Exercices : Portes logiques et algèbre de boole

corrigés

Exercice 1

1)

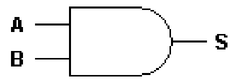
a.

$$S = \overline{\bar{A} + \bar{B}}$$

b.

A	B	S
0	0	0
0	1	0
1	0	0
1	1	1

$$S = \overline{\bar{A} + \bar{B}} = A.B$$

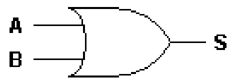
c. La fonction logique réalisée est : le ET logique (AND), son symbole est : 

2)

$$S = \overline{\bar{A}.\bar{B}}$$

A	B	S
0	0	0
0	1	1
1	0	1
1	1	1

$$S = \overline{\bar{A}.\bar{B}} = A + B$$

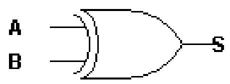
La fonction logique réalisée est : le OU logique (OR), son symbole est : 

3)

$$S = \overline{\overline{A.\overline{A.B}}.\overline{B.\overline{A.B}}}$$

A	B	S
0	0	0
0	1	1
1	0	1
1	1	0

$$S = \overline{\overline{A.\overline{A.B}}.\overline{B.\overline{A.B}}} = A.\bar{B} + \bar{A}.B = A\oplus B$$

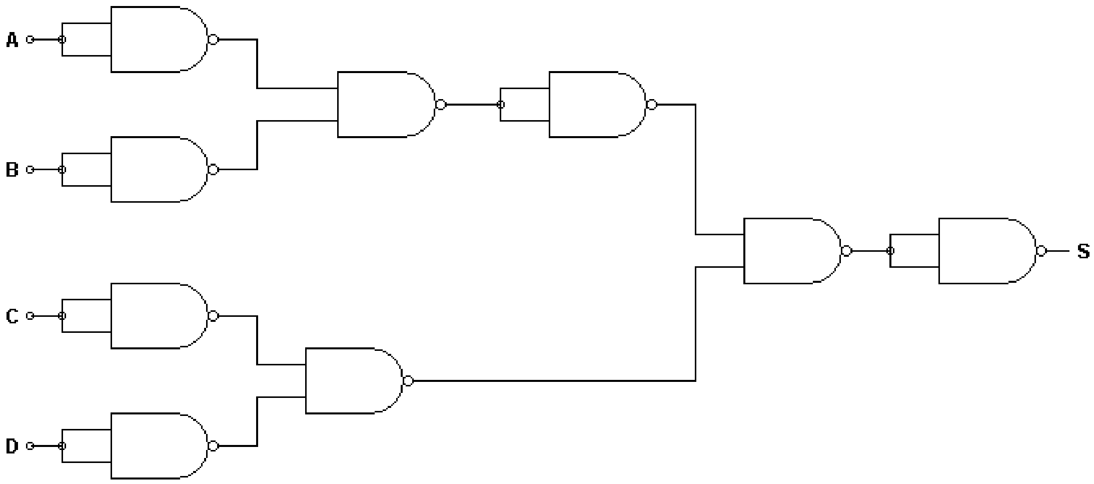
La fonction logique réalisée est : le OU exclusif (XOR), son symbole est : 

Exercice 2

1.

$$S = \overline{A+B}\,.\,\overline{\bar{C}.\bar{D}}$$

2.



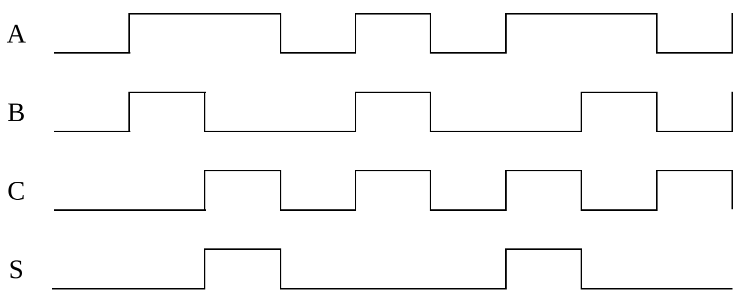
Exercice 3

1.

C	B	A	S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

2. $S = C.(A.\bar{B} + A.\bar{B}) = C.(A\oplus B)$

3.



