frac{1}{6}}} \right) \\
= \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{3}} \left(x^{\frac{-1}{12}} - \left(1 + \frac{1}{x} \right)^{\frac{1}{3}} \right)}{x^{\frac{1}{2}} \left(1 - x^{\frac{-1}{3}} \left(1 + \frac{1}{x} \right)^{\frac{1}{6}} \right)} \\
= \lim_{x \rightarrow +\infty} \left(x^{\frac{-1}{6}} \right) \left(\frac{x^{\frac{-1}{12}} - \left(1 + \frac{1}{x} \right)^{\frac{1}{3}}}{1 - x^{\frac{-1}{3}} \left(1 + \frac{1}{x} \right)^{\frac{1}{6}}} \right) \\
= (0) \left(\frac{0 - (1+0)^{\frac{1}{3}}}{1 - 0(1+0)^{\frac{1}{6}}} \right) = 0
\end{aligned}$$

$$4) \quad \lim_{x \rightarrow 0^+} \frac{1}{x} \left(\operatorname{Arctan} \left(\frac{1}{x} \right) - \frac{\pi}{2} \right)$$

D'abord voici un rappel sur l'arc tangente

$$\blacksquare \quad \lim_{x \rightarrow +\infty} \operatorname{Arctan}(x) = \frac{\pi}{2}$$

$$\blacksquare \quad \lim_{x \rightarrow -\infty} \operatorname{Arctan}(x) = \frac{-\pi}{2}$$

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$$\blacksquare \quad \lim_{x \rightarrow 0^\pm} \frac{\operatorname{Arctan}(x)}{x} = 1$$

$$\blacksquare \quad \forall x \geq 0 ; \operatorname{Arctan}(x) + \operatorname{Arctan} \left(\frac{1}{x} \right) = \frac{\pi}{2}$$

Donc on calcule la limite comme suit :

$$\begin{aligned}
\lim_{x \rightarrow 0^+} \frac{1}{x} \left(\operatorname{Arctan} \left(\frac{1}{x} \right) - \frac{\pi}{2} \right) \\
= \lim_{x \rightarrow 0^+} \frac{-\operatorname{Arctan}(x)}{x} = -1
\end{aligned}$$

Solution N° 91 :

1) Soit $x \in \mathbb{R}$ on procède comme suit :

$$\begin{aligned}
1 + x - x^2 \leq f(x) \\
\Rightarrow \underbrace{(1+x)}_{+\infty} \leq f(x) + x^2 \\
\Rightarrow \lim_{x \rightarrow +\infty} (f(x) + x^2) = +\infty
\end{aligned}$$

$$\begin{aligned}
2) \quad 1 + x - x^2 \leq f(x) \leq 1 + x - x^2 + x^4 \\
\Leftrightarrow x - x^2 \leq f(x) - 1 \leq x - x^2 + x^4 \\
\Leftrightarrow \underbrace{1-x}_1 \leq \frac{f(x)-1}{x} \leq \underbrace{1-x+x^3}_1 ; x > 0 \\
\Leftrightarrow \lim_{x \rightarrow 0^+} \left(\frac{f(x)-1}{x} \right) = 1
\end{aligned}$$

On suit le même procédé pour la deuxième limite :

$$\lim_{x \rightarrow 0^-} \left(\frac{f(x)-1}{x} \right) = 1$$

$$3) \quad 1 - x \leq \frac{f(x) - 1}{x} \leq 1 - x + x^3 ; x > 0$$

$$\Leftrightarrow \frac{1 - x}{x} \leq \frac{f(x) - 1}{x^2} \leq \frac{1 - x + x^3}{x} ; x > 0$$

$$\Leftrightarrow \frac{1}{x} - 1 \leq \frac{f(x) - 1}{x^2} \leq \frac{1}{x} - 1 + x^2 ; x > 0$$

$$\Leftrightarrow \underbrace{-1}_{-1} \leq \left(\frac{f(x) - 1 - x}{x^2} \right) \leq \underbrace{-1 + x^2}_{-1}$$

$$\Rightarrow \lim_{x \rightarrow 0^{\pm}} \left(\frac{f(x) - 1 - x}{x^2} \right) = -1$$

Solution N° 92 :

$$1) \quad \lim_{x \rightarrow +\infty} g(x) = \lim_{x \rightarrow +\infty} \left(\frac{3\sqrt{1+x^4} - x}{2+x} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{3\sqrt{x^4 \left(1 + \frac{1}{x^4}\right)} - x}{2+x} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{3\sqrt{x^4} \sqrt{\left(1 + \frac{1}{x^4}\right)} - x}{2+x} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{3x^2 \sqrt{\left(1 + \frac{1}{x^4}\right)} - x}{2+x} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(3x \sqrt{\left(1 + \frac{1}{x^4}\right)} - 1 \right)}{x \left(\frac{2}{x} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{3x \sqrt{1 + \frac{1}{x^4}} - 1}{\frac{2}{x} + 1} \right)$$

$$= \left(\frac{3(+\infty)\sqrt{1+0} - 1}{0 + 1} \right) = +\infty$$

$$\blacksquare \quad \lim_{\substack{x \rightarrow \frac{-\pi}{2} \\ x > \frac{-\pi}{2}}} g(x) = \lim_{\substack{x \rightarrow \frac{-\pi}{2} \\ x > \frac{-\pi}{2}}} \left(\frac{1 - \cos^3 x}{x \tan x \cos^2 x} \right)$$

$$= \lim_{x \rightarrow \frac{-\pi}{2}} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{\frac{x \cdot \sin x \cdot \cos^2 x}{\cos x}}$$

$$= \lim_{x \rightarrow \frac{-\pi}{2}} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{x \cdot \sin x \cdot \cos x}$$

$$= \lim_{x \rightarrow \frac{-\pi}{2}} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{x}{\sin x} \right) \left(\frac{1 + \cos x + \cos^2 x}{\cos x} \right)$$

$$= \lim_{x \rightarrow \frac{-\pi}{2}} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{1}{\frac{\sin x}{x}} \right) \left(\frac{1}{\cos x} + 1 + \cos x \right)$$

$$= \left(\frac{1 - 0}{\frac{\pi^2}{4}} \right) \left(\frac{\pi}{2} \right) \left(\frac{1}{0^+} + 1 + 0 \right) = +\infty$$

$$\blacksquare \quad \lim_{\substack{x \rightarrow \frac{-\pi}{2} \\ x < \frac{-\pi}{2}}} g(x) = \lim_{\substack{x \rightarrow \frac{-\pi}{2} \\ x < \frac{-\pi}{2}}} (2x + \pi) \tan x$$

$$= \lim_{\substack{t \rightarrow 0 \\ t = \frac{\pi}{2} + x}} 2t \cdot \tan \left(t - \frac{\pi}{2} \right)$$

$$= \lim_{t \rightarrow 0} 2t \cdot \frac{\sin \left(t - \frac{\pi}{2} \right)}{\cos \left(t - \frac{\pi}{2} \right)}$$

$$= \lim_{t \rightarrow 0} -2t \cdot \left(\frac{\cos t}{\sin t} \right)$$

$$= -2 \lim_{t \rightarrow 0} \left(\frac{t}{\sin t} \right) \cdot \cos t = -2$$

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$$2) \lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} \left(\frac{3\sqrt{1+x^4} - x}{2+x} \right)$$

$$= \left(\frac{3\sqrt{1+0^4} - 0}{2+0} \right) = \frac{3}{2} = g(0) \in \mathbb{R}$$

$$\blacksquare \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left(\frac{1 - \cos^3 x}{x \cdot \tan x \cdot \cos^2 x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{\frac{x \cdot \sin x \cdot \cos^2 x}{\cos x}}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x + \cos^2 x)}{x \cdot \sin x \cdot \cos x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right) \left(\frac{x}{\sin x} \right) \left(\frac{1 + \cos x + \cos^2 x}{\cos x} \right)$$

$$= \left(\frac{1}{2} \right) \left(\frac{1}{1} \right) \left(\frac{1+1+1^2}{1} \right) = \frac{3}{2} = g(0) \in \mathbb{R}$$

On remarque que :

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^-} g(x) = g(0)$$

Donc la fonction g est continue en 0.

Solution N° 93 :

Pour que la fonction f soit continue au point $x_0 = 2$ il faut et il suffit qu'on ait :

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x) = f(2)$$

$$\Leftrightarrow \lim_{x \rightarrow 2^+} \left(\frac{x^2 + x - a}{x - 2} \right) = \left(\frac{4+b}{3} \right)$$

$$\Leftrightarrow \lim_{x \rightarrow 2^+} \left(x + 3 + \frac{6-a}{x-2} \right) = \left(\frac{4+b}{3} \right)$$

$$\Leftrightarrow \begin{cases} Et & 6 - a = 0 \\ Et & 2 + 3 = \frac{4+b}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} Et & a = 6 \\ Et & b = 11 \end{cases}$$

Solution N° 94 :

Pour que la fonction f soit continue au point $x_0 = 1$ il faut et il suffit qu'on ait :

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1)$$

$$\Leftrightarrow \begin{cases} \lim_{x \rightarrow 1^+} \left(\frac{-2x^2 + 3x + 3}{x^2 + 1} \right) = \left(\frac{2+c}{3} \right) \\ \lim_{x \rightarrow 1^-} \left(\frac{3x^2 - 2bx + 1}{2x^2 + ax - a - 2} \right) = \left(\frac{2+c}{3} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} 2 = \left(\frac{2+c}{3} \right) \\ \lim_{x \rightarrow 1^-} \frac{3x^2 - 2bx + 1}{(x-1)(2x+a+2)} = \left(\frac{2+c}{3} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} c = 4 \\ \lim_{x \rightarrow 1^-} \left(\frac{3x^2 - 2bx + 1}{x-1} \right) \left(\frac{1}{2x+a+2} \right) = 2 \end{cases}$$

$$\lim_{\substack{x \rightarrow 1^- \\ c=4}} \left(3x + 3 - 2b + \frac{4-2b}{x-1} \right) \left(\frac{1}{2x+a+2} \right) = 2$$

$$\Leftrightarrow \begin{cases} c = 4 \\ \frac{3+3-2b}{2+a+2} = 2 \\ 4-2b = 0 \end{cases} \Leftrightarrow \begin{cases} a = -3 \\ b = 2 \\ c = 4 \end{cases}$$

Solution N° 95 :

Pour que la fonction f_n soit continue au point $x_0 = 2$ il faut et il suffit qu'on ait :

$$\lim_{x \rightarrow 2^+} f_n(x) = \lim_{x \rightarrow 2^-} f_n(x) = f_n(2)$$

$$\Leftrightarrow \lim_{x \rightarrow 2^+} \left(\frac{3x + b}{4} \right) = \left(\frac{(3 - x)^n - a}{x - 2} \right)$$

$$\Leftrightarrow \left(\frac{6 + b}{4} \right) = \lim_{\substack{t \rightarrow 0^- \\ t = x - 2}} \frac{(1 - t)^n - a}{t}$$

$$\Leftrightarrow \lim_{t \rightarrow 0^-} \left(\frac{\sum_{k=0}^{k=n} C_n^k (-t)^k - a}{t} \right) = \left(\frac{6 + b}{4} \right)$$

$$\Leftrightarrow \lim_{t \rightarrow 0^-} \left(\frac{1 - a}{t} + \sum_{k=1}^{k=n} C_n^k \cdot (-1)^k \cdot t^{k-1} \right) = \left(\frac{6 + b}{4} \right)$$

$$\Leftrightarrow \begin{cases} 1 - a = 0 \\ \lim_{t \rightarrow 0^-} \sum_{k=1}^{k=n} C_n^k \cdot (-1)^k \cdot t^{k-1} = \left(\frac{6 + b}{4} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} 1 - a = 0 \\ -n + \lim_{t \rightarrow 0^-} \sum_{k=2}^{k=n} C_n^k \cdot (-1)^k \cdot t^{k-1} = \left(\frac{6 + b}{4} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 1 \\ -n + \sum_{k=2}^{k=n} C_n^k \cdot (-1)^k \cdot 0^{k-1} = \left(\frac{6 + b}{4} \right) \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 1 \\ -n + 0 = \left(\frac{6 + b}{4} \right) \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = -4n - 6 \end{cases}$$

