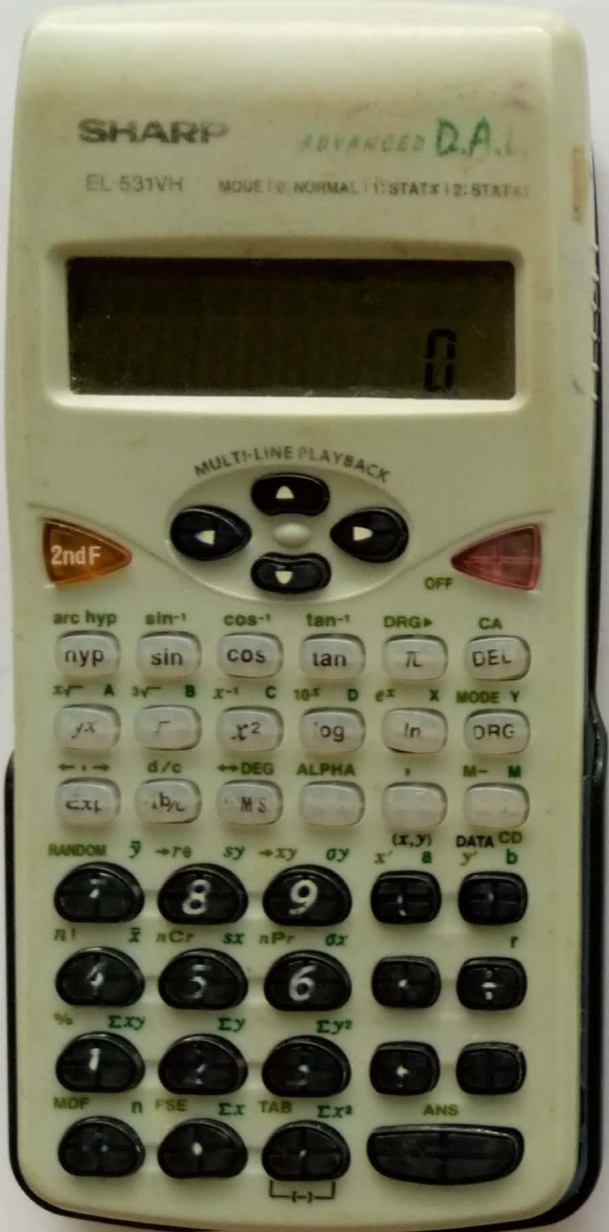

CALCULER L'ARGUMENT D'UN NOMBRE COMPLEXE À L'AIDE DE LA CALCULATRICE SHARP EL-531VH

$Re(z)$ $\boxed{2ndF}$ $\boxed{STO}$ $Im(z)$ $\boxed{8}$ $\boxed{2ndF}$ $\boxed{Exp}$ $\boxed{\div}$ $\boxed{180}$ $\boxed{2ndF}$ $\boxed{ab/c}$

après avoir tapé $\boxed{8}$ Vous aurez le module.

l'angle en degré.

Vous aurez L'angle en radian

Exple: $1r4 = \frac{\pi}{4}$

$2r4 = \frac{2\pi}{4} = \frac{\pi}{2}$

Exemple : $z = 1 + i$

- $Re(z) = 1$ et $Im(z) = 1$

1 $\boxed{2ndF}$ $\boxed{STO}$ 1 $\boxed{8}$ $\boxed{2ndF}$ $\boxed{Exp}$ $\boxed{\div}$ $\boxed{180}$ $\boxed{2ndF}$ $\boxed{ab/c}$

$r = 1{,}414\ldots$
$r = \sqrt{2}$
(module)

$\theta = 45°$
(L'angle en degré)

$1r4 = \frac{\pi}{4}$
(Angle en radian)

NOMBRES COMPLEXES

TRAVAUX DIRIGES 1 (CORRECTION)

