Corrigé sujet 5

1. $Z = \dfrac{\sqrt{2}+\sqrt{6}\mathrm{i}}{2+2\mathrm{i}} = \dfrac{\left(\sqrt{2}+\sqrt{6}\mathrm{i}\right)\left(2-2\mathrm{i}\right)}{\left(2+2\mathrm{i}\right)\left(2-2\mathrm{i}\right)} = \dfrac{2\sqrt{2}+2\sqrt{6}}{8} + \mathrm{i}\dfrac{-2\sqrt{2}+2\sqrt{6}}{8} = \dfrac{\sqrt{2}+\sqrt{6}}{4} + \mathrm{i}\dfrac{\sqrt{6}-\sqrt{2}}{4}$

2. $\lvert z_1 \rvert^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{6}\right)^2 = 2+6=8$, $\lvert z_1 \rvert = \sqrt{8} = 2\sqrt{2}$ et on a

$$\begin{cases} \cos\theta = \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2} \\ \sin\theta = \dfrac{\sqrt{6}}{2\sqrt{2}} = \dfrac{\sqrt{3}\times\sqrt{2}}{2\sqrt{2}} = \dfrac{\sqrt{3}}{2} \end{cases}$$; d'où $\theta = \dfrac{\pi}{3} + 2k\pi$, avec $k \in \mathbb{Z}$

Donc $z_1 = \sqrt{2} + i\sqrt{6} = 2\sqrt{2}\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) = 2\sqrt{2}\left(\cos\left(\pi/3\right) + i\sin\left(\pi/3\right)\right) = 2\sqrt{2}e^{i\pi/3}$

$\lvert z_2 \rvert^2 = 2^2 + 2^2 = 4+4=8$; $\lvert z_1 \rvert = \sqrt{8} = 2\sqrt{2}$ et on a

$$\begin{cases} \cos\theta = \dfrac{2}{2\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\ \sin\theta = \dfrac{2}{2\sqrt{2}} = \dfrac{\sqrt{2}}{2} \end{cases}$$; $\theta = \dfrac{\pi}{4} + 2k\pi$, avec $k \in \mathbb{Z}$

$z_2 = 2+2i = 2\sqrt{2}\left(\dfrac{\sqrt{2}}{2} + i\dfrac{\sqrt{2}}{2}\right) = 2\sqrt{2}\left(\cos\left(\pi/4\right) + i\sin\left(\pi/4\right)\right) = 2\sqrt{2}e^{i\pi/4}$.

On a donc $\lvert Z \rvert = \dfrac{\lvert z_1 \rvert}{\lvert z_2 \rvert} = 1$ et on a : $\arg Z = \arg z_1 - \arg z_2 = \dfrac{\pi}{3} - \dfrac{\pi}{4} = \dfrac{\pi}{12} + 2k\pi$, avec $k \in \mathbb{Z}$

Donc $Z = \cos\left(\pi/12\right) + i\sin\left(\pi/12\right)$, donc $\cos\left(\pi/12\right) = \mathrm{Re}(Z)$ et $\sin\left(\pi/12\right) = \mathrm{Im}(Z)$

et on a : $\cos\left(\pi/12\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ et $\sin\left(\pi/12\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Corrigé sujet 6

$z_3 = \dfrac{2}{z_2} = \dfrac{2}{1-i} = 1+i$

$\lvert z_1 \rvert = \sqrt{(-3)^2 + \sqrt{3}^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

