

MATHEMATICS (B)

(2015)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) If the equation $\log_{10}(ax) \log_{10}(bx) + 1 = 0$ with $a > 0, b > 0$ constants has a solution $x > 0$, it follows that $\frac{b}{a} \geq$ ①

or ② $\geq \frac{b}{a} >$ ③.

(2) If $\cos \theta = \sqrt{\frac{1}{2} + \frac{1}{2\sqrt{2}}}$ and $\sin \theta = -\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}$ with $0 \leq \theta < 2\pi$, it follows that $2\theta =$ π .

(3) If $y = \log_2(x + \sqrt{x^2 + 1})$, then $2^y - 2^{-y} =$ x .

(4) The function $f(x) = \log_2(\log_3(\log_2(\log_3(\log_2 x))))$ has the interval $x >$ as its maximum domain on real numbers.

(5) The total number of subtractions that result in 11111 remaining after a four-digit number has been subtracted from a five-digit number and the digits 1 through 9 have all been used is .

2. Draw a circle C on a plane. Put n distinct points on the circumference of C . Join any two of the n points with a chord. Suppose that no three chords intersect at any one point inside C . Let c_n be the number of such chords, i_n that of intersections of the chords inside C , and r_n that of regions in C bounded by an arc and/or some chords. Fill in the blanks with the answers to the following questions.

- (1) Find $c_1, c_2, c_3, c_4, c_5, c_6$ and write their values in this order.
- (2) Find $i_1, i_2, i_3, i_4, i_5, i_6$ and write their values in this order.
- (3) Find $r_1, r_2, r_3, r_4, r_5, r_6$ and write their values in this order.
- (4) Express c_n, i_n, r_n with binomial coefficients in terms of n .
 (For binomial coefficients $\binom{m}{k}$, note that $\binom{m}{k} = 0$ if $m < k$.)

(1) $c_1, \dots, c_6$:

①

,

②

,

③

,

④

,

⑤

,

⑥

.

(2) $i_1, \dots, i_6$:

①

,

②

,

③

,

④

,

⑤

,

⑥

.

(3) $r_1, \dots, r_6$:

①

,

②

,

③

,

④

,

⑤

,

⑥

.

(4) $c_n =$

①

, $i_n =$

②

, $r_n =$

③

.

3. Let C be the plane curve $y = f(x)$ defined by the cubic function $f(x) = x^3 - 4x^2 + ax + b$ with a, b real constants. Fill in the blanks with the answers to the following questions.

- (1) When C is tangent to the x -axis at $x = 3$, what are a and b ?
- (2) When (1) holds, find all x such that C has points in common with the x -axis.
- (3) When (1) holds, calculate the area S of the limited region bounded by C and the x -axis.

(1) $a =$, $b =$.

(2) $x =$.

(3) $S =$.