

(2016)

MATHEMATICS

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Answer the following questions and write your answers in the boxes provided.

1) Let $x = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$ and $y = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$. Calculate $x^2 - y^2$.

2) Solve the equation $x^3 - x^2 - 10x - 8 = 0$.

$x =$

3) Solve the equation $2 \sin^2 x - \cos x = 1$ ($0 \leq x < 2\pi$).

$x =$

4) Solve the equation $2^{2x+2} + 3 \cdot 2^x - 1 = 0$.

$x =$

5) Solve the inequality $(\log_3 x)^2 < \log_9 x^4$.

6) Solve the inequality $\sin 2x > \sqrt{2} \sin x$ ($0 \leq x < 2\pi$).

7) Let $\vec{a} = (1, 2, 3)$, $\vec{b} = (3, 2, 1)$, $\vec{c} = (5, 4, 3)$. Find the value of t such that $\vec{a} + t\vec{b}$ is parallel to \vec{c} .

$t =$

8) Let $O(0, 0)$ and $A(3, 1)$. Let A' be the symmetric point of A with respect to the line $y = 2x$. Calculate the area of the triangle OAA' .

9) The sequence $\{a_n\}$ satisfies the following conditions. Calculate $\sum_{n=1}^5 (a_n - 5)$.

$$a_1 = 3, \quad a_{n+1} = 2a_n \quad (n = 1, 2, 3, \dots)$$

10) Calculate $\lim_{x \rightarrow 0} (\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + x})$.

11) Let $f(x) = \frac{\cos x}{\sqrt{e^x}}$. Calculate $f'(0)$.

$$f'(0) =$$

12) Calculate $\int_1^2 (3x^2 - 4x) \log_e x \, dx$.

2 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Answer the following questions and write your answers in the boxes provided.

1) Let $A = \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} x & 3 \\ 3 & 6 \end{pmatrix}$. Find the value of x which satisfies $AB = BA$.

$x =$

2) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & x \\ 4 & y \end{pmatrix}$. Find the values of x and y which satisfy $BA = O$.

$x = \qquad y =$

3) Let A satisfying $A^2 = A - I$. Find A^{15} .

$A^{15} = \begin{pmatrix} & \\ & \end{pmatrix}$

3 Answer the following questions and write your answers in the boxes provided.

1) Calculate $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$.

2) Calculate $\int_0^{\frac{\pi}{4}} \cos^3 x \, dx$.

3) Calculate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin x + 2 \cos x)^3 \, dx$.