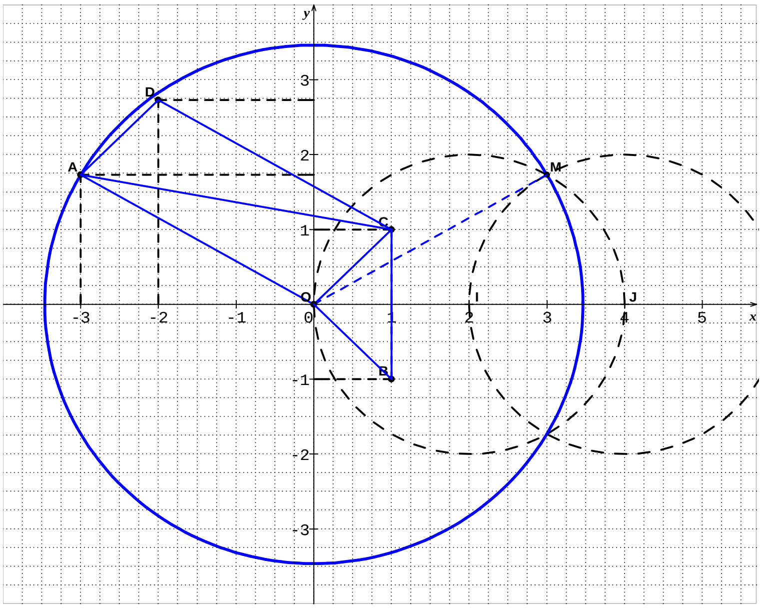
$\lvert z_2 \rvert = \sqrt{1+1} = \sqrt{2}$. $\theta_1 = \arg z_1 : \begin{cases} \cos\theta_1 = \dfrac{-3}{2\sqrt{3}} = \dfrac{-\sqrt{3}}{2} \\ \sin\theta_1 = \dfrac{\sqrt{3}}{2\sqrt{3}} = \dfrac{1}{2} \end{cases}$ donc

$\lvert z_3 \rvert = \left\lvert \dfrac{2}{z_2} \right\rvert = \dfrac{2}{\lvert z_2 \rvert} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$\theta_1 = \dfrac{2\pi}{3} + 2k\pi \,;\, k \in \mathbb{Z}$

$$\theta_2 = \arg z_2 : \begin{cases} \cos\theta_2 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\theta_1 = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \end{cases}$$, donc $\theta_2 = -\frac{\pi}{4} + 2k\pi \; ; k \in \mathbb{Z}$, comme $z_3 = \overline{z_2}$,

on a : $\arg z_2 = -\arg z_1 + 2k\pi$, donc $\theta_3 = \frac{\pi}{4} + 2k\pi \; ; k \in \mathbb{Z}$



On sait que $|z_2| = |z_3| = \sqrt{2}$, on déduit que le triangle BOC est isocèle en O.

$z_{\overrightarrow{BC}} = z_C - z_B = 1 + i - (1 - i) = 1 + i - 1 + i = 2i$, donc $BC = |z_{\overrightarrow{BC}}| = 2$

$BC^2 = 4$ et $OB^2 + OC^2 = \sqrt{2}^2 + \sqrt{2}^2 = 2 + 2 = 4$, on déduit que $BC^2 = OB^2 + OC^2$ et par conséquent
Le triangle BOC est rectangle isocèle en O .
OADC est un parallélogramme signifie que $z_{\overrightarrow{AD}} = z_{\overrightarrow{OC}}$, donc $z_D - z_A = z_C$ ou encore $z_D = z_C + z_A = 1 + i - 3 + i\sqrt{3} = -2 + (1 + \sqrt{3})i$

Corrigé sujet 7

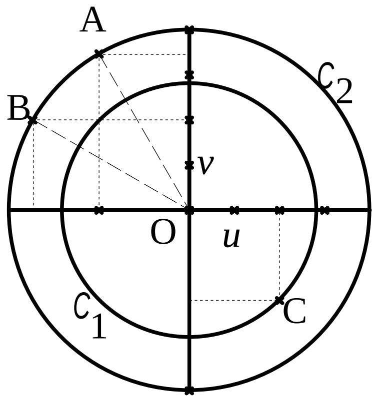
a) $z_A = 1 + i\sqrt{3}$; $z^2{}_A = 1 + 2i\sqrt{3} - 3$; $z^2{}_A = -2 + 2i\sqrt{3}$

$$z_C = \frac{z^2{}_A}{z_B} = \frac{-2(1 - i\sqrt{3})(\sqrt{2} - i\sqrt{2})}{(\sqrt{2} + i\sqrt{2})(\sqrt{2} - i\sqrt{2})} = \frac{-\sqrt{2} + \sqrt{6} + i(\sqrt{6} + \sqrt{2})}{2} \; ; \; z_C = \frac{\sqrt{6} - \sqrt{2}}{2} + i\frac{\sqrt{6} + \sqrt{2}}{2}$$

b) $z_A = 1 + i\sqrt{3}$; $|z_A| = \sqrt{1 + 3} = \sqrt{4} = 2$; $z_A = 2(\frac{1}{2} + i\frac{\sqrt{3}}{2}) = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$