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Solution N° 96 :

Voici les réponses sans développement :

$$\blacksquare \quad 1) \quad a = 2 \quad \text{et} \quad c = \frac{17 + 3b}{8}$$

$$\blacksquare \quad 2) \quad 4a + 2b - 3c = 17$$

Solution N° 97 :

$$\begin{aligned} 2) \quad \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \left(\frac{x\sqrt{x} - 1}{\sqrt{3x + 1} - \sqrt{x + 3}} \right) \\ &= \lim_{x \rightarrow 1} \left(\frac{(x\sqrt{x})^2 - 1^2}{(\sqrt{3x + 1})^2 - (\sqrt{x + 3})^2} \right) \\ &\quad \times \frac{(\sqrt{3x + 1} + \sqrt{x + 3})}{(x\sqrt{x} + 1)} \end{aligned}$$

$$= \lim_{x \rightarrow 1} \left(\frac{x^3 - 1}{2x - 2} \right) \left(\frac{\sqrt{4} + \sqrt{4}}{1\sqrt{1} + 1} \right)$$

$$= 2 \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{2(x - 1)}$$

$$= 2 \lim_{x \rightarrow 1} \frac{(x^2 + x + 1)}{2} = 2 \left(\frac{1^2 + 1 + 1}{2} \right) = 3 \in \mathbb{R}$$

Donc f admet un prolongement par continuité en $x_0 = 1$ et défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) ; & x \neq 1 \\ \tilde{f}(1) = 3 \end{cases}$$

$$2) \quad \lim_{x \rightarrow -2} \left(\frac{\sqrt{x + 6} + \sqrt{2x + 5} - 3}{4 - x^2} \right)$$

$$= \lim_{\substack{t \rightarrow 0 \\ t = x + 2}} \frac{\sqrt{t + 4} + \sqrt{2t + 1} - 3}{t(4 - t)}$$

$$\begin{aligned}
&= \lim_{\substack{t \rightarrow 0 \\ t=x+2}} \frac{(\sqrt{t+4} + \sqrt{2t+1})^2 - 3^2}{t(4-t)(\sqrt{t+4} + \sqrt{2t+1} + 3)} \\
&= \lim_{t \rightarrow 0} \left(\frac{3t - 4 + 2\sqrt{2t^2 + 9t + 4}}{t} \right) \\
&\quad \times \left(\frac{1}{(4-t)(\sqrt{t+4} + \sqrt{2t+1} + 3)} \right) \\
&= \frac{1}{24} \lim_{t \rightarrow 0} \left(\frac{3t - 4 + 2\sqrt{2t^2 + 9t + 4}}{t} \right) \\
&= \frac{1}{24} \lim_{t \rightarrow 0} \left(3 + 2 \left(\frac{\sqrt{2t^2 + 9t + 4} - 2}{t} \right) \right) \\
&= \frac{3}{24} + \frac{2}{24} \lim_{t \rightarrow 0} \left(\frac{\sqrt{2t^2 + 9t + 4} - 2}{t} \right) \\
&= \frac{3}{24} + \frac{2}{24} \lim_{t \rightarrow 0} \left(\frac{(\sqrt{2t^2 + 9t + 4})^2 - 2^2}{t(\sqrt{2t^2 + 9t + 4} + 2)} \right) \\
&= \frac{3}{24} + \frac{2}{24} \lim_{t \rightarrow 0} \frac{t(2t + 9)}{t(\sqrt{2t^2 + 9t + 4} + 2)} \\
&= \frac{3}{24} + \frac{2}{24} \lim_{t \rightarrow 0} \left(\frac{2t + 9}{\sqrt{2t^2 + 9t + 4} + 2} \right) \\
&= \frac{3}{24} + \frac{2}{24} \times \left(\frac{0 + 9}{\sqrt{4} + 2} \right) = \frac{5}{16} \in \mathbb{R}
\end{aligned}$$

Donc f est prolongeable par continuité en $x_0 = -2$ et ce prolongement est défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq -2 \\ \tilde{f}(-2) = \frac{5}{16} \end{cases}$$

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$$\begin{aligned}
3) \quad \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x + \sin x} \right) \\
&= \lim_{x \rightarrow 0} \frac{x \left(\frac{\tan x}{x} - \frac{\sin x}{x} \right)}{x \left(1 + \frac{\sin x}{x} \right)} \\
&= \lim_{x \rightarrow 0} \left(\frac{\tan x}{x} - \frac{\sin x}{x} \right) \left(\frac{1}{1 + \frac{\sin x}{x}} \right) \\
&= (1 - 1) \left(\frac{1}{1 + 1} \right) = 0 \in \mathbb{R}
\end{aligned}$$

Donc f admet un prolongement par continuité en $x_0 = 1$ et défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq 0 \\ \tilde{f}(0) = 0 \end{cases}$$

$$\begin{aligned}
4) \quad \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \left(\frac{\cos x - \sqrt{1 + \sin x}}{x} \right) \\
&= \lim_{x \rightarrow 0} \frac{\cos^2 x - (\sqrt{1 + \sin x})^2}{x(\cos x + \sqrt{1 + \sin x})} \\
&= \lim_{x \rightarrow 0} \left(\frac{\cos^2 x - 1 - \sin x}{x} \right) \left(\frac{1}{\cos x + \sqrt{1 + \sin x}} \right) \\
&= \lim_{x \rightarrow 0} \left(\frac{\cos^2 x - 1 - \sin x}{x} \right) \left(\frac{1}{\cos 0 + \sqrt{1 + \sin 0}} \right) \\
&= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\cos^2 x - 1 - \sin x}{x} \right) \\
&= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{-\sin^2 x - \sin x}{x} \right)
\end{aligned}$$

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$$= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{-\sin x}{x} \right) (1 + \sin x)$$

$$= \frac{1}{2} \times (-1) \times (0 + 1) = \frac{-1}{2} \in \mathbb{R}$$

Donc f est prolongeable par continuité en $x_0 = 0$ et ce prolongement est défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq 0 \\ \tilde{f}(0) = \frac{-1}{2} \end{cases}$$

Solution N° 98 :

$$1) \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (x^2 - 3x + 2) = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{(x-1)^2}{\sqrt{x^2-1}}$$

$$= \lim_{x \rightarrow 1^+} \frac{(x-1)(x-1)}{\sqrt{x-1}\sqrt{x+1}}$$

$$= \lim_{x \rightarrow 1^+} (x-1)\sqrt{x-1} \times \lim_{x \rightarrow 1^+} \left(\frac{1}{\sqrt{x+1}} \right)$$

$$= \lim_{\substack{t \rightarrow 0 \\ t=x-1}} t\sqrt{t} \times \frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}} \times 0 \times \sqrt{0} \right) = 0$$

$$2) \text{ comme } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = 0 \in \mathbb{R}$$

$$\text{Alors : } \lim_{x \rightarrow 1} f(x) = 0 \in \mathbb{R}$$

Donc f est prolongeable par continuité en $x_0 = 1$ et ce prolongement est défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq 1 \\ \tilde{f}(1) = 0 \end{cases}$$

$$3) \lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = \lim_{\substack{x \rightarrow -1 \\ x > -1}} (x^2 - 3x + 2) = 6$$

$$\lim_{\substack{x \rightarrow -1 \\ x > -1}} f(x) = \lim_{\substack{x \rightarrow -1 \\ x > -1}} \frac{(x-1)^2}{\sqrt{x^2-1}}$$

$$= \lim_{x \rightarrow -1} \frac{1}{\sqrt{x^2-1}} \times \lim_{x \rightarrow -1} (x-1)^2$$

$$= \lim_{\substack{t \rightarrow 1^+ \\ t=x^2}} \frac{1}{\sqrt{t-1}} \times (-1-1)^2$$

$$= 4 \lim_{t \rightarrow 1^+} \frac{1}{\sqrt{t-1}} = 4 \lim_{\substack{u \rightarrow 0^+ \\ u=\sqrt{t-1}}} \frac{1}{u}$$

$$= 4 \times \frac{1}{0^+} = +\infty \notin \mathbb{R}$$

$$\text{comme } \lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x) \neq \lim_{\substack{x \rightarrow -1 \\ x < -1}} f(x)$$

$$\text{Alors } \lim_{x \rightarrow -1} f(x) \notin \mathbb{R}$$

Donc la fonction f n'admet pas de prolongement par continuité en $x_0 = -1$

Solution N° 99 :

$$\lim_{x \rightarrow \frac{\pi}{4}} f(x) = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \tan x)^2}{\cos(2x)}$$

$$= \lim_{\substack{t \rightarrow 0 \\ t=x-\frac{\pi}{4}}} \frac{\left(1 - \tan\left(t + \frac{\pi}{4}\right)\right)^2}{\cos\left(2t + \frac{\pi}{2}\right)}$$

$$= \lim_{t \rightarrow 0} \frac{\left(1 - \frac{1 + \tan t}{1 - \tan t}\right)^2}{-\sin(2t)}$$

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$$\begin{aligned}
 &= \lim_{t \rightarrow 0} \frac{(1 - \tan t - \tan t - 1)^2}{(1 - \tan t)^2(-\sin 2t)} \\
 &= \lim_{t \rightarrow 0} \frac{4 \tan^2 t}{(1 - \tan t)^2(-\sin 2t)} \\
 &= 4 \lim_{t \rightarrow 0} \left(\frac{\tan t}{t} \right)^2 \left(\frac{2t}{-\sin 2t} \right) \left(\frac{t}{2(1 - \tan t)^2} \right) \\
 &= 4(1)^2 \left(\frac{-1}{1} \right) \left(\frac{0}{2(1 - \tan 0)^2} \right) = 0 \in \mathbb{R}
 \end{aligned}$$

Donc f est prolongeable par continuité en $x_0 = \frac{\pi}{4}$ et ce prolongement est défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq \frac{\pi}{4} \\ \tilde{f}\left(\frac{\pi}{4}\right) = 0 \end{cases}$$

