CHAPTER 1

THE MATHEMATICS TOOL BOX

1.1 INTRODUCTION

To have a common understanding for the mathematics used in the stability course, it is considered beneficial to refresh some theory that will be used in the following chapters of this manual.

1.2 STANDARD UNITS

THE METRIC AND CONVENTIONAL SYSTEM

A unit is a standard measure of quantity such as length, mass, energy etc. Compared to the SI (Système International d'Unités) or metric system the conventional system (also called Imperial or US system) is more complicated because several units are used for each quantity. For example, measurements of length can be expressed in miles, yard, feet and inches.

In the SI or metric system only one unit is used for each basic quantity like the metre is the basic unit of length and the kilogram is the basic unit of weight.
For stability calculations both systems are used in the offshore drilling industry

The basic units and derived quantities for each system are shown in Table 1.1.

Fundamental Quantities and Derived Quantities	Conventional System Unit	Symbol	Metric (SI) System Unit	Symbol
Acceleration	feet per sec.per sec.	ft/sec^2	metre per sec. per sec.	m/sec^2
Area	square feet	ft^2	square metre	m^2
Density	pound per cubic ft	lb/ft^3	kilogram per cubic m	kg/m^3
Force	pound force	Lb_f	Newton	N
Frequency	hertz	Hz	hertz	Hz
Length	foot	Ft	metre	m
Mass	pound	lb.	kilogram	kg
Power	foot-pound per sec. horse power watt	ft-lb./sec hp W	joule per second kilogram-metre per sec watt	J/s kg.m/sec W
Pressure	pound per square inch	psi	Pascal (N/m^2) kilogram/square cm	Pa kg/cm^2
Temperature	degree Fahrenheit	ºF	degree Celsius	ºC
Weight	short tons (2000 lb.)	St	tonne (1000 kg)	t
Velocity	foot per second	ft/sec	metre per second	m/s
Volume	cubic foot gallon barrel	cu.ft or ft^3 gal bbl	cubic meter	m^3
Work	foot-pound	ft-lb.	joule Newton-metre	J N-m

Table 1.1

One of the most important prerequisites to remember when calculating is to keep track of the units you are using for the data values.

If you calculate in SI units, e.g. m (meter), T (ton) and m^3 (cubic meter), then all data has to be converted to these units to make sense. The same goes for calculations in conventional units.

Therefore, it is handy to know how to convert the unit values when working in the offshore industry, where many MOU's are having documentation in conventional units, but receive cargo information in SI units or vice versa.

1.3 QUANTITY AND MEASUREMENTS FOR STABILITY CALCULATIONS

LENGTH

The basic unit of length is the metre or foot.
1 metre (m) = 10 decimetre (dm) = 100 centimetre (cm) = 1000 millimetre.
1 foot (ft) = 12 inches (in).
1 metre = 3.281 ft =39.37 in.
1 yard = 3 ft = 0.9144 m.
1 ft = 0.305 m.

AREA

For metric units in most cases the area is measured in m^2.
For conventional units the area is measured in ft^2 or in^2.
1 m^2 = 10,000 cm^2 = 10.76 ft^2.
1 $yard^2$ = 9 ft^2 = 0.836 m^2.
1 ft^2 = 144 in^2 = 0.093 m^2.

VOLUME

Volume is measured in m^3 or in ft^3.
1 m^3 =10^6 cm^3
1 m^3 = 35.31 ft^3 = 264.12 gallon = 6.29 bbl
1 litre = 1000 cm^3 = 0.264 gallon
1 ft^3 = 1728 in^3 = 0.028 m^3
1 gallon (US) = 0.0038 m^3 = 0.134 ft^3.
1 barrel (bbl) = 0.159 m^3 = 5.61 ft^3 = 42 gallon.

WEIGHT/ MASS

The basic units for weight are the kilogram (kg) and pounds (lbs.).
1 kg = 1000 gram.
1 kg = 2.2046 lbs.
1 litre of fresh water weights 1 kg.
1 ft^3 of fresh water = 62.5 lbs. versus 1 ft^3 of seawater = 64 lbs.
1 lb. = 16 ounces (oz) = 0.454 kg.
1 metric tonne (t) = 1000 kilograms.
1 m^3 fresh water = the weight of 1000 kg.
1 metric tonne = 2204.6 lbs. = 1.102 st.
1 short ton (st) = 2000 lbs. = 0.9072 metric tonne.
1 long ton = 2240 lbs.= 1016 kg.