Exercice 1 : $z_1 = 3-3i$ et $z_2 = 2+i$

$z_1 + z_2 = 3-3i+2+i$

$\boxed{z_1 + z_2 = 5-2i}$

$z_1 - z_2 = 3-3i-(2+i)$
$= 3-3i-2-i$

$\boxed{z_1 - z_2 = 1-4i}$

$z_1 \times z_2 = (3-3i)(2+i)$
$= 6+3i-6i+3$

$\boxed{z_1 \times z_2 = 9-3i}$

$z_1^2 = (3-3i)$
$= 9 - 2\times 3 \times 3i + (3i)^2$
$= 9 - 18i - 9$

$\boxed{z_1^2 = -18i}$

$\frac{1}{z_2} = \frac{1}{2+i}$
$= \frac{2-i}{2^2-i^2}$
$= \frac{2-i}{5}$

$\boxed{\frac{1}{z_2} = \frac{2}{5} - \frac{1}{5}i}$

$\frac{z_1}{z_2} = \frac{3-3i}{2+i}$
$= \frac{(3-3i)(2-i)}{(2+i)(2-i)}$
$= \frac{6-3i-6i-3}{5}$
$= \frac{3-9i}{5}$

$\boxed{\frac{z_1}{z_2} = \frac{3}{5} - \frac{9}{5}i}$

Exercice 2 : Calculs

$i^{25} = i^{4\times 6+1} = i$

$i^{99} = i^{4\times 2+3} = -i$

$i^{2014} = i^{4\times 503+2} = -1$

$i^{2022} = i^{4\times 505+2} = -1$

$\forall n \in \mathbb{N}$; $i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = -1$

$i^{4n+3} = -i$

Exercice 4 : module et Argument

$* \; z_1 = \sqrt{6} - i\sqrt{2}$

- $|z_1| = \sqrt{\sqrt{6}^2 + (-\sqrt{2})^2} = 2\sqrt{2}$

soit θ l'angle

- $\arg(z_1) \begin{cases} \cos\theta = \frac{\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{-\sqrt{2}}{2\sqrt{2}} = \frac{-1}{2} \end{cases} \Rightarrow \theta = -\frac{\pi}{6}$

$* \; z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

- $|z_2| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

soit θ l'angle

- $\arg(z_2) \begin{cases} \cos\theta = \frac{1/2}{1} = \frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta = \frac{\pi}{3}$

$* \; z_3 = i(\sqrt{6} - i\sqrt{2}) = \sqrt{2} + i\sqrt{6}$

- $|z_3| = 2\sqrt{2}$
- $\arg(z_3) = \frac{\pi}{3}$

$* \; z_4 = \frac{-1+i\sqrt{3}}{1+i}$

$z_4 = \frac{\sqrt{3}-1}{2} + \left(\frac{1+\sqrt{3}}{2}\right)i$

- $|z_4| = \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{1+\sqrt{3}}{2}\right)^2} = \sqrt{2}$
- $\arg(z_4) = \frac{5\pi}{12}$

* $z_5 = (-1-i)^4$

posons : $z_5' = -1-i$

- $|z_5'| = \sqrt{2}$
- $\arg(z_5') = -\frac{3\pi}{4}$

Alors : $|z_5| = |z_5'|^4$

$|z_5| = \sqrt{2}^4 = 4$

$\arg(z_5) = [\arg(z_5')]^4 = 4\arg(z_5')$

$\arg(z_5) = -\frac{3\pi}{4} \times 4 = -3\pi$.

* $z_6 = \frac{(1-i)^4}{(\sqrt{3}-i)^3} = \frac{z'}{z''}$

- $|z'| = |1-i|^4 = 4$
- $|z''| = |\sqrt{3}-i|^3 = 8$

Donc : $|z_6| = \frac{1}{2}$

- $\arg(z_6) = \arg\left(\frac{z'}{z''}\right)$

$\arg(z_6) = \arg(z') - \arg(z'')$

$\left.\begin{array}{l} \bullet \arg(z') = -\pi \\ \bullet \arg(z'') = -\frac{\pi}{2} \end{array}\right\} \arg(z_6) = -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$