Solution N° 100 :

$$1) \lim_{\substack{x \rightarrow 1 \\ x > 1}} f(x) = \lim_{\substack{x \rightarrow 1 \\ x > 1}} \left(\frac{x^2 - x + 1}{3x^2 + 6} \right) = \frac{1}{9}$$

$$\lim_{\substack{x \rightarrow 1 \\ x < 1}} f(x) = \lim_{x \rightarrow 1} \left(\frac{1}{x^2 - x} \right) \sin\left(\frac{\pi x}{2}\right)$$

$$= \lim_{x \rightarrow 1^-} \frac{1}{x(x-1)} \times \lim_{x \rightarrow 1^-} \sin\left(\frac{\pi x}{2}\right)$$

$$= \lim_{\substack{t \rightarrow 0^- \\ t = x-1}} \left(\frac{1}{t(t+1)} \right) \times 1$$

$$= \lim_{t \rightarrow 0^-} \left(\frac{1}{t+1} \right) \times \lim_{t \rightarrow 0^-} \left(\frac{1}{t} \right)$$

$$= \frac{1}{1+1} \times \frac{1}{0^-} = -\infty \notin \mathbb{R}$$

On remarque que :

$$\lim_{x \rightarrow 1^+} f(x) \neq \lim_{x \rightarrow 1^-} f(x)$$

Donc la fonction f n'admet pas de prolongement par continuité en $x_0 = 1$

$$2) \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi x}{2}\right)}{x(x-1)}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin\left(\frac{\pi x}{2}\right)}{\frac{\pi x}{2}} \right) \times \lim_{x \rightarrow 0} \left(\frac{\frac{\pi}{2}}{x-1} \right)$$

$$= 1 \times \frac{\frac{\pi}{2}}{0-1} = \frac{-\pi}{2} \in \mathbb{R}$$

Donc f est prolongeable par continuité en $x_0 = 0$ et ce prolongement est défini ainsi :

$$\begin{cases} \tilde{f}(x) = f(x) & ; \quad x \neq 0 \\ \tilde{f}(0) = \frac{-\pi}{2} \end{cases}$$

Solution N° 101 :

$$1) \text{ Soit : } \varphi(x) = x^3 - 3x^2 + 15x - 7$$

On a la fonction φ est continue sur $\mathbb{R}$ donc φ est continue sur $]0,1[\subset \mathbb{R}$.

Et on a $\varphi(0) = -7 < 0$ et $\varphi(1) = 6 > 0$

Donc $\varphi(0) \times \varphi(1) < 0$

Alors d'après le TVI on en déduit que :

$$\exists \alpha \in]0,1[; \varphi(\alpha) = 0$$

$$\text{c-à-d } \exists \alpha \in]0,1[: \alpha^3 - 3\alpha^2 + 15\alpha - 7 = 0$$

$$2) \text{ soit } \varphi(x) = 1 + \sin x - x^2$$

On a φ est continue sur $\mathbb{R}$ donc φ est continue sur $\left]0, \frac{\pi}{2}\right[\subset \mathbb{R}$

$$\text{On a } \varphi(0) = 1 + \sin 0 - 0^2 = 1 > 0$$

$$\text{Et } \varphi\left(\frac{\pi}{2}\right) = 1 + \sin\left(\frac{\pi}{2}\right) - \frac{\pi^2}{4} = 2 - \frac{\pi^2}{4} < 0$$

$$\text{Donc } \varphi(0) \cdot \varphi\left(\frac{\pi}{2}\right) < 0$$

D'où d'après le TVI on en déduit que :

$$\exists \alpha \in \left]0, \frac{\pi}{2}\right[\quad ; \quad \varphi(\alpha) = 0$$

$$\Leftrightarrow \exists \alpha \in \left]0, \frac{\pi}{2}\right[\quad ; \quad 1 + \sin \alpha - \alpha^2 = 0$$

$$3) \text{ Soit } \varphi(x) = x^{17} - x^{11} - 1$$

On a φ est continue sur $\mathbb{R}$ car c'est un polynôme. Donc φ est continue sur $]1, 2[\subset \mathbb{R}$. Et on a $\varphi(1) = -1 < 0$

$$\text{Et encore } \varphi(2) = 2^{11}(2^6 - 1) - 1 > 0$$

$$\text{Donc : } \varphi(1) \times \varphi(2) < 0$$

D'où d'après le TVI on en déduit que :

$$\exists \alpha \in]1, 2[\quad ; \quad \varphi(\alpha) = 0$$

$$\Leftrightarrow \exists \alpha \in]1, 2[\quad ; \quad \alpha^{17} = \alpha^{11} + 1 = 0$$

$$4) \text{ soit } \varphi(x) = \sqrt{x^3 + 5x + 4} - 100$$

On a la fonction φ est continue sur $\mathbb{R}^+$

Donc φ est continue sur $]20, 21[\subset \mathbb{R}^+$

$$\begin{aligned} \text{On a } \varphi(21) &= \sqrt{21^3 + 5 \times 21 + 4} - 100 \\ &= \sqrt{9370} - 100 < 0 \end{aligned}$$

$$\begin{aligned} \text{Et on a } \varphi(22) &= \sqrt{22^3 + 5 \times 22 + 4} - 100 \\ &= \sqrt{10762} - 100 > 0 \end{aligned}$$

$$D'où : \varphi(21) \cdot \varphi(22) < 0$$

Donc d'après le TVI on en déduit que :

$$\exists \alpha \in]21; 22[\quad ; \quad \varphi(\alpha) = 0$$

$$\Leftrightarrow \exists \alpha \in]21; 22[\quad ; \quad \sqrt{\alpha^3 + 5\alpha + 4} = 100$$

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$$5) \text{ soit } \varphi(x) = \cos x - \frac{2}{(x+1)^2}$$

On a φ est continue sur $\mathbb{R} \setminus \{-1\}$ donc φ est continue sur $]0, 1[\subset \mathbb{R} \setminus \{-1\}$.

$$\text{On a } \varphi(0) = \cos 0 - \frac{2}{(0+1)^2} = -1 < 0$$

$$\text{et on a } \varphi(1) = \cos 1 - \frac{1}{2} > 0 \text{ car } \pi > 3$$

$$D'où \varphi(0) \times \varphi(1) < 0$$

Donc d'après le TVI on en déduit que :

$$\exists \alpha \in]0, 1[\quad : \quad \varphi(\alpha) = 0$$

$$\Leftrightarrow \exists \alpha \in]0, 1[\quad : \quad \cos \alpha = \frac{2}{(\alpha+1)^2}$$

$$6) \text{ soit } \varphi(x) = x^2 \cos x + x \sin x + 1$$

φ est continue sur $D_\varphi = \mathbb{R}$

Donc φ est continue sur $]0, \pi[\subset \mathbb{R}$

$$\text{On a } \varphi(0) = 0 + 0 + 1 = 1 > 0$$

$$\begin{aligned} \text{Et on a } \varphi(\pi) &= \pi^2 \cos \pi + \pi \sin \pi + 1 \\ &= 1 - \pi^2 = (1 - \pi)(1 + \pi) < 0 \end{aligned}$$

$$D'où \varphi(0) \cdot \varphi(\pi) < 0$$

Donc d'après le TVI on en déduit que :

$$\exists \alpha \in]0, \pi[\quad ; \quad \varphi(\alpha) = 0$$

$$\Leftrightarrow \exists \alpha \in]0, \pi[\quad ; \quad \alpha^2 \cos \alpha + \alpha \sin \alpha + 1 = 0$$

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Solution N° 102 :

On pose : $t = \frac{2-x}{3+x}$

L'équation (E) devient alors :

$$t^{\frac{1}{4}} + t^{\frac{-1}{4}} = 2 \quad ; \quad t > 0$$

$$\Rightarrow \left(t^{\frac{1}{4}} + t^{\frac{-1}{4}}\right)^2 = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^{\frac{1}{2}} + t^{\frac{-1}{2}} + 2 \cdot t^{\frac{1}{4}} \cdot t^{\frac{-1}{4}} = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^{\frac{1}{2}} + t^{\frac{-1}{2}} + 2 = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^{\frac{1}{2}} + t^{\frac{-1}{2}} = 2 \quad ; \quad t > 0$$

$$\Rightarrow \left(t^{\frac{1}{2}} + t^{\frac{-1}{2}}\right)^2 = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^1 + t^{-1} + 2 \cdot t^{\frac{1}{2}} \cdot t^{\frac{-1}{2}} = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^1 + t^{-1} + 2 = 4 \quad ; \quad t > 0$$

$$\Rightarrow t^1 + t^{-1} = 2 \quad ; \quad t > 0$$

$$\Rightarrow t + \frac{1}{t} = 2 \quad ; \quad t > 0$$

$$\Rightarrow \frac{t^2 + 1}{t} = 2 \quad ; \quad t > 0$$

$$\Rightarrow t^2 - 2t + 1 = 0 \quad ; \quad t > 0$$

$$\Rightarrow (t - 1)^2 = 0 \quad ; \quad t > 0$$

$$\Rightarrow t = 1 > 0 \quad ; \quad t > 0$$

$$\Rightarrow \frac{2-x}{3+x} = 1$$

$$\Rightarrow 2 - x = 3 + x$$

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$$\Rightarrow x = \frac{-1}{2}$$

Inversement on a encore l'égalité :

$$\sqrt[4]{\frac{2+\frac{1}{2}}{3-\frac{1}{2}}} + \sqrt[4]{\frac{3-\frac{1}{2}}{2+\frac{1}{2}}} = \sqrt[4]{1} + \sqrt[4]{1} = 2$$

Donc la fraction $\frac{-1}{2}$ est la seule solution pour cette équation.

2) en utilisant des puissances rationnelles on se rend compte que (E) devient :

$$\frac{1}{3}(x+3)^{\frac{1}{3}} + (x^{-2} + 3x^{-3})^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3}(x+3)^{\frac{1}{3}} + (x^{-2}(1+3x^{-1}))^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3}(x+3)^{\frac{1}{3}} + x^{\frac{-2}{3}}(1+3x^{-1})^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3}(x+3)^{\frac{1}{3}} + x^{\frac{-2}{3}}(x^{-1}(x^1+3))^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3}(x+3)^{\frac{1}{3}} + x^{\frac{-2}{3}} \cdot x^{\frac{-1}{3}}(x+3)^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3}(x+3)^{\frac{1}{3}} + x^{-1}(x+3)^{\frac{1}{3}} = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow (x+3)^{\frac{1}{3}}\left(\frac{1}{3} + x^{-1}\right) = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow (x+3)^{\frac{1}{3}}\left(\frac{1}{3} + \frac{1}{x}\right) = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow (x+3)^{\frac{1}{3}}\left(\frac{x+3}{3x}\right) = \frac{1}{2}x^{\frac{1}{3}}$$

$$\Leftrightarrow (x+3)^{\frac{1}{3}} \cdot (x+3)^1 \cdot (3x)^{-1} = \frac{1}{2} x^{\frac{1}{3}}$$

$$\Leftrightarrow (x+3)^{\frac{4}{3}} \cdot (3x)^{-1} = \frac{1}{2} x^{\frac{1}{3}}$$

$$\Leftrightarrow \frac{1}{3} (x+3)^{\frac{4}{3}} \cdot x^{-1} \cdot x^{\frac{-1}{3}} = \frac{1}{2}$$

$$\Leftrightarrow (x+3)^{\frac{4}{3}} \cdot x^{\frac{-4}{3}} = \frac{3}{2}$$

$$\Leftrightarrow \left(\frac{x+3}{x}\right)^{\frac{4}{3}} = \frac{3}{2}$$

$$\Leftrightarrow \left(1 + \frac{3}{x}\right)^{\frac{4}{3}} = \frac{3}{2}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } \left(1 + \frac{3}{x}\right) = \left(\frac{3}{2}\right)^{\frac{3}{4}} \\ \text{ou bien } -\left(1 + \frac{3}{x}\right) = \left(\frac{3}{2}\right)^{\frac{3}{4}} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } \frac{3}{x} = \left(\frac{3}{2}\right)^{\frac{3}{4}} - 1 \\ \text{ou bien } \frac{-3}{x} = \left(\frac{3}{2}\right)^{\frac{3}{4}} + 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } \frac{x}{3} = \frac{1}{\left(\frac{3}{2}\right)^{\frac{3}{4}} - 1} \\ \text{ou bien } \frac{-x}{3} = \frac{1}{\left(\frac{3}{2}\right)^{\frac{3}{4}} + 1} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x = \left(\frac{3}{\left(\frac{3}{2}\right)^{\frac{3}{4}} - 1}\right) \\ \text{ou bien } x = \left(\frac{-3}{\left(\frac{3}{2}\right)^{\frac{3}{4}} + 1}\right) \end{cases}$$