FORCE

1 Newton (N) = 1 kg.m/sec^2 = 0.2248 pound-force (lb_f).
1 pound-force (lb_f) = lb_m *32.15 ft/s^2 = 4.448 Newton.

MAERSK TRAINING

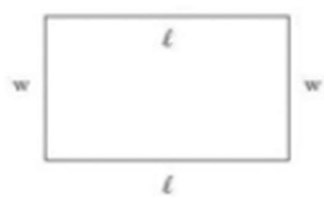
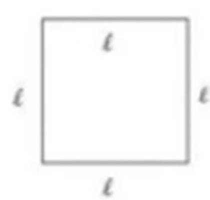
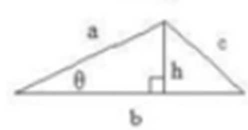
DEFINITION
GRAVITY, WEIGHT AND MASS
Gravity is the force that tends to draw all bodies towards the centre of earth.
Mass is the physical quantity of matter in a body.
Weight is the vertical force experienced by a mass as a result of the gravity force.
Weight can be considered to be proportional to mass at the earth surface.

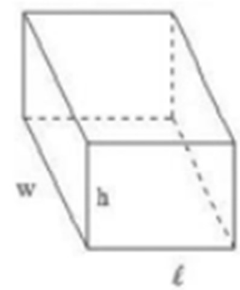
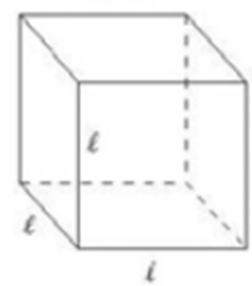
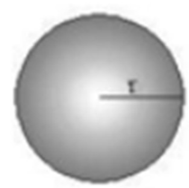
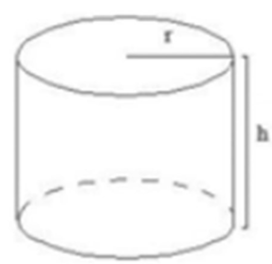
FORCE
Force is the push or pull on a body. It is the cause of the motion of a body. By definition force is the product of mass and acceleration.
In the metric system the unit of force is the **Newton (N)**. One Newton is the force required to accelerate a mass of 1 kilogram at a metre per second squared (kg.m/s²).
The unit of force for the conventional system is the **pound force** that is the force required to accelerate a mass of 1 pound at 32.15 feet per second squared (32.15 ft/ s²).

1.4 GEOMETRICAL SHAPES

Most items when calculating stability has a reference to the volume and weight of items. It is therefore important to be able to recognise a geometrical shape to calculate the area, the volume and the geometrical centre of such a shape.

The standard shapes we refer to are:

2-D Shape	Perimeter	Area	Sketch
Rectange	$2(l+w)$	lw	
Square	$4l$	l^2	
Circle	$2\pi r$	πr^2	
Triangle	$a+b+c$	$\frac{1}{2}bh$ or $\frac{1}{2}ab\sin\theta$	

3-D Shape	Surface Area	Volume	Sketch
Rectangular Prism	$2(lw + lh + wh)$	lwh	
Cube	$6l^2$	l^3	
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$	
Cylinder	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$	

