2019年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2019

学科試験　問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数　学

MATHEMATICS

注意　☆試験時間は60分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

(2019)

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	

1 Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 - 2x^2 - x + 2 = 0$.

$x =$

2) Solve the equation $\cos x - 2\cos^2 x = 0$ $(0 \leq x \leq \pi)$.

$x =$

3) Express $|\sqrt{8} - 3| + |2 - \sqrt{2}|$ without the absolute value symbols.

4) Solve the equation $\log_2(x-1) = \log_4(x-1)$.

$x =$

5) Find the maximum value m of the function $f(x) = \cos x + \cos(x + \frac{\pi}{3})$ $(0 \leq x < 2\pi)$. Also, at what values of x does $f(x)$ have the maximum?

$m =$ $x =$

6) By using $\lim_{t \to 0}(1+t)^{\frac{1}{t}} = e$, calculate $\lim_{h \to 0}(1+2h)^{\frac{1}{h}}$.

7) Find the intersection point of the line $\frac{x-1}{6} = \frac{y-1}{2} = \frac{z-2}{3}$ and the plane $x + 2y - 4z + 1 = 0$.

$x =$ $y =$ $z =$

8) Find the tangent line to the curve $y = \log_e x$ which goes through the point $(0, 0)$.

$y =$

9) Calculate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

10) Calculate $\displaystyle\lim_{x \to -\infty} \frac{2x+1}{\sqrt{x^2+1}}$.

11) Let $f(x) = \log_e \dfrac{\sqrt{x-1}}{x+1}$. Calculate $f'(x)$.

$f'(x) =$

12) Calculate $\displaystyle\int_{-\pi}^{\pi} \sin 3x \sin x \; dx$.

2 For $A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$, answer the following questions and write your answers in the boxes provided.

1) Calculate A^n.

$A^n = \begin{pmatrix} & \end{pmatrix}$

2) Calculate $S = \sum_{k=1}^{n} A^k$.

$S = \begin{pmatrix} & \end{pmatrix}$

3) Calculate the inverse S^{-1} of the matrix $S = \sum_{k=1}^{n} A^k$.

$S^{-1} = \begin{pmatrix} & \end{pmatrix}$

3 For any natural number $k > 0$, let $I_{2k+1} = \frac{2k}{2k+1} \cdot \frac{2k-2}{2k-1} \cdots \frac{4}{5} \cdot \frac{2}{3}$ and $I_{2k} = \frac{2k-1}{2k} \cdot \frac{2k-3}{2k-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$. Answer the following questions and write your answers in the boxes provided.

1) Caluculate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$.

2) Find a_k which satisfies $I_{2k+1} \cdot I_{2k} = \frac{\pi}{2} \cdot a_k$.

$a_k =$

3) Find b_k which satisfies $I_{2k-1} \cdot I_{2k} = \frac{\pi}{2} \cdot b_k$.

$b_k =$

4) Calculate $\displaystyle\lim_{k\to\infty} \frac{1}{k} \left\{ \frac{(2k)(2k-2)\cdots 4 \cdot 2}{(2k-1)(2k-3)\cdots 3 \cdot 1} \right\}^2$ by assuming $I_{2k+1} < I_{2k} < I_{2k-1}$.