ESATIC 2024 - 2025

Signal Mathematics LICENCE 2 RTEL

Exercise 1
Let the periodic function $f : \mathbb{R} \to \mathbb{R}$ of period 2π, be given by .

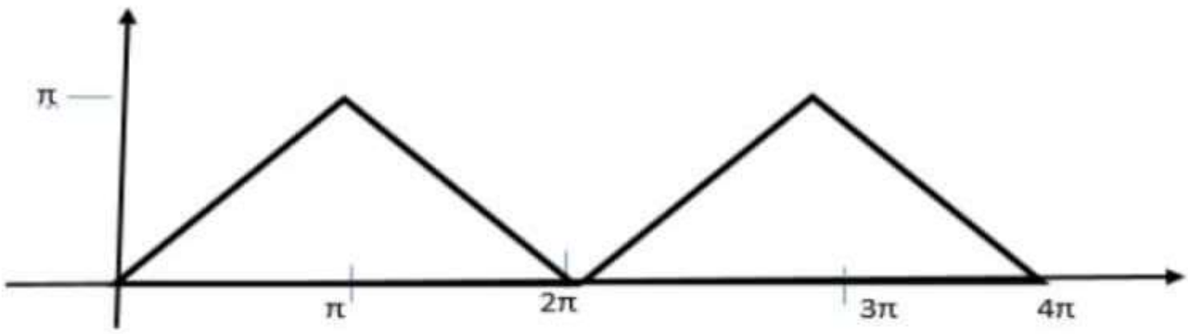
$$f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ \pi, & \pi \leq x \leq 2\pi \end{cases}$$

1.Plot the graph of f.
2.Calculate the Fourier coefficients of f. Then write the Fourier series for f.
Exercise 2. Compute the Fourier series of the function of period 4 given by

$$f(x) = \begin{cases} 0 & \text{if } -2 \leq x < 0 \\ x & \text{if } 0 \leq x < 2 \end{cases}$$

1. Plot the graph of f.
2.Calculate the Fourier coefficients of f. Then write the Fourier series for f.

Exercise 3. Let the periodic function $f : \mathbb{R} \to \mathbb{R}$ of period 2π, be given by the following graph.



1- Determine the function f
2- Give the Fourier series for f.
Exercise 4 Let f be a periodic function of period 2π such that

$$f(x) = \begin{cases} 0 & \text{if} -\pi < x \leq 0 \\ \sin x & \text{if } 0 < x \leq \pi \end{cases}$$

1- Give the Fourier series for f.

2- Assume that there is a Fourier series converging to f. Find the sum of the series for $x = \pi$. Then deduce the sum $\sum_{n\geq 1} \frac{1}{4n^2-1}$.

Exercise 5 Find the complex form of the Fourier series for the function

$$f(x) = e^{-x} \qquad -1 < x < 1 \ .$$

Exercise 6 Assume that there is a Fourier series converging to f. Show that the 2π-periodic function $f(t)$ defined by:

$$f(x) = \begin{cases} -1 & \text{if } x \in]0, \pi[\\ 1 & \text{if } x \in]\pi, 2\pi[\end{cases}$$

can be written in the form:

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{2i}{\pi(2n+1)} e^{i(2n+1)x}.$$

Exercise 7 (cours) Let f be 2π -periodic function defined by

$$f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 \leq x < \pi \end{cases}$$

1. Plot the graph of f on $[-3\pi, 3\pi]$.
2. Assume that there is a Fourier series converging to f , Find the Fourier series of f, and deduce the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}.$$

Exercise 8 (cours) Let f be a 2π-periodic function defined by

$$f(x) = x^2, \qquad \text{if } x \in]-\pi, \pi[$$

1. Plot the graph of f.
2. Assume that there is a Fourier series converging to f . Prove that

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cos nx.$$

3. Deduce that sum the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

3.Prove by applying Parseval's equation that:

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$