1.5 COORDINATE SYSTEM

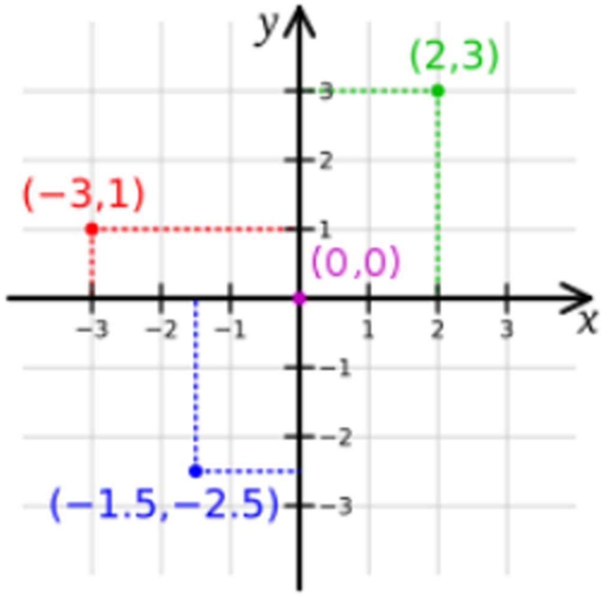


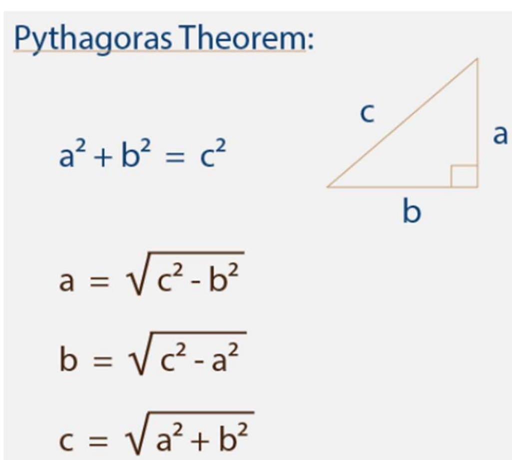
Fig. 1.2 Coordinate System

Illustration of a Cartesian Coordinate plane. Four points are marked and labelled with their coordinates: (2, 3) in green, (-3, 1) in red, (-1.5, -2.5) in blue, and the origin (0, 0) in purple.

1.6 PYTHAGORAS

The trigonometry used in the stability calculations are for the majority relating to the right triangle, where one of the three angles is 90 degrees.

To solve the relation between the side lengths in a right triangle Pythagoras found:



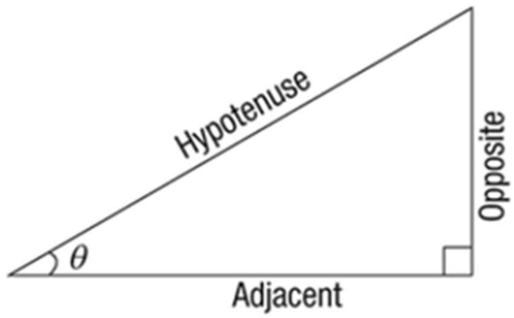
EXAMPLE

A right triangle matching the sketch above has the side length a=3m, side b=4m
Find the length of side c:

Side $c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

1.7 TRIGONOMETRICAL FUNCTIONS

The basic trigonometrical functions can be defined with rations created by dividing the lengths of the sides of a right triangle in a specific order. The label *hypotenuse* always remains the same – it's the longest side. But the designations of *opposite* and *adjacent* can change - depending on which angle you're referring to at the time. The *opposite* side is always that doesn't help making up the angle, and the *adjacent* side is always one of the sides in the angle. **(Fig. 1.3)**



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Fig. 1.3 Trigonometrical Functions

1.8 VECTOR

Vector is a variable quantity, such as force, that has magnitude and direction.
A vector may be graphically represented by a straight-line arrow with a direction and a length to indicate the magnitude. (**Fig. 1.4)**

To calculate a force, we need to know:

① The Point of Application
② The Direction
③ The Magnitude

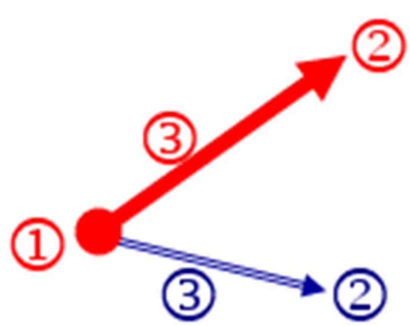


Fig. 1.4 FORCES

RESOLVING A VECTOR INTO A HORIZONTAL AND VERTICAL COMPONENT

A vector not orientated in the horizontal nor the vertical direction but having an angle to these two orientations can be resolved to find the horizontal and vertical component. This can be done when we know, what angle the vector has to either the horizontal or the vertical plane. **(Fig 1.5)**

The angle with horizontal plane is ϴ
The angle with vertical plane is β
(ϴ = 90 – β)

The vector to resolve is **F:**

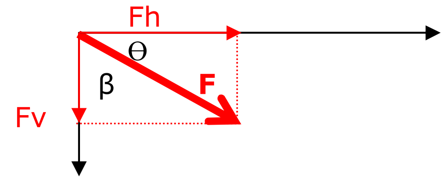


FIG. 1.5 RESOLVING A FORCE

Using the trigonometrical functions for a right triangle, we find:

Fh = F x Cos (ϴ) or Fh = F x Sin (β)
Fv = F x Cos (β) or Fv = F x Sin (ϴ)

RESULTANT FORCE VECTOR

The combined effect of two or more forces acting on a body is called the **resultant force.** To know the details of forces we need to know the point of application, the direction and the magnitude. This is graphically displayed in **(Fig 1.6)**.