$z_B = \sqrt{2} + i\sqrt{2}$; $|z_B| = \sqrt{2+2} = \sqrt{4} = 2$. $z_B = 2(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = 2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = 2e^{i\frac{\pi}{4}}$

$$z_C = \frac{z^2{}_A}{z_B} = \frac{4e^{2i\frac{\pi}{3}}}{2e^{i\frac{\pi}{4}}} = 2e^{2i\frac{\pi}{3} - i\frac{\pi}{4}} = 2e^{5i\frac{\pi}{12}} = 2(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12})$$



.On a $z_C = \frac{\sqrt{6}-\sqrt{2}}{2} + i\frac{\sqrt{6}+\sqrt{2}}{2} = 2(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12})$ d'où

$\cos\frac{5\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ et $\sin\frac{5\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

Corrigé sujet 8

a- $|z_1| = \sqrt{4+4\times 3} = \sqrt{16} = 4$; $z_1 = 4(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = 2e^{i\frac{2\pi}{3}}$;

$\arg z_1 = \frac{2\pi}{3} + 2k\pi$, $k \in Z$

b- $|z_3| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$; $z_3 = 2\sqrt{2}(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}) = 2(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}) = 2e^{-i\frac{\pi}{4}}$;

$\arg z_3 = -\frac{\pi}{4} + 2k'\pi$, $k' \in Z$.

c- $z_2 = 4\,e^{\frac{5i\pi}{6}} = 4(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}) = 4(-\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) = 4(-\frac{\sqrt{3}}{2} + \frac{i}{2}) = -2\sqrt{3} + 2i$.

d- $|z_2| = \sqrt{12+4} = \sqrt{16} = 4$; $\left|\frac{z_2}{z_1}\right| = \frac{|z_2|}{|z_1|} = \frac{4}{4} = 1$;

$\arg\frac{z_2}{z_1} = \arg z_2 - \arg z_1 = \frac{5\pi}{6} - \frac{2\pi}{3} = \frac{5\pi}{6} - \frac{4\pi}{6} = \frac{\pi}{6} + 2k\pi, k \in Z$. $\frac{z_2}{z_1} = \frac{4e^{i\frac{5\pi}{6}}}{4e^{i\frac{2\pi}{3}}} = e^{i\frac{5\pi}{6} - \frac{2\pi}{6}} = e^{i\frac{\pi}{6}}$,

donc le point B

est l'image du point A par la rotation de centre O et d'angle $\frac{\pi}{6}$.

Corrigé sujet 9

1. $z_2 = iz_1 = i(-1 - i\sqrt{3}) = -i - i^2\sqrt{3} = \sqrt{3} - i$

2.a . Soient z_1 et z_2 des arguments respectifs de z_1 et z_2 :

$|z_1| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$|z_2| = |iz_1| = |i| \times |z_1| = |z_1| = 2$. On peut aussi utiliser les propriétés du module d'un nombre complexe :

$$\left.\begin{array}{l}\cos\theta_1 = \dfrac{-1}{2}\\[2mm] \sin\theta_1 = \dfrac{-\sqrt{3}}{2}\end{array}\right\}\ \text{donc}$$

$$\theta_1 = \frac{5\pi}{3} + 2k\pi \quad où\ k \in Z$$

$$\left.\begin{array}{l}\cos\theta_2 = \dfrac{\sqrt{3}}{2}\\[2mm] \sin\theta_2 = \dfrac{-1}{2}\end{array}\right\}\ \text{donc}$$