3) en utilisant les puissances rationnelles l'équation devient :

$$(E) : 2 \cdot x^{\frac{4}{3}} - \frac{3x^1}{x^{\frac{1}{3}}} - 20 = 0$$

$$\Leftrightarrow 2x^{\frac{4}{3}} - 3 \cdot x^{(1-\frac{1}{3})} - 20 = 0$$

$$\Leftrightarrow 2x^{\frac{4}{3}} - 3x^{\frac{2}{3}} - 20 = 0$$

$$\Leftrightarrow 2t^2 - 3t - 20 = 0 ; t = t^{\frac{2}{3}} > 0$$

$$\Leftrightarrow t = \frac{3 \pm \sqrt{13}}{4} ; \Delta = 13$$

$$\Leftrightarrow t = \frac{3 + \sqrt{13}}{4} > 0$$

$$\Leftrightarrow x^{\frac{2}{3}} = \frac{3 + \sqrt{13}}{4} > 0$$

$$\Leftrightarrow x = \left(\frac{3 + \sqrt{13}}{4}\right)^{\frac{3}{2}} > 0$$

$$4) (E) : \sqrt[3]{x^3 - 3x^2 + x + 1} = x - 1$$

$$\Leftrightarrow (x^3 - 3x^2 + x + 1) = (x - 1)^3$$

$$\Leftrightarrow x^3 - 3x^2 + x + 1 = x^3 + 3x - 3x^2 - 1$$

$$\Leftrightarrow -2x + 2 = 0$$

$$\Leftrightarrow x = 1$$

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Solution N° 103 :

$$\begin{aligned}
 1) \quad \lim_{x \rightarrow 0} \left(\frac{\sqrt[3]{x+8} - 2}{x} \right) &= \lim_{x \rightarrow 0} \frac{(x+8)^{\frac{2}{3}} + 2(x+8)^{\frac{1}{3}} + 4}{(x+8)^{\frac{2}{3}} + 2(x+8)^{\frac{1}{3}} + 4} \\
 &= \lim_{x \rightarrow 0} \frac{\left((x+8)^{\frac{1}{3}}\right)^3 - 2^3}{x \left((x+8)^{\frac{2}{3}} + 2(x+8)^{\frac{1}{3}} + 4\right)} \\
 &= \lim_{x \rightarrow 0} \frac{x}{x \left((x+8)^{\frac{2}{3}} + 2(x+8)^{\frac{1}{3}} + 4\right)} \\
 &= \lim_{x \rightarrow 0} \frac{1}{\left((x+8)^{\frac{2}{3}} + 2(x+8)^{\frac{1}{3}} + 4\right)} \\
 &= \frac{1}{\left((0+8)^{\frac{2}{3}} + 2(0+8)^{\frac{1}{3}} + 4\right)} = \frac{1}{12}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \lim_{x \rightarrow 0^+} \left(\frac{\sqrt[3]{x^2} - x}{x} \right) &= \lim_{x \rightarrow 0^+} \left(\frac{x^{\frac{2}{3}} - x}{x} \right) \\
 &= \lim_{x \rightarrow 0^+} \frac{x \left(x^{-\frac{1}{3}} - 1 \right)}{x} = \lim_{x \rightarrow 0^+} \left(x^{-\frac{1}{3}} - 1 \right) \\
 &= \lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt[3]{x}} - 1 \right) = \frac{1}{0^+} - 1 = +\infty
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \lim_{x \rightarrow +\infty} \left(\frac{\sqrt[3]{x} - 1}{\sqrt[3]{x^2} + 1} \right) &= \lim_{x \rightarrow +\infty} \left(\frac{x^{\frac{1}{3}} - 1}{x^{\frac{2}{3}} + 1} \right) \\
 &= \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{3}} \left(1 - x^{-\frac{1}{3}} \right)}{x^{\frac{2}{3}} \left(1 + x^{-\frac{2}{3}} \right)}
 \end{aligned}$$

$$= \lim_{x \rightarrow \infty} \left(\frac{1 - \frac{1}{x^{\frac{1}{3}}}}{1 + \frac{1}{x^{\frac{2}{3}}}} \right) = \frac{1 - 0}{1 + 0} = 1$$

$$\begin{aligned}
 4) \quad \lim_{x \rightarrow 2} \left(\frac{x - \sqrt[3]{x+6}}{3 - \sqrt{2x+5}} \right) &= \lim_{x \rightarrow 2} \frac{x - (x+6)^{\frac{1}{3}}}{3 - (2x+5)^{\frac{1}{2}}} \\
 &= \lim_{x \rightarrow 2} \left(\frac{x^3 - \left((x+6)^{\frac{1}{3}}\right)^3}{3^2 - \left((2x+5)^{\frac{1}{2}}\right)^2} \right) \\
 &\quad \times \left(\frac{3 + (2x+5)^{\frac{1}{2}}}{x^2 + x(x+6)^{\frac{1}{3}} + (x+6)^{\frac{2}{3}}} \right) \\
 &= \lim_{x \rightarrow 2} \left(\frac{x^3 - x - 6}{4 - 2x} \right) \times \left(\frac{3 + 9^{\frac{1}{2}}}{4 + 2(8)^{\frac{1}{3}} + 8^{\frac{2}{3}}} \right) \\
 &= \frac{1}{2} \lim_{x \rightarrow 2} \left(\frac{-1}{2} x^2 - x - \frac{3}{2} \right) \\
 &= \frac{1}{2} \left(\frac{-4}{2} - 2 - \frac{3}{2} \right) = \frac{-11}{4}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x} - x \right) &= \lim_{x \rightarrow +\infty} (x^3 + x)^{\frac{1}{3}} - x \\
 &= \lim_{x \rightarrow +\infty} \left(\frac{\left((x^3 + x)^{\frac{1}{3}}\right)^3 - x^3}{(x^3 + x)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2} \right) \\
 &= \lim_{x \rightarrow +\infty} \frac{x}{\left((x^3)^{\frac{2}{3}} \left(1 + \frac{1}{x^2}\right)^{\frac{2}{3}} + x(x^3 + x)^{\frac{1}{3}} + x^2\right)} \\
 &= \lim_{x \rightarrow +\infty} \frac{x}{x \left(x \left(1 + \frac{1}{x^2}\right)^{\frac{2}{3}} + (x^3 + x)^{\frac{1}{3}} + x \right)}
 \end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{x \left(1 + \frac{1}{x^2} \right)^{\frac{2}{3}} + (x^3 + x)^{\frac{1}{3}} + x} \right)$$

$$= \left(\frac{1}{(+\infty)(1 + 0)^{\frac{2}{3}} + \infty + \infty} \right) = \frac{1}{+\infty} = 0$$

6) $\lim_{x \rightarrow +\infty} (x - \sqrt[3]{x} - \sqrt{x})$

$$= \lim_{x \rightarrow +\infty} \left(x^1 - x^{\frac{1}{3}} - x^{\frac{1}{2}} \right)$$

$$= \lim_{x \rightarrow +\infty} x^1 \left(1 - x^{-\frac{2}{3}} - x^{-\frac{1}{2}} \right)$$

$$= \lim_{x \rightarrow +\infty} x^1 \left(1 - \frac{1}{\sqrt[3]{x^2}} - \frac{1}{\sqrt{x}} \right)$$

$$= (+\infty)(1 - 0 - 0) = +\infty$$

Solution N° 104 :

1) $x^8 - 25 = 0$

$$\Leftrightarrow t^2 = 25 \quad ; \quad t = x^4$$

$$\Leftrightarrow t = \pm\sqrt{25}$$

$$\Leftrightarrow t = +\sqrt{25} \quad ; \quad \text{car } t = x^4 \geq 0$$

$$\Leftrightarrow x^4 = 5$$

$$\Leftrightarrow u^2 = 5 \quad ; \quad u = x^2 \geq 0$$

$$\Leftrightarrow u = \pm\sqrt{5}$$

$$\Leftrightarrow u = +\sqrt{5} \quad \text{car } u = x^2 \geq 0$$

$$\Leftrightarrow x^2 = \sqrt{5} = 5^{\frac{1}{2}}$$

$$\Leftrightarrow x = \pm\sqrt{\sqrt{5}} = \pm\sqrt[4]{5}$$

$$\Leftrightarrow S = \{-\sqrt[4]{5} ; \sqrt[4]{5}\}$$

2) $x^7 = \sqrt{3} \Leftrightarrow x^7 = 3^{\frac{1}{2}}$

$$\Leftrightarrow (x^7)^{\frac{1}{7}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{7}}$$

$$\Leftrightarrow x = 3^{\frac{1}{14}} = \sqrt[14]{3}$$

$$\Leftrightarrow S = \{\sqrt[14]{3}\}$$

3) $x^4 = 16 \Leftrightarrow t^2 = 16 \quad ; \quad t = x^2 > 0$

$$\Leftrightarrow t = \pm\sqrt{16} = \pm 4$$

$$\Leftrightarrow t = 4 \quad \text{car } t = x^2 > 0$$

$$\Leftrightarrow x^2 = 4 \quad \Leftrightarrow (x - 2)(x + 2) = 0$$

$$\Leftrightarrow x = \pm 2$$

4) $x^3 + 8 = 0 \Leftrightarrow x^3 = -8$

$$\Leftrightarrow (x^3)^{\frac{1}{3}} = (-8)^{\frac{1}{3}}$$

$$\Leftrightarrow x = -2$$

5) $\sqrt[5]{x} = \sqrt[6]{7} \Leftrightarrow x^{\frac{1}{3}} = 7^{\frac{1}{6}}$

$$\Leftrightarrow \left(x^{\frac{1}{3}}\right)^3 = \left(7^{\frac{1}{6}}\right)^3$$

$$\Leftrightarrow x = \left(7^{\frac{1}{6}}\right)^3$$

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$$6) (3x - 4)^5 = 32 \Leftrightarrow ((3x - 4)^5)^{\frac{1}{5}} = 32^{\frac{1}{5}}$$