The various scenarios of two forces acting on a body are shown in **Fig. 1.6.**
The resultant force of two forces acting in the same directions is the sum of the two forces
The resultant force of two forces acting opposite each other, in a straight line, is the difference between the two forces
The resultant force of two forces acting under an angle is worked out on a parallelogram of forces.

① Force one
② Force two
③ Resultant force

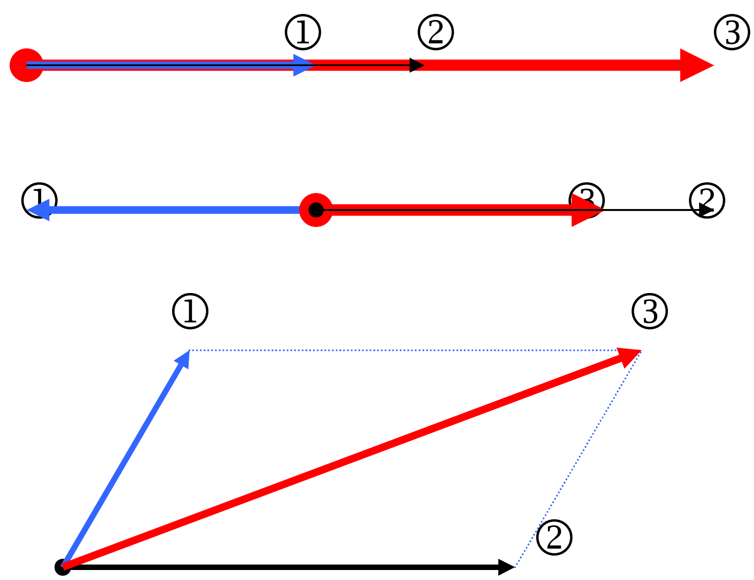
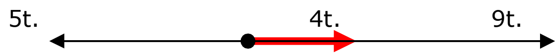
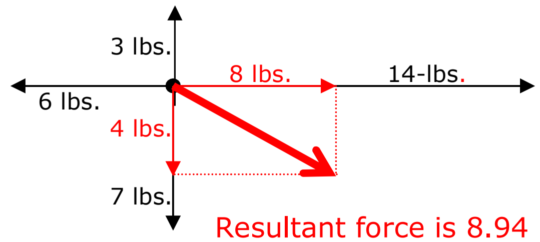


Fig. 1.6 RESULTANT FORCE

EXAMPLES OF RESULTANT FORCE



Resultant force is 4t.



Resultant force is 8.94 lbs (by use of Pythagoras Theorem).

1.8 MOMENT OF FORCES

In the stability calculations, the majority of weights or forces are working on a lever arm to a "rotation point", which we want to calculate the impact on.
A force **F** applied to a body at a distance **d** from its axis or point **A** results in a rotation about the axis.

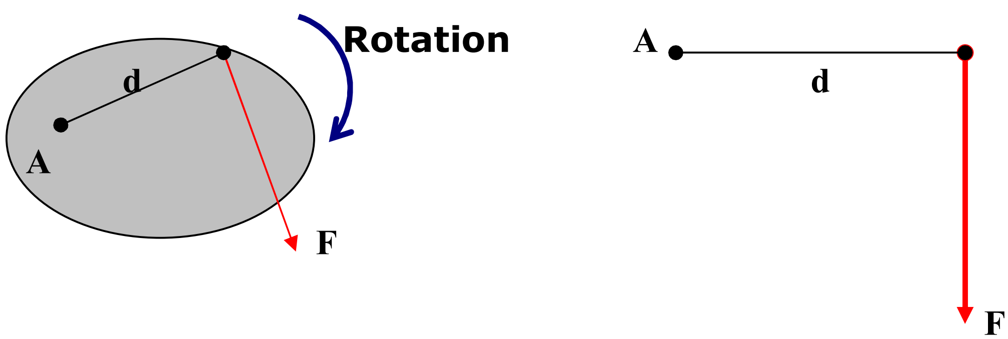
Moment is the tendency to produce rotation by a force **F** at a distance **d** about a point or axis.

The effect of the rotation depends on:
1) The magnitude of the force. **F**
2) The length **d** of the lever or arm. (**Fig. 1.7)**

Moment = Force x Distance

M = F x d Substitute F for Weight (W)

M = W x d



M = F x d
or M = W x d

Fig. 1.7 Moment of Forces

Moments can be applied about any imaginary point. Stability calculations do not use just one moment but a combination of many moments.