Exercice 3 : Forme Algébrique

$z_1 = \frac{2-i}{3+4i}$

$= \frac{(2-i)(3-4i)}{3^2+4^2}$

$= \frac{6-8i-3i-4}{25}$

$= \frac{2-11i}{25}$

$\boxed{z_1 = \frac{2}{25} - \frac{11}{25}i}$

$z_2 = \frac{1}{2-i\sqrt{3}}$

$= \frac{2+i\sqrt{3}}{2^2+\sqrt{3}^2}$

$= \frac{2+i\sqrt{3}}{7}$

$\boxed{z_2 = \frac{2}{7} + \frac{\sqrt{3}}{7}i}$

$z_3 = (1-2i)(4+5i)$

$= 4+5i-8i+10$

$\boxed{z_3 = 14-3i}$

Exercice 5 : $z = (1+i)(1-i\sqrt{3})$

1/ Forme Algebrique

$z = (1+i)(1-i\sqrt{3})$

$= 1-i\sqrt{3}+i+\sqrt{3}$

$\boxed{z = (1+\sqrt{3}) + (1-\sqrt{3})i}$

2/ a/ Forme Trigo et Forme expo.

$|z| = 2\sqrt{2}$

$\arg(z) = -\frac{\pi}{12} = \theta$

* Forme Trigo $(z = |z|(\cos\theta + i\sin\theta))$

$\boxed{z = 2\sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)}$

* Forme expo $(z = |z|e^{i\theta})$

$\boxed{z = 2\sqrt{2}\,e^{-i\frac{\pi}{12}}}$

b/ deduction de Forme Trigo et expo de $\frac{1-i\sqrt{3}}{1+i}$

• $\left|\frac{1-i\sqrt{3}}{1+i}\right| = \frac{2}{\sqrt{2}} = \sqrt{2}$

• $\arg\left(\frac{1-i\sqrt{3}}{1+i}\right) = \arg(1-i\sqrt{3}) - \arg(1+i) = -\frac{\pi}{3} - \frac{\pi}{4} = -\frac{7\pi}{12}$

• Forme Trigo : $\boxed{\frac{1-i\sqrt{3}}{1+i} = \sqrt{2}\left(\cos\left(-\frac{7\pi}{12}\right) + i\sin\left(-\frac{7\pi}{12}\right)\right)}$

- Forme expo : $\boxed{\dfrac{1-i\sqrt{3}}{1+i} = \sqrt{2}\, e^{-i\frac{7\pi}{12}}}$

Exercice 7 : $a = \sqrt{2}(1+i)$; $b = \sqrt{3}+i$ et $c = ba^3$

1/ Modules et arguments

$|a| = |\sqrt{2} + i\sqrt{2}| = 2$

$\arg(a) = \dfrac{\pi}{4}$

$|b| = 2$

$\arg(b) = \dfrac{\pi}{6}$

2/ deduction du module et de l'Argument de c.

$C = ba^3$

- $|c| = |b| \cdot |a|^3 = 2 \times 2^3 = 16$
- $\arg(c) = \arg(ba^3)$

$= \arg(b) + [\arg(a)]^3$

$= \arg(b) + 3\arg(a)$

$= \dfrac{\pi}{6} + \dfrac{3\pi}{4}$

$\boxed{\arg(c) = \dfrac{11\pi}{12}}$

3. Deduisons les valeurs exactes de $\cos\left(\frac{11\pi}{12}\right)$ et $\sin\left(\frac{11\pi}{12}\right)$

on sait que : $\arg(c) = \dfrac{11\pi}{12}$

$c = ba^3 = 4\sqrt{2}(-\sqrt{3}-1) + 4\sqrt{2}(-1+\sqrt{3})i$

forme Trigo de C : $C = 16\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) = 16\cos\frac{11\pi}{12} + \underset{16}{i}\sin\frac{11\pi}{12}$

Alors :

$$\begin{cases} 16\cos\dfrac{11\pi}{12} = 4\sqrt{2}(-\sqrt{3}-1) \\ 16\sin\dfrac{11\pi}{2} = 4\sqrt{2}(-1+\sqrt{3}) \end{cases} \Rightarrow \boxed{\begin{cases} \cos\dfrac{11\pi}{12} = \dfrac{-\sqrt{6}-\sqrt{2}}{4} \\ \sin\dfrac{11\pi}{12} = \dfrac{\sqrt{6}-\sqrt{2}}{4} \end{cases}}$$