$$\theta_2 = \frac{-\pi}{6} + 2k\pi \quad où\ k \in Z$$

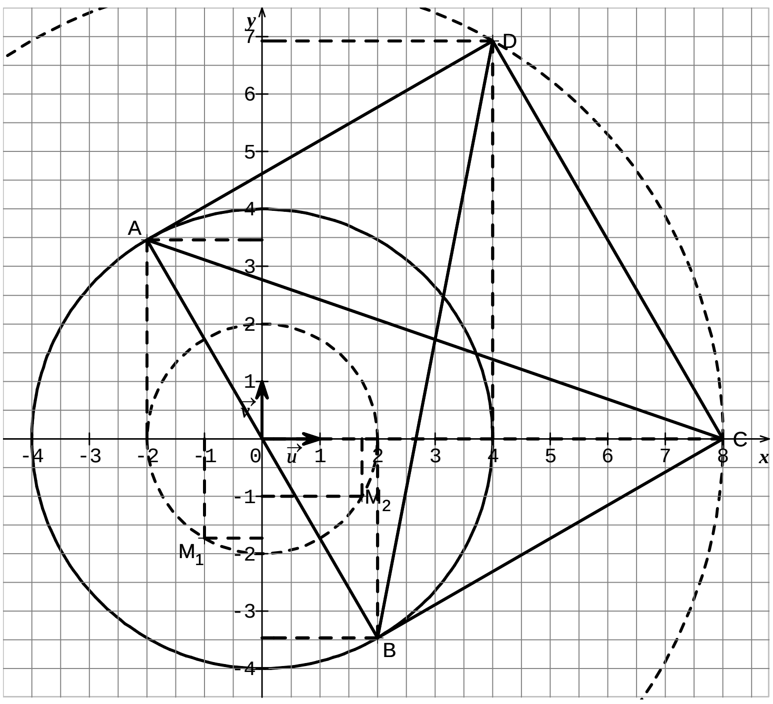
2.b.

3.a.

$$2\overline{z_1} = 2(-1+i\sqrt{3}) = -2+i\sqrt{3} = -2+2i\sqrt{3} = z_A$$

$$-z_A = -(-2+2i\sqrt{3}) = 2-2i\sqrt{3} = z_B$$

3.b. voir dessin ci-contre



3.c.

$$z_{\overrightarrow{AB}} = z_B - z_A = 2-2i\sqrt{3} - (-2+2i\sqrt{3})$$
$$= 2-2i\sqrt{3}+2-2i\sqrt{3} = 4-4i\sqrt{3}$$

$$AB = |z_{\overrightarrow{AB}}| = |z_B - z_A| = \sqrt{4^2+(4\sqrt{3})^2}.$$
$$= \sqrt{16+48} = \sqrt{64} = 8 \quad AB = 8cm$$

$$z_{\overrightarrow{BC}} = z_C - z_B = 8-2+2i\sqrt{3} = 6+2i\sqrt{3}$$

$$BC = |z_{\overrightarrow{BC}}| = |z_C - z_B| = \sqrt{6^2+(2\sqrt{3})^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}.\ \mathrm{BC} = 2\sqrt{13}\,\mathrm{cm}$$

$$z_{\overrightarrow{AC}} = z_C - z_A = 8+2-2i\sqrt{3} = 10+2i\sqrt{3}.$$

$$AC = |z_{\overrightarrow{AC}}| = |z_C - z_A| = \sqrt{10^2+(2\sqrt{3})^2} = \sqrt{100+12} = \sqrt{112} = \sqrt{16\times 7} = 4\sqrt{7}.\ \mathrm{AC} = 4\sqrt{7}\,\mathrm{cm}.$$

$$\left.\begin{array}{l}AB^2+BC^2=64+52=112\\ AC^2=112\end{array}\right\}\ \text{donc}\ \mathrm{AC}^2 = \mathrm{AB}^2+\mathrm{BC}^2$$

d'après la réciproque du théorème de Pythagore ABC est rectangle en B.

3.d. Pour que ABCD soit un rectangle il suffit que ABCD soit un parallélogramme puisque ABC est un triangle rectangle. Pour que ABCD soit un rectangle il suffit donc que $\overrightarrow{AB} = \overrightarrow{DC}$.

Soient z_A, z_B, z_C, z_D les affixes respectifs de A,B, C, D.

De $\overrightarrow{AB} = \overrightarrow{DC}$. On en déduit : $z_{\overrightarrow{AB}} = z_{\overrightarrow{DC}}$ or $z_{\overrightarrow{AB}} = 4-4i\sqrt{3}$ et $z_{\overrightarrow{DC}} = z_C - z_D = 8 - z_D$

soit $4-4i\sqrt{3} = 8-z_D$, $8-4+4i\sqrt{3} = z_D$, soit $4+4i\sqrt{3} = z_D$; $4 + 4i\sqrt{3}$ est donc l'affixe du point D.