2020 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE GOVERNMENT (MEXT) SCHOLARSHIP 2020

学科試験　　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学　（A）

MATHEMATICS(A)

注意　☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

MATHEMATICS(A) (2020)

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) For sets A, B, and C, we assume $C \subset A$. The number of elements in set A is 66, that in set A but not in set C is 47, that in set B but not in set C is 42, that in set C but not in set B is 8, and that in set A but not in either set B or set C is 31. Then, the number of elements included in set A, B, or C is $\boxed{\text{[1-1]}}$.

(2) Consider a graph of the function $y = x^2$ in xy-plane. The minimum distance between point $(0,\ 4)$ on the y-axis and points on the graph is $\boxed{\text{[1-2]}}$. You should rationalize the denominator in the answer.

(3) Consider a graph of the function $y = 2x^2 - 6x + 2$ in xy-plane. Next, consider another graph that is symmetrical to the previous graph with respect to the line $x = 2$. The graph that is symmetrical to the latter graph with respect to the line $y = 3$ is described as

$$y = a_2x^2 + a_1x + a_0.$$

Then, we have $a_2 = \boxed{\text{[1-3]}}$, $a_1 = \boxed{\text{[1-4]}}$, and $a_0 = \boxed{\text{[1-5]}}$.

(4) Let x and y be integers. We assume that $x + y$ is a multiple of 2, $x + 4y \leq 17$, and $3x + 2y \leq 21$. Then, $x + 2y$ is maximum when $x = \boxed{\text{[1-6]}}$ and $y = \boxed{\text{[1-7]}}$. The maximum value is $\boxed{\text{[1-8]}}$.

(5) Consider two graphs of the functions $y = \frac{1}{8}x^2 - 2$ and $y = \frac{1}{2}x^2 - 8$ in xy-plane. We describe a common tangent line of the two graphs in xy-plane as

$$y = a_1 x + a_0.$$

We assume that the x-coordinates of both tangential points are positive. Then, we have $a_1 = \boxed{\text{[1-9]}}$ and $a_0 = \boxed{\text{[1-10]}}$.

(6) When we set $t = \cos x$ for a function $f(x) = \cos 2x + \cos 3x$, $f(x)$ has an expression in t as follows:

$$\boxed{\text{[1-11]}}\ t^3 + \boxed{\text{[1-12]}}\ t^2 + \boxed{\text{[1-13]}}\ t + \boxed{\text{[1-14]}}.$$

(7) For a set $A = \{2, 3, 5\}$, we randomly choose one element in A three times. Let B_k be a number of the k-th trial in the three trials and let $C = B_1 \times B_2 \times B_3$ be the product of $\{B_i\}$. The probability that C is an odd number is $\dfrac{\boxed{\text{[1-15]}}}{27}$ and the probability that C is a multiple of 5 is $\dfrac{\boxed{\text{[1-16]}}}{27}$.

(8) A function $f(x) = x(x-6)^2$ has the extreme values at $\boxed{\text{[1-17]}}$ and $\boxed{\text{[1-18]}}$, where $\boxed{\text{[1-17]}} < \boxed{\text{[1-18]}}$. If we define $g(x) = |f(x)|$ and we consider the numbers of different real solutions of the equation $g(x) = a$ of x according to a constant a, then the maximum number of real solutions is $\boxed{\text{[1-19]}}$.

(9) For eight data $1, 1, 3, 5, 6, 8, 9, 15$, the sample mean is $\boxed{\text{[1-20]}}$. If we define a deviation as the difference of each data from the sample mean, the sum of squares of the deviations is $\boxed{\text{[1-21]}}$ and the mean is $\boxed{\text{[1-22]}}$.

2. Take two points B and C on the circumference of a circle, whose center is denoted by O. We assume that the three points O, B, and C are not collinear. We consider a straight line that is tangential with the circumference on point B. We define point A on the tangential line, such that $\angle ABC > \frac{\pi}{2}$. Furthermore, line CA is a bisector of both angles $\angle BAO$ and $\angle BCO$. We assume that the lengths of edges AB and CB are 2 and 1, respectively. We denote the intersection of lines OB and CA by D. And we denote the lengths of edges BD and OD by x and y, respectively. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. They should be simplified as much as possible.

(1) From the assumption that edge AD is a bisector of angle $\angle BAO$, the length OA is described as $\boxed{\text{[2-1]}}$ by using x, y.

(2) From the assumption that edge CD is a bisector of angle $\angle BCO$, y is described as $\boxed{\text{[2-2]}}$ by using x.

(3) Therefore, we have

$$\begin{aligned} x &= 4 - \boxed{\text{[2-3]}}, \\ y &= \frac{\boxed{\text{[2-4]}}}{3} - 4. \end{aligned}$$

3. We divide the sequence of natural numbers $1, 2, 3, \ldots$ as follows:

$$\underbrace{1}_{\text{first group}} \;\Big|\; \underbrace{2, 3, 4}_{\text{second group}} \;\Big|\; \underbrace{5, 6, 7, 8, 9}_{\text{third group}} \;\Big|\; \cdots$$

Here the n-th group ($n = 1, 2, 3, \ldots$) has $(2n-1)$ elements. Let a_n be the first number in the n-th group and let S_n be the total sum of the numbers in the n-th group. Answer the following questions for the sequences $\{a_n\}, \{S_n\}$.

(1) For the sequence $\{a_n\}$, the n-th term is

$$a_n = \boxed{\text{[3-1]}}\, n^3 + \boxed{\text{[3-2]}}\, n^2 + \boxed{\text{[3-3]}}\, n + \boxed{\text{[3-4]}}.$$

(2) 2678 is in the $\boxed{\text{[3-5]}}$-th group, and in the group it is the $\boxed{\text{[3-6]}}$-th term.

(3) For the sequence $\{S_n\}$, the n-th term is

$$S_n = \boxed{\text{[3-7]}}\, n^3 + \boxed{\text{[3-8]}}\, n^2 + \boxed{\text{[3-9]}}\, n + \boxed{\text{[3-10]}}.$$