$$\Leftrightarrow 3x - 4 = (2^5)^{\frac{1}{5}} = 2$$

$$\Leftrightarrow 3x - 4 = 2$$

$$\Leftrightarrow x = 2$$

$$7) \sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 4 = 0$$

$$\Leftrightarrow x^{\frac{2}{3}} - 5 \cdot x^{\frac{1}{3}} + 4 = 0$$

$$\Leftrightarrow t^2 - 5t + 4 = 0 ; t = x^{\frac{1}{3}} \in \mathbb{R}$$

$$\Leftrightarrow t = \frac{5 \pm \sqrt{9}}{2} ; \Delta = 9 > 0$$

$$\Leftrightarrow t = 4 \text{ ou bien } t = 1$$

$$\Leftrightarrow x^{\frac{1}{3}} = 4 \text{ ou bien } x^{\frac{1}{3}} = 1$$

$$\Leftrightarrow \left(x^{\frac{1}{3}}\right)^3 = 4^3 \text{ ou bien } \left(x^{\frac{1}{3}}\right)^3 = 1^3$$

$$\Leftrightarrow x = 64 \text{ ou bien } x = 1$$

$$\Leftrightarrow S = \{1; 64\}$$

$$8) 9x - 7 \cdot \sqrt[3]{x} - 2 = 0$$

$$\Leftrightarrow 9x - 7x^{\frac{1}{3}} - 2 = 0$$

$$\Leftrightarrow 9t^3 - 7t - 2 = 0 ; t = x^{\frac{1}{3}} \in \mathbb{R}$$

A ce stade, il faut remarquer que 1 est une racine simple du polynôme de gauche donc on peut effectuer la division de ce polynôme par $(t - 1)$

$$\Leftrightarrow (t - 1)(9t^2 + 9t + 2) = 0$$

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$$\Leftrightarrow (t - 1) \cdot 9 \left(t + \frac{2}{3}\right) \left(t + \frac{1}{3}\right) = 0$$

$$\Leftrightarrow \begin{cases} \text{ou bien } t = 1 \\ \text{ou bien } t = -2/3 \\ \text{ou bien } t = -1/3 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x^{1/3} = 1 \\ \text{ou bien } x^{1/3} = -2/3 \\ \text{ou bien } x^{1/3} = -1/3 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{ou bien } x = 1 \\ \text{ou bien } x = (-2/3)^3 \\ \text{ou bien } x = (-1/3)^3 \end{cases}$$

$$\Leftrightarrow S = \left\{1; \frac{-8}{27}; \frac{-1}{27}\right\}$$

$$9) x^5 - 5x^2 - 24 = 0$$

$$\Leftrightarrow t^2 - 5t - 24 = 0 ; t = x^2 \geq 0$$

$$\Leftrightarrow t = \frac{5 \pm \sqrt{121}}{2} ; \Delta = 121 > 0$$

$$\Leftrightarrow t = \frac{5 + \sqrt{121}}{2} \text{ car } t = x^2 \geq 0$$

$$\Leftrightarrow x^2 = 8 ; x \in \mathbb{R}$$

$$\Leftrightarrow x = \pm\sqrt{8}$$

$$\Leftrightarrow \text{Solutions} = \{-\sqrt{8}; \sqrt{8}\}$$

Solution N° 105 :

$$1) \lim_{x \rightarrow 0} \frac{\text{Arctan}(3x)}{x} = 3 \lim_{x \rightarrow 0} \frac{\text{Arctan}(3x)}{3x} = 3$$

$$2) \lim_{|x| \rightarrow +\infty} \text{Arctan}(x^4 - x)$$

$$= \lim_{\substack{t \rightarrow +\infty \\ t=x^4-x}} \operatorname{Arctan}(t) = \frac{\pi}{2}$$

$$\begin{aligned} 3) \quad & \lim_{x \rightarrow +\infty} \left(x \cdot \operatorname{Arctan}(x) - \frac{\pi x}{2} \right) \\ &= \lim_{x \rightarrow +\infty} x \left(\operatorname{Arctan}(x) - \frac{\pi}{2} \right) \\ &= \lim_{x \rightarrow +\infty} -x \cdot \operatorname{Arctan}\left(\frac{1}{x}\right) \\ &= \lim_{\substack{t \rightarrow 0^+ \\ t=\frac{1}{x}}} \frac{-\operatorname{Arctan}(t)}{t} = -1 \end{aligned}$$

$$\begin{aligned} 4) \quad & \lim_{x \rightarrow 0} \frac{\operatorname{Arctan}(x^2 + 4x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{\operatorname{Arctan}(x^2 + 4x)}{(x^2 + 4x)} \cdot \left(\frac{x^2 + 4x}{x} \right) \\ &= \lim_{\substack{t \rightarrow 0 \\ t=x^2+4x}} \frac{\operatorname{Arctan}(t)}{t} \times \lim_{x \rightarrow 0} \left(\frac{x^2 + 4x}{x} \right) \\ &= \lim_{t \rightarrow 0} \frac{\operatorname{Arctan}(t)}{t} \times \lim_{x \rightarrow 0} (x + 4) \\ &= 1 \times (0 + 4) = 4 \end{aligned}$$

$$\begin{aligned} 5) \quad & \lim_{x \rightarrow -\infty} (x^2 + 1) \operatorname{Arctan}\left(\frac{1}{x}\right) \\ &= \lim_{x \rightarrow -\infty} \left(\frac{x^2 + 1}{x} \right) \cdot \left(\frac{\operatorname{Arctan}\left(\frac{1}{x}\right)}{\frac{1}{x}} \right) \\ &= \lim_{x \rightarrow -\infty} \left(x + \frac{1}{x} \right) \times \lim_{\substack{t \rightarrow 0^- \\ t=\frac{1}{x}}} \frac{\operatorname{Arctan} t}{t} \\ &= (-\infty + 0) \times (1) = -\infty \end{aligned}$$

$$6) \quad \text{Soit } l = \lim_{x \rightarrow 1^+} \left(\frac{\operatorname{Arctan}\left(\frac{1}{1-x^2}\right) + \frac{\pi}{2}}{x-1} \right)$$

$$\text{Rappel : } \forall x \leq 0 ; \operatorname{Arctan}(x) + \operatorname{Arctan}\left(\frac{1}{x}\right) = \frac{-\pi}{2}$$

$$\begin{aligned} \text{Si } x \rightarrow 1^+ \quad & \text{Alors } x^2 \rightarrow 1^+ \\ & \Rightarrow 1 - x^2 \rightarrow 0^- \\ & \Rightarrow 1 - x^2 < 0 \end{aligned}$$

$$\Rightarrow \operatorname{Arctan}(1 - x^2) + \operatorname{Arctan}\left(\frac{1}{1 - x^2}\right) = \frac{-\pi}{2}$$

$$D'où \quad l = \lim_{x \rightarrow 1^+} \frac{-\operatorname{Arctan}(1 - x^2)}{x - 1}$$

$$= \lim_{x \rightarrow 1^+} \frac{\operatorname{Arctan}(1 - x^2)}{(1 - x^2)} \cdot (1 + x)$$

$$= \lim_{\substack{t \rightarrow 0^- \\ t=1-x^2}} \frac{\operatorname{Arctan}(t)}{t} \cdot (1 + 1)$$

$$= 1 \times (1 + 1) = 2$$

$$7) \quad l = \lim_{x \rightarrow 1^+} \left(\frac{x - 2\sqrt{\operatorname{Arctan}(x) - \frac{\pi}{4}} - 1}{x - 1} \right)$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{x - 1}{x - 1} - 2 \left(\frac{\sqrt{\operatorname{Arctan}(x) - \frac{\pi}{4}}}{x - 1} \right) \right)$$

$$= \lim_{x \rightarrow 1^+} \left(1 - 2 \left(\sqrt{\frac{\operatorname{Arctan}(x) - \frac{\pi}{4}}{x - 1}} \cdot \frac{1}{\sqrt{x - 1}} \right) \right)$$

$$\text{Soit } \alpha = \operatorname{Arctan}(x) - \frac{\pi}{4}$$

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$$\Rightarrow \tan \alpha = \tan \left(\operatorname{Arctan}(x) - \frac{\pi}{4} \right)$$

$$= \frac{\tan(\operatorname{Arctan}(x)) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan(\operatorname{Arctan}(x)) \cdot \tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{x - 1}{1 + x}$$

$$\text{Donc } \alpha = \operatorname{Arctan}\left(\frac{x - 1}{1 + x}\right) = \operatorname{Arctan}(x) - \frac{\pi}{4}$$

$$\text{Ainsi : } \lim_{x \rightarrow 1^+} \left(\frac{\operatorname{Arctan}(x) - \frac{\pi}{4}}{x - 1} \right)$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} \right) \left(\operatorname{Arctan}\left(\frac{x - 1}{x + 1}\right) \right)$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} \right) \left(\frac{x - 1}{x + 1} \right) \cdot \frac{\operatorname{Arctan}\left(\frac{x - 1}{x + 1}\right)}{\left(\frac{x - 1}{x + 1}\right)}$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{1}{x + 1} \right) \cdot \lim_{\substack{t \rightarrow 0^+ \\ t = \frac{x-1}{x+1}}} \left(\frac{\operatorname{Arctan} t}{t} \right)$$

$$= \left(\frac{1}{1 + 1} \right) \cdot 1 = \frac{1}{2}$$

Remarque : plus tard dans la leçon de dérivation on écrira directement que :

$$\lim_{x \rightarrow 1^+} \left(\frac{\operatorname{Arctan}(x) - \frac{\pi}{4}}{x - 1} \right)$$

$$= \lim_{x \rightarrow 1^+} \left(\frac{\operatorname{Arctan}(x) - \operatorname{Arctan}(1)}{x - 1} \right)$$

$$= (\operatorname{Arctan} x)'_{/x=1} = \left(\frac{1}{1 + x^2} \right)_{/x=1} = \frac{1}{2}$$

Revenons maintenant à la limite l qu'on devrait calculer par une méthode légitime :

$$l = \lim_{x \rightarrow 1^+} \left(1 - 2 \sqrt{\frac{\operatorname{Arctan}(x) - \frac{\pi}{4}}{x - 1}} \cdot \frac{1}{\sqrt{x - 1}} \right)$$

$$= 1 - 2 \sqrt{\frac{1}{2}} \cdot \left(\lim_{x \rightarrow 1^+} \frac{1}{\sqrt{x - 1}} \right)$$

$$= 1 - \sqrt{2} \cdot \lim_{\substack{t \rightarrow 0^+ \\ t = \sqrt{x-1}}} \left(\frac{1}{t} \right)$$

$$= 1 - \sqrt{2} \times \left(\frac{1}{0^-} \right) = -\infty$$

$$8) \lim_{x \rightarrow 1^\pm} \frac{\operatorname{Arctan}(\sqrt[3]{x - 1})}{x - 1}$$

$$= \lim_{x \rightarrow 1^\pm} \frac{\operatorname{Arctan}\left((x - 1)^{\frac{1}{3}}\right)}{x - 1}$$

$$= \lim_{x \rightarrow 1^\pm} \frac{\operatorname{Arctan}\left((x - 1)^{\frac{1}{3}}\right)}{(x - 1)^{\frac{1}{3}}} \times \frac{(x - 1)^{\frac{1}{3}}}{(x - 1)^1}$$

$$= \lim_{\substack{t \rightarrow 0^\pm \\ t = (x-1)^{\frac{1}{3}}}} \frac{\operatorname{Arctan}(t)}{t} \times \lim_{x \rightarrow 1^+} (x - 1)^{\frac{1}{3} - 1}$$

$$= \lim_{t \rightarrow 0^\pm} \frac{\operatorname{Arctan}(t)}{t} \times \lim_{x \rightarrow 1^+} \left(\frac{1}{x - 1} \right)^{\frac{2}{3}}$$

$$= \lim_{t \rightarrow 0^\pm} \frac{\operatorname{Arctan}(t)}{t} \times \lim_{t = (x-1)^{2/3}} \frac{1}{t}$$

$$= 1 \times \frac{1}{0^+} = +\infty$$

Solution N° 106 :

