ENTRANCE EXAMINATION TO THE CYCLES OF TECHNICIAN SUPERIOR AND TECHNICIAN OF THE AFRICAN SCHOOL OF THE METEOROLOGY AND THE CIVIL AVIATION (EAMAC)

SESSION 201[illegible]

TEST OF : MATHEMATICS

DURATION : 3 HOURS

Exercise 1 (5*pts*)

One considers in $\mathbb{C}$ the sequence of general term z_n defined by:

$$\begin{cases} z_0 = 1 \\ 2z_{n+1} = z_n + i \end{cases}$$

1. Show that, for each integer n, no vanishing, the module r_n of z_n is lower than 1.

2. We set $z_n = x_n + iy_n$, x_n and y_n being real numbers and $u_n = z_n - i$
   a. Find a recurrence relation between u_{n+1} and u_n.
   b. Deduce that the sequence (x_n) is a geometric sequence which converges towards 0 and the sequences of general terms y_n and r_n converge towards 1.

3. Calculate the minor term n_0 such that, for each n equal or higher than n_0, one has : $|z_n - i| < 10^{-5}$.

Exercise 2 (4*pts*)

1. Solve the differencial equation : $9y'' + 4y = 4\sqrt{3}$.

2. a. Determine among the solutions of this equation the solution h such that : $h'\left(\frac{\pi}{2}\right) = \dfrac{4}{3}$ and $h''\left(\frac{\pi}{4}\right) = \dfrac{4}{9}$.
   b. Write $h(x)$ in the form $A + B\cos(\omega x + \varphi)$ where A, B, ω and φ are four reales that one will specify.

3. Solve in $\mathbb{R}$ the equation $h(x) = 0$.

Exercise 3 (5*pts*)

f is the numerical function defined on $\mathbb{R}_+^*$ by : $f(x) = \ln\left[\dfrac{e}{2}\left(x + \dfrac{1}{x}\right)\right]$

One calls (C) the representative curve of f in an orthonormal reference $\left(O, \vec{i}, \vec{j}\right)$.

1. a. Study f, then show that f has a minimum; that one must specify

2. b. Show that the curve (Γ) of equation $y = \ln\left(\frac{e}{2}x\right)$ is an asymptote of the curve (C).
   c. Draw the curves (C) and (Γ) in the same orthonormal reference $\left(O, \vec{i}, \vec{j}\right)$.

3. a. Establish that for any x in $\mathbb{R}_+^*$, $f'(x) < 1$
   b. Deduce the sign of $(f(x) - x)$ and the position of the curve (C) according to the line (D) of equation $y = x$

4. a. Deduce from previous results that the sequence (u_n) which verifies :

$$\begin{cases} u_0 \in \mathbb{R}_+^* \\ u_{n+1} = f(u_n) = \ln\left[\frac{e}{2}\left(u_n + \frac{1}{u_n}\right)\right] \end{cases}$$

is decreasing and undervalued by 1 for starting by row $n = 1$ (for starting by row $n = 0$ if $u_0 \geq 1$).
   b. Show that the sequence (u_n) converges towards 1.

Exercise 4 (6*pts*)

1. A ballot box contains two white balls and n black balls, indistinguishable by touch.
   A player extracts simultaneously two balls from the ballot box and one notes A_2 the event:
   A_2 : « the player extracted two white balls ».
   Determine n so that the probability of $p(A_2)$ is equal to $1/15$.
   In the following part of the exercise, we will take $n = 4$.

2. A player extracts simultaneously two balls from the ballot box and one notes :
   A_0: « The player extracts extracts two black balls » ;
   A_1 : « The player extracts one black ball and one white ball» ;
   A_2 : « The player extracts extracts two white balls».
   a. Calculate $p(A_0)$ and $p(A_1)$.
   b. With this pulling, the player gets three points for each white ball extracted and two points for each black ball extracted. Let X be the random variable associated to the numbers of points obtained.
   Determine the law of probability of the random variable X and calculate its mean $E(X)$.

3. After this first pulling, the player drawn back the black balls and leaves the white ones, then extracts simultaneously two balls from the ballot box.
   a. Give $p(B_0/A_2)$ and deduce $p(B_0 \cap A_2)$; Calculate $p(B_0/A_1)$ and $p(B_0 \cap A_1)$.
   Deduce that $p(B_0) = \frac{41}{75}$.
   b. Show that $p(B_2) = \frac{2}{75}$. Deduce $p(B_1)$.