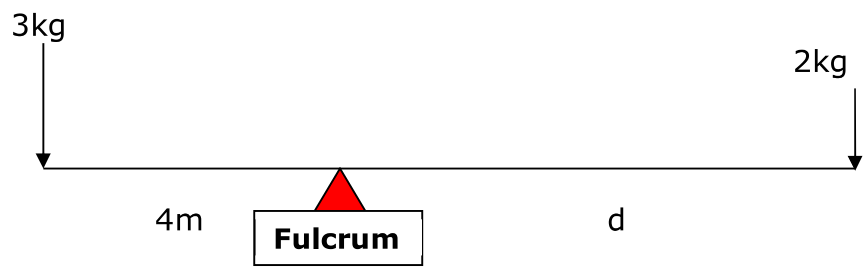
- The resultant moment is the total effect of all the combined moments.
- The rotation direction about the point or axis is clockwise or anti-clock wise.

- To find the resultant all clockwise moments are added together and deducted from the sum of all the anti-clockwise moments.
- To calculate the moment of force any point can be used as reference point to find the resultant moment.

EXAMPLES OF MOMENTS OF FORCES

The seesaw is a good example to use to explain the calculation of the moments of forces. For the purpose of stability calculations, we use weight as the forces.

Example 1. Find the unknown distance d in metres for the seesaw in equilibrium.



The system is in equilibrium i.e. the anti-clockwise moment = the clockwise moment or:

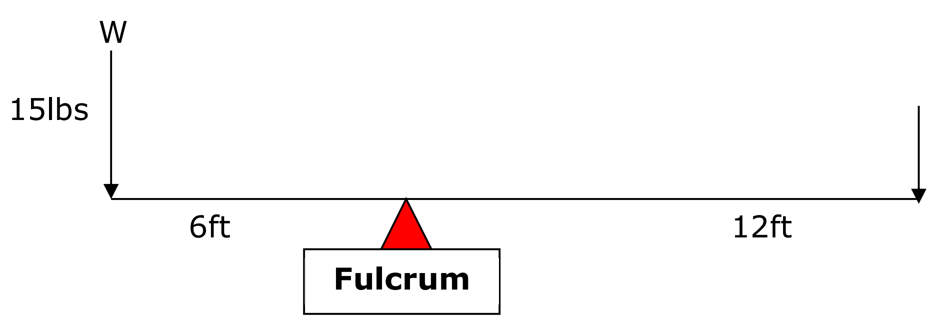
$$W \times d = W \times d$$

$$3kg \times 4m = 2kg \times d$$

$$d = \frac{3kg \times 4m}{2kg}$$

$$d = 6m$$

Example 2. Find the unknown weight W in lbs. for seesaw in equilibrium.



$$W \times d = W \times d$$
$$W \times 6ft = 15lbs \times 12\ ft$$

$$W = \frac{15lbs \times 12\ ft}{6}$$

$$W = 30lbs$$

1.9 THE CENTRE OF GRAVITY (G)

- The **centre of gravity (G)** is the geometrical centre of a homogeneous body.
- The force of gravity acts through the centre of gravity.
- The force of gravity acts vertically downward.
- The force of the gravity is equal to the weight of the body.
- The centre of gravity is the point about which a body will balance.

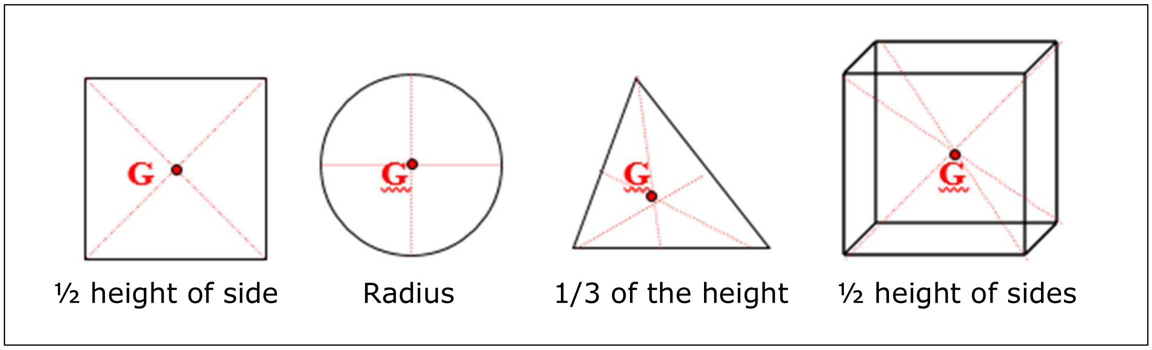
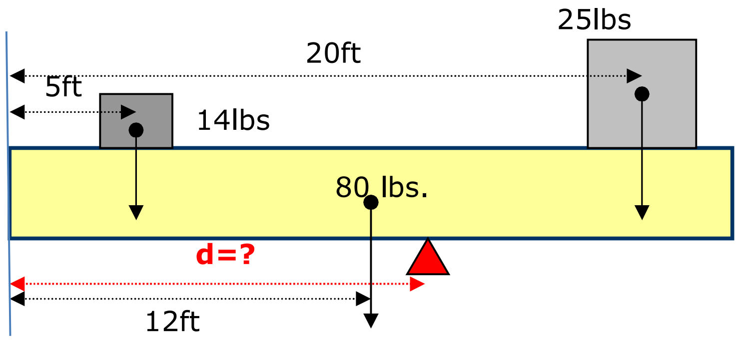