Voici d'abord un petit rappel :

■ $\forall x \in \mathbb{R} ; \tan(\text{Arctan}(x)) = x$

■ $\forall x \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[; \text{Arctan}(\tan x) = x$

1) $\text{Arctan}\left(\tan\left(\frac{41\pi}{17}\right)\right)$

$$= \text{Arctan}\left(\tan\left(2\pi + \frac{7\pi}{17}\right)\right)$$

$$= \text{Arctan}\left(\tan\left(\frac{7\pi}{17}\right)\right) = \frac{7\pi}{17} \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[$$

2) $\text{Arctan}\left(\tan(5\text{Arctan}\sqrt{3})\right)$

$$= \text{Arctan}\left(\tan\left(5 \times \frac{\pi}{3}\right)\right)$$

$$= \text{Arctan}\left(\tan\left(\frac{5\pi}{3}\right)\right)$$

$$= \text{Arctan}\left(\tan\left(2\pi - \frac{\pi}{3}\right)\right)$$

$$= \text{Arctan}\left(\tan\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3} \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[$$

3) $\tan(\text{Arctan}(2016)) = 2016 \in \mathbb{R}$

4) $\tan(-\text{Arctan}(5)) = -\tan(\text{Arctan}(5))$
 $= -5 \in \mathbb{R}$

5) $\text{Arctan}\left(\tan\left(-\frac{79\pi}{3}\right)\right)$

$$= \text{Arctan}\left(\tan\left(-26\pi - \frac{\pi}{3}\right)\right)$$

$$= \text{Arctan}\left(\tan\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3} \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[$$

6) $\text{Arctan}\left(\frac{1}{\tan\left(\frac{3\pi}{11}\right)}\right)$

$$= \text{Arctan}\left(\frac{\cos\left(\frac{3\pi}{11}\right)}{\sin\left(\frac{3\pi}{11}\right)}\right)$$

$$= \text{Arctan}\left(\frac{\sin\left(\frac{\pi}{2} - \frac{3\pi}{11}\right)}{\cos\left(\frac{\pi}{2} - \frac{3\pi}{11}\right)}\right)$$

$$= \text{Arctan}\left(\frac{\sin\left(\frac{5\pi}{11}\right)}{\cos\left(\frac{5\pi}{11}\right)}\right)$$

$$= \text{Arctan}\left(\tan\left(\frac{5\pi}{11}\right)\right) = \frac{5\pi}{11} \in \left] -\frac{\pi}{2} ; \frac{\pi}{2} \right[$$

7) Soit $\alpha = \text{Arctan}\left(\frac{2}{3}\right) - \text{Arctan}\left(\frac{3}{7}\right)$

$$\tan(\alpha) = \tan\left(\text{Arctan}\left(\frac{2}{3}\right) - \text{Arctan}\left(\frac{3}{7}\right)\right)$$

$$= \frac{\tan\left(\text{Arctan}\left(\frac{2}{3}\right)\right) - \tan\left(\text{Arctan}\left(\frac{3}{7}\right)\right)}{1 + \tan\left(\text{Arctan}\left(\frac{2}{3}\right)\right) \cdot \tan\left(\text{Arctan}\left(\frac{3}{7}\right)\right)}$$

$$= \frac{\frac{2}{3} - \frac{3}{7}}{1 + \frac{2}{7}} = \frac{5}{27}$$

Ainsi : $\alpha = \text{Arctan}\left(\frac{5}{27}\right)$

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8) $\tan(2\text{Arctan}(3))$

$$= \tan(\text{Arctan}(3) + \text{Arctan}(3))$$

$$= \frac{\tan(\text{Arctan}(3)) + \tan(\text{Arctan}(3))}{1 - \tan(\text{Arctan}(3))\tan(\text{Arctan}(3))}$$

$$= \frac{3 + 3}{1 - 9} = \frac{-3}{4} \in \mathbb{R}$$

Solution N° 107 :

1) $\lim_{x \rightarrow 1} \left(\frac{\sqrt[3]{x+7} - 2}{\sqrt[4]{x} - 1} \right)$

$$= \lim_{x \rightarrow 1} \frac{(\sqrt[3]{x+7} - 2)(\sqrt[4]{x} + 1)}{(\sqrt[4]{x} - 1)(\sqrt[4]{x} + 1)}$$

$$= \lim_{x \rightarrow 1} \left(\frac{(x+7)^{\frac{1}{3}} - 2}{x^{\frac{1}{4}} - 1} \right) \times (\sqrt[4]{1} + 1)$$

$$= 2 \lim_{x \rightarrow 1} \left(\frac{(x+7)^{\frac{1}{3}} - 2}{\left(x^{\frac{1}{4}}\right)^2 - 1^2} \right) \times (x^{\frac{1}{4}} + 1)$$

$$= 2 \lim_{x \rightarrow 1} \left(\frac{(x+7)^{\frac{1}{3}} - 2}{x - 1} \right) \times \left(1^{\frac{1}{4}} + 1\right)$$

$$= 4 \lim_{x \rightarrow 1} \left(\frac{(x+7)^{\frac{1}{3}} - 2}{x - 1} \right) \times \frac{(x+7)^{\frac{2}{3}} + 2(x+7)^{\frac{1}{3}} + 4}{(x+7)^{\frac{2}{3}} + 2(x+7)^{\frac{1}{3}} + 4}$$

$$= 4 \lim_{x \rightarrow 1} \frac{\left((x+7)^{\frac{1}{3}}\right)^3 - 2^3}{(x-1)\left((x+7)^{\frac{2}{3}} + 2(x+7)^{\frac{1}{3}} + 4\right)}$$

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$$= 4 \lim_{x \rightarrow 1} \left(\frac{1}{(x+7)^{\frac{2}{3}} + 2(x+7)^{\frac{1}{3}} + 4} \right)$$

$$= 4 \times \left(\frac{1}{(1+7)^{\frac{2}{3}} + 2(1+7)^{\frac{1}{3}} + 4} \right) = \frac{1}{3}$$

2) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1} - 2x \right)$

$$= \lim_{x \rightarrow +\infty} (x^3 + x^2 + 1)^{\frac{1}{3}} - 2x$$

$$= \lim_{x \rightarrow +\infty} \left(x^3 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right) \right)^{\frac{1}{3}} - 2x$$

$$= \lim_{x \rightarrow +\infty} (x^3)^{\frac{1}{3}} \cdot \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} - 2x$$

$$= \lim_{x \rightarrow +\infty} x \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} - 2x$$

$$= \lim_{x \rightarrow +\infty} x \left(\left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} - 2 \right)$$

$$= (+\infty) \left((1 + 0 + 0)^{\frac{1}{3}} - 2 \right) = -\infty$$

3) $\lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^2 - x} - x - 1 \right)$

$$= \lim_{x \rightarrow +\infty} (x^2 - x)^{\frac{1}{3}} - x - 1$$

$$= \lim_{x \rightarrow +\infty} \left(x^3 \left(\frac{1}{x} - \frac{1}{x^3} \right) \right)^{\frac{1}{3}} - x - 1$$

$$= \lim_{x \rightarrow +\infty} x \left(\frac{1}{x} - \frac{1}{x^3} \right)^{\frac{1}{3}} - x - 1$$

$$= \lim_{x \rightarrow +\infty} x \left(\left(\frac{1}{x} - \frac{1}{x^3} \right)^{\frac{1}{3}} - 1 - \frac{1}{x} \right)$$

$$= (+\infty) \left((0 - 0)^{\frac{1}{3}} - 1 - 0 \right) = -\infty$$

$$4) \lim_{x \rightarrow 4} \frac{(5-x)^{\frac{1}{3}} - 1}{2 - (x+4)^{\frac{1}{3}}}$$

$$= \lim_{x \rightarrow 4} \frac{\left((5-x)^{\frac{1}{3}} \right)^3 - 1^3}{2^3 - \left((x+4)^{\frac{1}{3}} \right)^3} \times \frac{2^2 + 2(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}{(5-x)^{\frac{2}{3}} + (5-x)^{\frac{1}{3}} + 1}$$

$$= \lim_{x \rightarrow 4} \left(\frac{4-x}{4-x} \right) \times \frac{2^2 + 2(8)^{\frac{1}{3}} + (8)^{\frac{2}{3}}}{(5-4)^{\frac{2}{3}} + (5-4)^{\frac{1}{3}} + 1}$$

$$= 1 \times \frac{4+4+4}{(1)^{\frac{2}{3}} + (1)^{\frac{1}{3}} + 1} = 4$$

$$5) \lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left((x^3 + x^2 + 1)^{\frac{1}{3}} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{\left((x^3 + x^2 + 1)^{\frac{1}{3}} \right)^3 - x^3}{(x^3 + x^2 + 1)^{\frac{2}{3}} + x(x^3 + x^2 + 1)^{\frac{1}{3}} + x^2}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 + 1}{\left(x^3 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right) \right)^{\frac{2}{3}} + x \left(x^3 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right) \right)^{\frac{1}{3}} + x^2}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 + 1}{x^2 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + x^2 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + x^2}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 \left(1 + \frac{1}{x^2} \right)}{x^2 \left(\left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1 + \frac{1}{x^2}}{\left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + 1} \right)$$

$$= \left(\frac{1 + 0}{(1 + 0 + 0)^{\frac{2}{3}} + (1 + 0 + 0)^{\frac{1}{3}} + 1} \right) = \frac{1}{3}$$

$$6) \lim_{x \rightarrow -\infty} \left(\sqrt[3]{x^4 + 5} + 2x \right)$$

$$= \lim_{x \rightarrow -\infty} (x^4 + 5)^{\frac{1}{3}} + 2x$$

$$= \lim_{\substack{t \rightarrow +\infty \\ t = -x}} (t^4 + 5)^{\frac{1}{3}} - 2t$$

$$= \lim_{t \rightarrow +\infty} \left(t^4 \left(1 + \frac{5}{t^4} \right) \right)^{\frac{1}{3}} - 2t$$

$$= \lim_{t \rightarrow +\infty} t^{\frac{4}{3}} \left(1 + \frac{5}{t^4} \right)^{\frac{1}{3}} - 2t$$

$$= \lim_{t \rightarrow +\infty} \left(t^{\frac{4}{3}} \right) \left(\left(1 + \frac{5}{t^4} \right)^{\frac{1}{3}} - \frac{2}{t^{\frac{1}{3}}} \right)$$

$$= (+\infty) \left((1 + 0)^{\frac{1}{3}} - 0 \right) = +\infty$$

Solution N° 108 :