Exercice 4

1) $\overline{A}\,(A+\overline{B})(\overline{A}+B) = \overline{A}\,(\overline{A}+B)\,(A+\overline{B})$

$= \overline{A}\,(A+\overline{B})$

$= \overline{A}\;\overline{B}$

2) $(B + A\cdot B + C)\cdot(A+\overline{B}+\overline{A}\cdot\overline{C}) = (B + C)\cdot(A+\overline{B}+\overline{C})$

$= AB + B\,\overline{C} + A\,C + \overline{B}\,C$

$= AB + B\,\overline{C} + A\,\overline{B}\,C + ABC + \overline{B}\,C$

$= A\,B + B\,\overline{C} + \overline{B}\,C$

3) $$AB + ACD + \overline{B}D = AB + ACD\underbrace{(B+\overline{B})}_{=1} + \overline{B}D = AB + ABCD + A\overline{B}CD + \overline{B}D$$

$$= AB\underbrace{(1+CD)}_{=1} + \overline{B}D\underbrace{(1+AC)}_{=1} = AB + \overline{B}D$$

4) $$(\overline{A}+B)(A+C)(B+C) = (\overline{A}+B)(A+C)(B+C+\underbrace{\overline{A}A}_{=0}) = (\overline{A}+B)(A+C)(B+C+\overline{A})(B+C+A)$$

$$= (\overline{A}+B+\underbrace{0.C}_{=0})(A+C+\underbrace{0.B}_{=0}) = (\overline{A}+B)(A+C)$$

5) $$AB + \overline{B}C = AB\underbrace{(1+C)}_{=1} + \overline{B}C\underbrace{(1+A)}_{=1} = AB + \overline{B}C + ABC + A\overline{B}C = AB + \overline{B}C + AC$$

$$= AB + \underbrace{\overline{B}B}_{=0} + \overline{B}C + AC = (A+\overline{B})B + (A+\overline{B})C = (A+\overline{B})(B+C)$$

6) $$\overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}}.\overline{\overline{A}B} = (\overline{A}+B)(A+\overline{B}) = \underbrace{\overline{A}A}_{=0} + \overline{A}\overline{B} + BA + \underbrace{B\overline{B}}_{=0} = AB + \overline{A}\,\overline{B}$$

7) $$\overline{(A+B)(\overline{A}+C)} = \overline{A+B} + \overline{\overline{A}+C} = (\overline{A}.\overline{B}) + (A.\overline{C}) = \underbrace{(\overline{A}+A)}_{=1}(\overline{A}+\overline{C})(\overline{B}+A)(\overline{B}+\overline{C})$$

$$= (\overline{A}+\overline{C})(A+\overline{B})(\underbrace{\overline{A}A}_{=0} + \overline{B} + \overline{C}) = (\overline{A}+\overline{C})(A+\overline{B})(\overline{A}+\overline{B}+\overline{C})(A+\overline{B}+\overline{C})$$

$$= (\overline{A} + \underbrace{0.\overline{B}}_{=0} + \overline{C})(A + \overline{B} + \underbrace{0.\overline{C}}_{=0}) = (A+\overline{B})(\overline{A}+\overline{C})$$

Exercice 5

$$E = \overline{a}bc + ac + a\overline{b}\overline{c} + \overline{a}\overline{b}$$

$$E = \overline{a}\left(bc + \overline{b}\right) + a\left(c + \overline{b}\overline{c}\right)$$

$$E = \overline{a}\left(c + \overline{b}\right) + a\left(c + \overline{b}\right)$$

$$E = \left(\overline{a} + a\right)\left(c + \overline{b}\right)$$

$$E = 1.\left(c + \overline{b}\right)$$

$$E = c + \overline{b}$$

$F = (\bar{a}+b) . (a+b+d).\bar{d}$

$F = (\bar{a}+b) . (a. \bar{d} + b. \bar{d} + d. \bar{d})$

$F = (\bar{a}+b) . (a. \bar{d} + b. \bar{d} + 0)$

$F = (\bar{a}+b) . (a. \bar{d} + b. \bar{d})$

$F = \bar{a}.a. \bar{d} + \bar{a}.b. \bar{d} + b.a. \bar{d} + b.b. \bar{d}$

$F = 0. \bar{d} + \bar{a}.b. \bar{d} + b.a. \bar{d} + b. \bar{d}$

$F = \bar{a}.b. \bar{d} + a.b. \bar{d} + b. \bar{d}$

$F = b. \bar{d} . (\bar{a}+a+1)$

$F = (b. \bar{d}) .1$

$F = b. \bar{d}$

$G =(a+b).(a+c) + (b+c).(b+a) + (c+a).(c+b)$

$G = a.a + a.c + b.a + b.c + b.b + b.a + b.c + c.a + c.c + c.b + a.c + a.b$

$G = a + a.c + a.b + b.c + b + a.b + b.c + a.c + c + b.c + a.c + a.b$

$G = a + a.c + a.b + b.c + b + c$

$G = a (1 + c + b) + b (c + 1) + c$

$G = a.1 + b.1 + c$

$G = a + b + c$

$H = a.b.c + a.\bar{b}.c + a.b.\bar{c}$

$H = a . (b.c + \bar{b}.c + b. \bar{c})$

$H = a . [b . (c + \bar{c}) + \bar{b}.c]$

$H = a . [b . 1 + \bar{b}.c]$

$H = a . (b + \bar{b}.c)$

$H = a . [(b + \bar{b}) . (b + c)]$

$H = a . [1 . (b + c)]$

$H = a . (b + c)$