Fig. 1.8 Centre of gravity of homogeneous body

The position of G for various areas and shapes are shown in **Fig.1.8**
The position of G can be expressed in the vertical, transverse, and longitudinal plane with respect to any reference point or line.

Example 3. Find the Horizontal distance of the position of G from the left side of the body.

The same principle of the seesaw is used. Because G is the point about which the body will balance G can be considered to be the position of the fulcrum.
Find the distance (d) of the centre of gravity from the left side for a body with various weights.



Total moments clockwise:

M_{cw} = (14 lbs x 5ft) + (80 lbs x 12ft) + (25 lbsx20ft) = 1530 lb-ft

Total moments anti clockwise M_{aw} = (14 lbs. + 80 lbs. +25 lbs.) d = 119 d

Clockwise and anti-clockwise moments are equal.

119 d = 1530 lbs.-ft

d = 12.86 ft

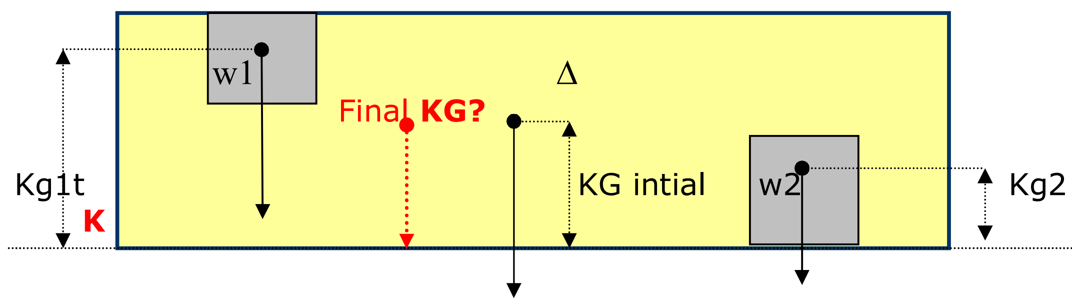
Calculation of the combined vertical centre of gravity for distributed loads:
We can use the same method to see that all weights in the gravity system has each a lever arm and contributes proportionally (Clockwise and anti-clockwise) to the total resulting moment around the reference plane.

The reference plane for the gravity system is then the "horizontal Keel plane" named **K.**

Mk initial = Δ_i x KG initial, Mk1 = w1 x Kg1, Mk2 = w2 x Kg2 and so on for "n" loads.

Mk Total = Mk initial + Mk1 + Mk2...+Mkn and the **Total weight**= Δ_i + w1 + w2 +...wn

$$\text{Final KG} = \frac{\text{Mk Total in tm}}{\text{Total of all Weights in t}} = \frac{\Delta_i \times KG_i + w1 \times Kg1 + w2 \times Kg2 + \ldots wn \times Kgn}{\Delta_i + w1 + w2 + \ldots wn}$$