En utilisant les puissances rationnelles, l'équation (E) devient :

$$x^{\frac{2}{3}} - 3(x^2 - x)^{\frac{1}{3}} + 2(x - 1)^{\frac{2}{3}} = 0$$

D'abord l'équation est définie si :

$$x \in \mathbb{R}^+ ; (x^2 - x) \in \mathbb{R} ; (x - 1) \in \mathbb{R}^+$$

$$\text{c-à-d} : x \geq 1$$

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$$(E) \Leftrightarrow x^{\frac{2}{3}} - 3(x^2(1-x^{-1}))^{\frac{1}{3}} + 2(x(1-x^{-1}))^{\frac{2}{3}} = 0$$

$$\Leftrightarrow x^{\frac{2}{3}} - 3(x^2)^{\frac{1}{3}} \left(1 - \frac{1}{x}\right)^{\frac{1}{3}} + 2x^{\frac{2}{3}} \left(1 - \frac{1}{x}\right)^{\frac{2}{3}} = 0$$

$$\Leftrightarrow x^{\frac{2}{3}} - 3x^{\frac{2}{3}} \left(1 - \frac{1}{x}\right)^{\frac{1}{3}} + 2x^{\frac{2}{3}} \left(1 - \frac{1}{x}\right)^{\frac{2}{3}} = 0$$

$$\Leftrightarrow x^{\frac{2}{3}} \left(1 - 3\left(1 - \frac{1}{x}\right)^{\frac{1}{3}} + 2\left(1 - \frac{1}{x}\right)^{\frac{2}{3}}\right) = 0$$

$$\Leftrightarrow \left(1 - 3\left(1 - \frac{1}{x}\right)^{\frac{1}{3}} + 2\left(1 - \frac{1}{x}\right)^{\frac{2}{3}}\right) = 0$$

$$\Leftrightarrow 1 - 3t + 2t^2 = 0 \quad ; \quad t = \left(1 - \frac{1}{x}\right)^{\frac{1}{3}}$$

$$\Leftrightarrow t = \frac{3 \pm \sqrt{1}}{4} \quad ; \quad \Delta = 1$$

$$\Leftrightarrow t \in \left\{1; \frac{1}{2}\right\}$$

$$\Leftrightarrow t = \frac{1}{2} \quad \text{sinon} \quad \begin{cases} \text{si } t = 1 \\ \text{alors } \frac{-1}{x} = 0 \\ \text{(contradiction)} \end{cases}$$

$$\Leftrightarrow \left(1 - \frac{1}{x}\right)^{\frac{1}{3}} = \frac{1}{2}$$

$$\Leftrightarrow 1 - \frac{1}{x} = \frac{1}{8} \quad \Leftrightarrow x = \frac{8}{7}$$

■ un passage aux puissances rationnelles nous permet d'écrire :

$$(F) : (1+x)^{\frac{2}{3}} - 4(1-x)^{\frac{2}{3}} - 4(1-x^2)^{\frac{1}{3}} = 0$$

Cette équation est définie si on aurait :

$$1+x \geq 0 \quad ; \quad 1-x \geq 0 \quad ; \quad (1-x^2) \in \mathbb{R}$$

c-à-d on devrait avoir $x \in [-1; 1]$

$$(1+x)^{\frac{2}{3}} - 4(1-x)^{\frac{2}{3}} - 4(1-x)^{\frac{1}{3}}(1+x)^{\frac{1}{3}} = 0$$

$$\Leftrightarrow (1+x)^{\frac{2}{3}} \left(1 - \frac{(1-x)^{\frac{2}{3}}}{(1+x)^{\frac{2}{3}}} - \frac{4(1-x)^{\frac{1}{3}}(1+x)^{\frac{1}{3}}}{(1+x)^{\frac{2}{3}}}\right) = 0$$

$$\Leftrightarrow (1+x)^{\frac{2}{3}} \left(1 - \left(\frac{1-x}{1+x}\right)^{\frac{2}{3}} - 4\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}\right) = 0$$

$$\Leftrightarrow \left(1 - \left(\frac{1-x}{1+x}\right)^{\frac{2}{3}} - 4\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}\right) = 0$$

$$\Leftrightarrow 1 - t^2 - 4t = 0 \quad ; \quad t = \left(\frac{1-x}{1+x}\right)^{\frac{1}{3}} \in \mathbb{R}$$

$$\Leftrightarrow t = \frac{-4 \pm 2\sqrt{5}}{2} \quad ; \quad \Delta = 20$$

$$\Leftrightarrow \begin{cases} \text{oubien } \left(\frac{1-x}{1+x}\right)^{\frac{1}{3}} = -(2+\sqrt{5}) \\ \text{oubien } \left(\frac{1-x}{1+x}\right)^{\frac{1}{3}} = -2+\sqrt{5} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{oubien } \frac{1-x}{1+x} = -(2+\sqrt{5})^3 \\ \text{oubien } \frac{1-x}{1+x} = (-2+\sqrt{5})^3 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{oubien } \frac{1-x}{1+x} = -38 - 17\sqrt{5} \\ \text{oubien } \frac{1-x}{1+x} = 17\sqrt{5} - 38 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{oubien} & x = \frac{-38 - 17\sqrt{5}}{37 - 17\sqrt{5}} > 1 \\ \text{oubien} & x = \frac{39 - 17\sqrt{5}}{17\sqrt{5} - 37} \in [-1; 1] \end{cases}$$

$$\Leftrightarrow \text{Solutions}(F) = \left\{ \frac{39 - 17\sqrt{5}}{17\sqrt{5} - 37} \right\}$$

$$2) \lim_{x \rightarrow +\infty} x^{\frac{3}{2}} (\sqrt[4]{x+1} - \sqrt[4]{x-1})$$

$$= \lim_{x \rightarrow +\infty} x^{\frac{3}{2}} \left((x+1)^{\frac{1}{4}} - (x-1)^{\frac{1}{4}} \right)$$

$$= \lim_{x \rightarrow +\infty} x^{\frac{3}{2}} x^{\frac{1}{4}} \left(\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} - \left(1 - \frac{1}{x}\right)^{\frac{1}{4}} \right)$$

$$= \lim_{x \rightarrow +\infty} x^{\frac{7}{4}} \left(\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} - \left(1 - \frac{1}{x}\right)^{\frac{1}{4}} \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x^{\frac{7}{4}} \left(\left(\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} \right)^2 - \left(\left(1 - \frac{1}{x}\right)^{\frac{1}{4}} \right)^2 \right)}{\left(1 + \frac{1}{x}\right)^{\frac{1}{4}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{4}}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x^{\frac{7}{4}} \left(\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} - \left(1 - \frac{1}{x}\right)^{\frac{1}{2}} \right)}{(1+0)^{\frac{1}{4}} + (1-0)^{\frac{1}{4}}}$$

$$= \frac{1}{2} \lim_{x \rightarrow +\infty} x^{\frac{7}{4}} \left(\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} - \left(1 - \frac{1}{x}\right)^{\frac{1}{2}} \right) \times \frac{\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}}}{\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}}}$$

$$= \frac{1}{2} \lim_{x \rightarrow +\infty} \frac{x^{\frac{7}{4}} \left(\left(\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} \right)^2 - \left(\left(1 - \frac{1}{x}\right)^{\frac{1}{2}} \right)^2 \right)}{\left(1 + \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}}}$$

$$= \frac{1}{2} \lim_{x \rightarrow +\infty} \frac{x^{\frac{7}{4}} \left(\frac{2}{x} \right)}{(1+0)^{\frac{1}{2}} + (1-0)^{\frac{1}{2}}}$$

$$= \frac{1}{4} \lim_{x \rightarrow +\infty} x^{\frac{7}{4}} \left(\frac{2}{x} \right) = \frac{1}{2} \lim_{x \rightarrow +\infty} \left(x^{\frac{3}{4}} \right) = +\infty$$

■ d'abord on doit calculer la limite :

$$\lim_{x \rightarrow +\infty} \left(\sqrt[4]{x^4 + x} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left((x^4 + x)^{\frac{1}{4}} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left((x^4)^{\frac{1}{4}} \left(1 + \frac{1}{x^3} \right)^{\frac{1}{4}} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left(x \left(1 + \frac{1}{x^3} \right)^{\frac{1}{4}} - x \right)$$

$$= \lim_{x \rightarrow +\infty} x \left(\left(1 + \frac{1}{x^3} \right)^{\frac{1}{4}} - 1 \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(\left(\left(1 + \frac{1}{x^3} \right)^{\frac{1}{4}} \right)^2 - 1^2 \right)}{\left(1 + \frac{1}{x^3} \right)^{\frac{1}{4}} + 1}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} - 1 \right)}{(1+0)^{\frac{1}{4}} + 1}$$

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$$\begin{aligned}
&= \frac{1}{2} \lim_{x \rightarrow +\infty} x \left(\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} - 1 \right) \\
&= \frac{1}{2} \lim_{x \rightarrow +\infty} \frac{x \left(\left(\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} \right)^2 - 1^2 \right)}{\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} + 1} \\
&= \frac{1}{2} \lim_{x \rightarrow +\infty} \frac{x \left(\frac{1}{x^3} \right)}{\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} + 1} \\
&= \frac{1}{2} \lim_{x \rightarrow +\infty} \left(\frac{\frac{1}{x^2}}{\left(1 + \frac{1}{x^3} \right)^{\frac{1}{2}} + 1} \right) \\
&= \frac{1}{2} \left(\frac{0}{(1+0)^{\frac{1}{2}} + 1} \right) = 0 \\
&\Rightarrow \lim_{x \rightarrow +\infty} \left(\sqrt[4]{x^4 + x} - x \right) = 0 \\
&\Rightarrow \lim_{x \rightarrow +\infty} \left(\sqrt[4]{x^4 + x} - x - 2 \right) = -2
\end{aligned}$$

3) Soit $l = \lim_{x \rightarrow +\infty} \left(\sqrt[3]{x^3 + x^2 + 1 + mx} \right)$

si $m \geq 0$ alors $l = (+\infty) + m(+\infty) = +\infty$

Si $m < 0$ alors $n = -m > 0$

D'où : $l = \lim_{x \rightarrow +\infty} (x^3 + x^2 + 1)^{\frac{1}{3}} - nx$

$$= \lim_{x \rightarrow +\infty} \frac{\left((x^3 + x^2 + 1)^{\frac{1}{3}} \right)^3 - (nx)^3}{(x^3 + x^2 + 1)^{\frac{2}{3}} + nx(x^3 + x^2 + 1)^{\frac{1}{3}} + n^2 x^2}$$

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$$\begin{aligned}
&= \lim_{x \rightarrow +\infty} \frac{(1 - n^3)x^3 + x^2 + 1}{x^2 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + nx^2 \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + n^2 x^2} \\
&= \lim_{x \rightarrow +\infty} \frac{x^2 \left((1 - n^3)x + 1 + \frac{1}{x^2} \right)}{x^2 \left(\left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + n \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + n^2 \right)}
\end{aligned}$$

$$= \lim_{x \rightarrow +\infty} \frac{(1 - n^3)x + 1 + \frac{1}{x^2}}{\left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{2}{3}} + n \left(1 + \frac{1}{x} + \frac{1}{x^3} \right)^{\frac{1}{3}} + n^2}$$

Si $(1 - n^3) = 0$ alors $l = \frac{1}{1 + n + n^2} = \frac{1}{3}$

Si $(1 - n^3) > 0$ alors $l = \frac{+\infty}{1 + n + n^2}$

Si $(1 - n^3) < 0$ alors $l = \frac{-\infty + 1 + 0}{1 + n + n^2}$

Conclusion : $l = \begin{cases} -\infty & \text{si } m < -1 \\ \frac{1}{3} & \text{si } m = -1 \\ +\infty & \text{si } m > -1 \end{cases}$

