2015年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2015

学科試験　　問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数　　学

MATHEMATICS

注意　☆試験時間は 60 分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

(2015)

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1 Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 + 4x^2 + 4x + 1 = 0$.

$x =$

2) Solve the equation $\cos 2x + \cos x = 0 \quad (0 \leq x \leq 2\pi)$.

$x =$

3) Solve the inequality $3^{x+1} + \dfrac{1}{3^x} < 4$.

4) Solve the inequality $\log_2 \sqrt{2x-1} < \log_4 x$.

5) Calculate $\displaystyle\sum_{n=0}^{120} \frac{1}{\sqrt{n+1}+\sqrt{n}}$.

6) How many integers n are there such that $1 \leq n \leq 200$ and n is not divisible by 2 nor 5 ?

7) Let $\vec{a} = (1,\ 3,\ -2)$, $\vec{b} = (2,\ 1,\ 1)$, $\vec{c} = (-3,\ 1-s,\ t)$. Find the values of s and t such that $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$.

$s =$ $t =$

8) Find the tangent line to the curve $y = e^{\frac{x}{2}}$ which goes through the point $(0,\ 0)$.

$y =$

9) The sequence $\{a_n\}$ satisfies the following conditions. Calculate $\displaystyle\sum_{n=1}^{\infty} a_n$.

$$a_1 = 5, \qquad a_{n+1} = \frac{3}{4}a_n \quad (n = 1,\ 2,\ 3,\ \cdots)$$

10) Calculate $\displaystyle\lim_{x\to 0} \frac{1-\sqrt{1-\sin x}}{x}$.

11) Let $f(x) = x - \log_e(2x+1)$. Solve the inequality $f'(x) < 0$.

12) Calculate $\displaystyle\int_0^{\pi} x \sin x \ dx$.

2 Let $A = a\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $(a > 0)$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ satisfy $A^4 + I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.
Answer the following questions and write your answers in the boxes provided.

1) Find a.

$a =$

2) Find the minimum positive integer n such that $A^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$n =$

3) Find A^{2014}.

$A^{2014} = \begin{pmatrix} & \\ & \end{pmatrix}$

3 Let n be a positive integer. Answer the following questions and write your answers in the boxes provided.

1) Let $f(x) = x(1-x)^n$. Solve the equation $f'(x) = 0$ in the open interval $0 < x < 1$.

$x =$

2) Let a_n be the maximum value of $f(x) = x(1-x)^n$ in the closed interval $0 \leq x \leq 1$. Calculate $\lim_{n\to\infty}(n+1)a_n$.

3) Calculate $\displaystyle\int_0^1 x(1-x)^n\,dx$.