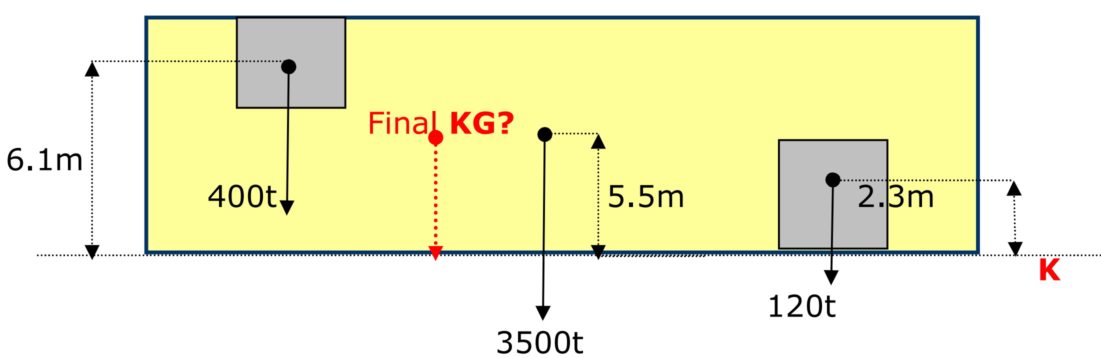


Side view sketch

In the "gravity world" the common centre of gravity shifts in the direction of the heaviest load to settle the weight system in a state of equilibrium.

Example 4. Find the position of G on a vessel with various weights.

K is the keel of the vessel. KG is the vertical distance from keel to G. KG is often too referred to as the Vertical Centre of Gravity (VCG).
A square shaped barge with a weight of 3500t lightship displacement and has a corresponding lightship KG of 5.5m. A load of 120t is placed at 2.3m above the keel and a second load of 400t is loaded 6.1 m above the keel. Find the new KG.
The reference point to find the vertical distance is the keel



Side view sketch

$$KG = \frac{\text{Total Moment of all Weights in tm}}{\text{The Total of all Weights in t}}$$

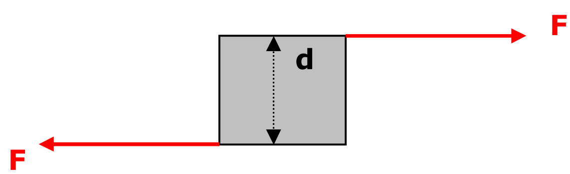
$$KG = \frac{(3500t \times 5.5m) + (400t \times 6.1m) + (120t \times 2.3m)}{3500t + 400t + 120t}$$

$$KG = \frac{21966tm}{4020t}$$

$$KG = 5.46m$$

1.10 A FORCE COUPLE

A force couple (Fig. 1.9) is the combination of a pair of equal and opposite parallel forces acting on a body. A couple has the tendency to produce rotation. The magnitude or moment of a couple is one of the forces multiplied times the distance between the parallel forces.

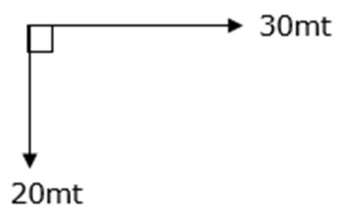


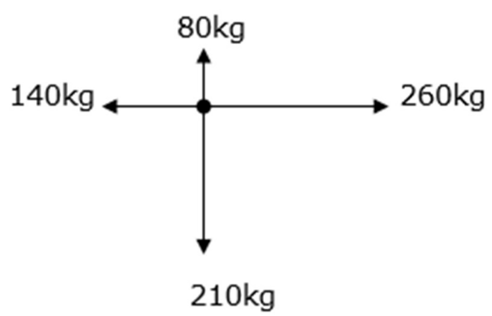
Couple moment (Units)= F x d (tonne –m) or **F x d (ton-ft)**

Fig 1.9 Couple

EXERCISES - CHAPTER 1
Class room exercises

1) Calculate the resultant force for the following examples.



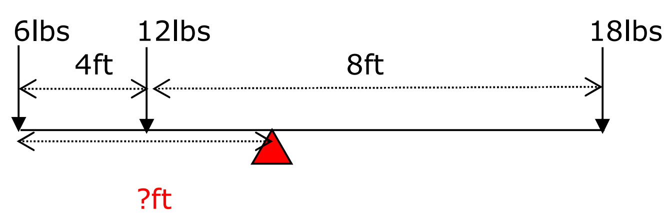


2) Define (Force)Moment = ………….. ?

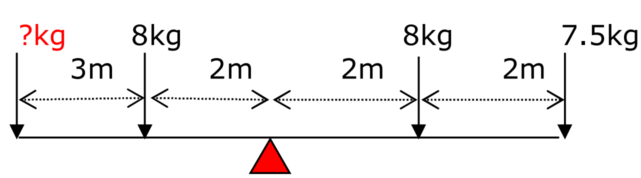
3) 15000mt = ………. lbs? 250st = …… oz? 12m = ………….inch?