Solution N° 109 :

1) soit $x \geq -1$ une solution de l'équation :

$$(E) : \sqrt{x+1} - \sqrt[3]{x} = 1$$

$$\Leftrightarrow (x+1)^{\frac{1}{2}} - x^{\frac{1}{3}} = 1 \quad ; \quad x \geq -1$$

$$\Leftrightarrow (x+1)^{\frac{1}{2}} = x^{\frac{1}{3}} + 1 \quad ; \quad x \geq -1$$

$$\Leftrightarrow \left((x+1)^{\frac{1}{2}} \right)^2 = \left(x^{\frac{1}{3}} + 1 \right)^2 \quad ; \quad x \geq -1$$

$$\Leftrightarrow x+1 = x^{\frac{2}{3}} + 2x^{\frac{1}{3}} + 1 \quad ; \quad x \geq -1$$

$$\Leftrightarrow x - x^{\frac{2}{3}} - 2x^{\frac{1}{3}} = 0 \quad ; \quad x \geq -1$$

$$\Leftrightarrow t^3 - t^2 - 2t = 0 \quad ; \quad t = x^{\frac{1}{3}} \in \mathbb{R}$$

$$\Leftrightarrow t(t-2)(t+1) = 0 \quad ; \quad t \geq -1$$

$$\Leftrightarrow \begin{cases} \text{oubien } t = 0 \\ \text{oubien } t = 2 \\ \text{oubien } t = -1 \end{cases} \quad ; \quad t \geq -1$$

$$\Leftrightarrow \begin{cases} \text{oubien } x^{\frac{1}{3}} = 0 \\ \text{oubien } x^{\frac{1}{3}} = 2 \\ \text{oubien } x^{\frac{1}{3}} = -1 \end{cases} \quad ; \quad x \geq -1$$

$$\Leftrightarrow \begin{cases} \text{oubien } x = 0 \\ \text{oubien } x = 8 \\ \text{oubien } x = -1 \end{cases} \quad ; \quad x \geq -1$$

$$\Leftrightarrow x \in \{-1 ; 0 ; 8\} \quad ; \quad x \geq -1$$

Remarque : On peut très facilement vérifier que chaque éventualité de l'ensemble $\{-1 ; 0 ; 8\}$ vérifie bien (E)

$$\text{Donc } \text{solution}(E) = \{-1 ; 0 ; 8\}$$

2) On considère la fonction φ définie par : $\forall x \in [0,2] ; \varphi(x) = h(x+1) - h(x)$
On a φ est continue sur $[0,2]$ car h et x et $x+1$ sont continues sur $[0,2]$ et que la composition $h(x+1)$ est bien définie. Donc φ est continue sur $[0,1] \subset [0,2]$. Or on a :

$$\begin{aligned} \varphi(0) \cdot \varphi(1) &= (h(1) - h(0))(h(2) - h(1)) \\ &= (h(1) - h(0))(h(0) - h(1)) \\ &= -(h(1) - h(0))^2 \leq 0 \end{aligned}$$

$$\text{Ainsi} : \begin{cases} \varphi \text{ continue sur } [0,1] \\ \varphi(0) \times \varphi(1) \leq 0 \end{cases}$$

$$\Rightarrow \exists \alpha \in [0,1] \quad ; \quad \varphi(\alpha) = 0 \quad ; \quad (TVI)$$

$$\Rightarrow \exists \alpha \in [0,1] \quad ; \quad h(\alpha+1) = h(\alpha)$$

$$3) \lim_{x \rightarrow +\infty} x \left(\sqrt[3]{1 + \frac{1}{x}} - 1 \right)$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(\sqrt[3]{1 + \frac{1}{x}} - 1 \right) \left(\left(1 + \frac{1}{x}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x}\right)^{\frac{1}{3}} + 1 \right)}{\left(\left(1 + \frac{1}{x}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x}\right)^{\frac{1}{3}} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(\left(\sqrt[3]{1 + \frac{1}{x}} \right)^3 - 1^3 \right)}{\left(\left(1 + \frac{1}{x}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x}\right)^{\frac{1}{3}} + 1 \right)}$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x \cdot \frac{1}{x}}{\left(1 + \frac{1}{x}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x}\right)^{\frac{1}{3}} + 1} \right)$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{1}{\left(1 + \frac{1}{x}\right)^{\frac{2}{3}} + \left(1 + \frac{1}{x}\right)^{\frac{1}{3}} + 1} \right)$$

$$= \left(\frac{1}{(1+0)^{\frac{2}{3}} + (1+0)^{\frac{1}{3}} + 1} \right) = \frac{1}{3}$$

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Solution N° 110 :

Voici les résultats de mes calculs :

$$1) \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\tan^2(2x)} \right) = \frac{1}{8}$$

$$2) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2 + x + 1} - 1}{4x} \right) = \frac{1}{8}$$

$$3) \lim_{x \rightarrow 0} \left(\frac{\cos x - \sqrt{1 + \sin x}}{x} \right) = \frac{-1}{2}$$

$$4) \lim_{x \rightarrow \frac{2}{\pi}} \left(\frac{\pi x - 2}{1 - \sin\left(\frac{1}{x}\right)} \right) = -\infty$$

$$5) \lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{1 - 2 \cos x}{\pi - 3x} \right) = \frac{-\sqrt{3}}{3}$$

$$6) \lim_{x \rightarrow 0} \left(\frac{2}{\sin^2 x} - \frac{1}{1 - \cos x} \right) = \frac{1}{2}$$

$$7) \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{1 + \sin x} - \cos x}{\sqrt{x}(2x - \pi)} \right) = 0$$

$$8) \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + 3x + 2} + x + 1 \right) = \frac{-1}{2}$$

$$9) \lim_{x \rightarrow 2} \left(\frac{x^2 + x - 6}{\sqrt{x} - \sqrt{2}} \right) = 10\sqrt{2}$$

$$10) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{3x^2 + 1} - \sqrt{x^2 + x}}{x} \right) = \sqrt{3} - 1$$

$$11) \lim_{x \rightarrow 0} \left(\frac{\sqrt{x + 4} - 2}{x - x^2} \right) = \frac{1}{4}$$

$$12) \lim_{x \rightarrow +\infty} \left(\sqrt{\frac{x^3}{x + 1}} - x \right) = \frac{-1}{2}$$

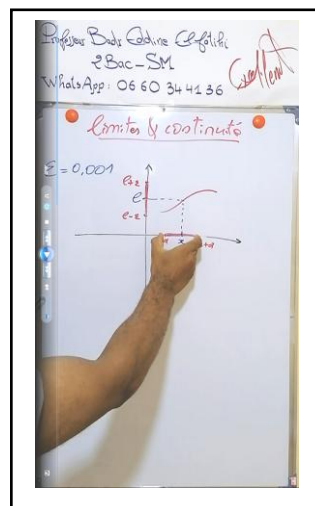
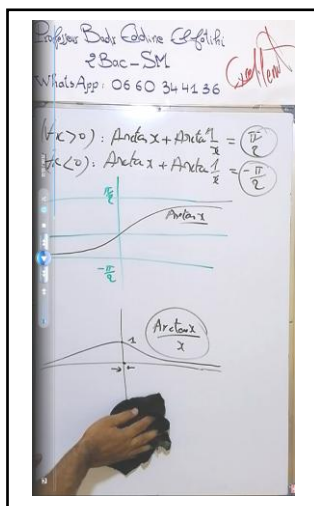
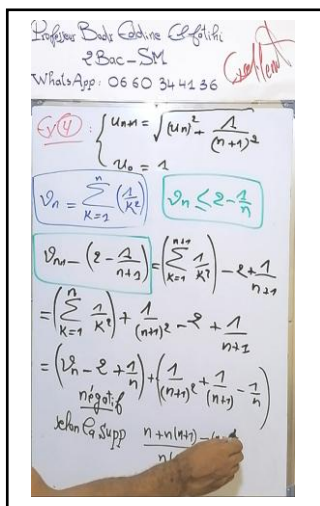
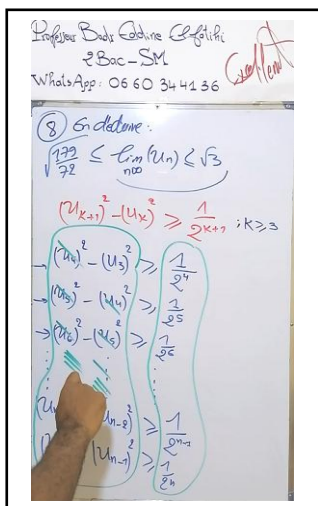
عروض السنة الدراسية : 2023/2022

عرض الدروس :

- 12 ساعة فيديوهات لشرح كل درس بالتفاصيل و بعض التمارين المحولة
- يرسل كل درس في بطاقة ذاكرة من فئة 32Go أو 64Go
- بإمكانكم اقتناء فقط الدروس التي تمثل عائق في البرنامج
- بإمكان التلاميذ المترشحين الأحرار و الرسميون الاستفادة من العرض
- ينصح بمشاهدة كل حصة مرتين على الأقل لضبط المفاهيم جيدا

عرض الإمتحانات التجريبية المصححة :

- يتكون من 12 امتحان تجريبي متوسط الصعوبة كلها مصححة بالتفاصيل
- يرسل العرض في بطاقة ذاكرة من فئة 64Go (12 امتحان في بطاقة واحدة)
- الشروحات صالحة للتحضير للمباريات الوطنية (20 ساعة شرح تقريبا)
- الإرسال يتم عبر البريد (خدمة أمانة المغرب)



Radio - SM

12/07/2022

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Groupe WhatsApp pour discussions

2BAC SM : Biof 2022/2023

leçon	support	Durée	Prix
Limites et continuité	Carte mémoire 32Go	12h cours et exercices	300 DH
Suites numériques	Carte mémoire 32Go	12h cours et exercices	300 DH
Dérivation, étude de fonctions	Carte mémoire 32Go	12h cours et exercices	300 DH
Logarithme népérien	Carte mémoire 32Go	12h cours et exercices	300 DH
L'exponentielle	Carte mémoire 32Go	12h cours et exercices	300 DH
Le calcul d'intégrales	Carte mémoire 32Go	12h cours et exercices	300 DH
Nombres complexes	Carte mémoire 64Go	14h cours et exercices	300 DH
Arithmétiques	Carte mémoire 32Go	12h cours et exercices	300 DH
Structures algébriques	Carte mémoire 64Go	14h cours et exercices	300 DH
Équations différentielles	Carte mémoire 32Go	12h cours et exercices	300 DH
Calcul de probabilités	Carte mémoire 32Go	12h cours et exercices	300 DH
12 Examens Blancs Corrigés	Carte mémoire 64Go	20h Examens corrigés	500 DH

التوصيل سوف يتم عن طريق خدمة AMANA للإرساليات (بريد المغرب)



SÉRIES D'EXERCICES

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Projet de livre 2022-2023

Tome 2 : Suites Numériques

- **Monotonie d'une suite numérique**
- **Suites arithmétiques et géométriques**
- **Limite d'une suite numérique**
- **Critères de convergence**
- **Suites récurrentes**
- **Démontrer une proposition par récurrence**
- **Suites adjacentes**
- **Sommes finies et infinies**

Professeur Badr Eddine EL FATIHI

Ouarzazate 2022

Pour Octobre 2022