<u>Find the unknown weight or distance for equilibrium.</u>

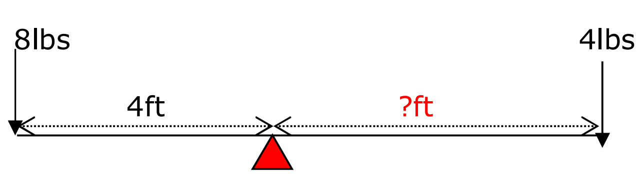
4)



5)

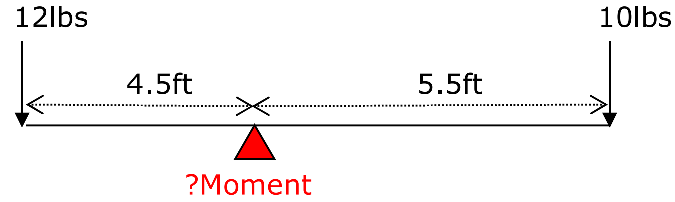


6)

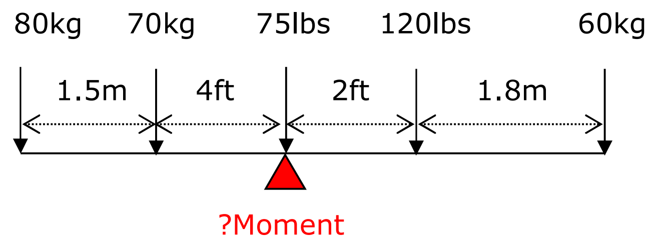


Find the resultant moment about the fulcrum

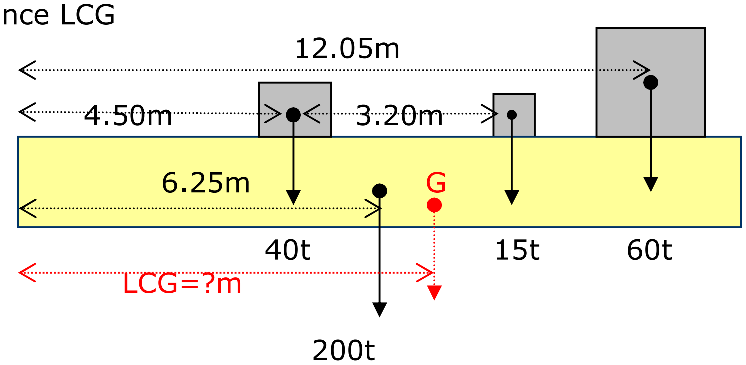
7)



8)

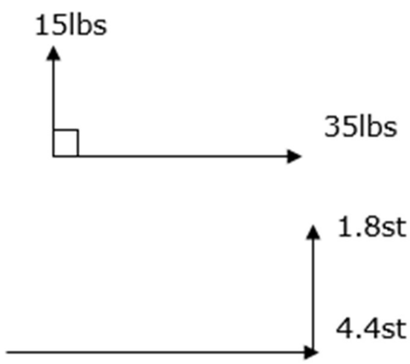


9) Find the distance LCG



Home work / Reflection Exercises

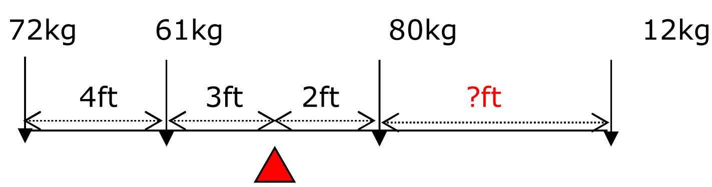
H1.1)
Calculate the resultant force for the following examples.



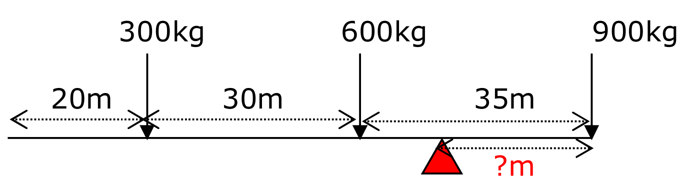
H1.2)
Convert:
350 US Gallon = … litre?
5.5m^2 = ……inch^2
3000st = ……….mt?

H1.3)

<u>Find the unknown weight or distance for equilibrium.</u>



H1.4)



H1.5) <u>Find the resultant moment about the fulcrum</u>